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HOVERCRAFT UNIT REPORT 9

DECEMBER 1969

NATIONAL PHYSICAL LABORATORY

HOVERCRAFT UNIT

HEAVE SUSPENSION CHARACTERISTICS AND POWER REQUIREMENTS OF A PLENUM AIR CUSHION

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SUMMARY

This report presents a preliminary analysis of the heave suspension characteristics of a plenum chamber air cushion system and derives a tentative cushion power criterion as a function of craft response.

CONTENTS

1. Introduction

The air cushion normally used in current marine hovercraft is based on the plenum chamber principle. The cushion is contained by peripheral skirts or hard structure sidewalls which are designed to ensure a minimum loss of air.

The dynamic behaviour of this form of air cushion is not understood especially with regard to the dependence of the heave suspension characteristics on the geometric and design parameters of the fan/duct/cushion system.

In addition, the volume flow and cushion power requirement for marine hovercraft is usually decided from considerations of minimum leakage in static conditions. It can be shown, however, that the critical cushion power requirement is dictated by operation over waves when the rates of change of cushion volume are large. It follows, therefore, that cushion power requirement can be related to the heave response of the craft and hence to the suspension characteristics.

This report presents a preliminary analysis of the dynamic heave characteristics of a plenum chamber air cushion based on simplified and convenient assumptions and derives a tentative cushion power criterion as a function of craft response.

2. Theory

2.1 General

A significant contribution to the heave response of a hovercraft derives from the changes in cushion pressure which are caused as the craft traverses a surface of varying shape. Changes of cushion pressure are a function of the relative displacement and rate of displacement between the craft and the surface. For example, if the cushion volume is decreasing as the craft and surface move towards each other, the air feed to the cushion will change and the escape of air flow from the cushion and hence cushion pressure, will increase, if the condition of continuity of volume flow through the cushion system is to be satisfied.

If this is not the case, changes in air density must occur which will cause variations in absolute cushion pressure and hence very large changes in cushion pressure relative to atmosphere. For example, ± 2% change of cushion volume and hence density in a "sealed" cushion * would result in a change of about ± 45 pounds per square foot in cushion pressure relative to atmosphere, which would correspond to ± 1 to 2 'g' acceleration in heave on most current hovercraft.

For this reason, the air cushion must be a "ventilated" system so that rates of change of cushion volume over waves can be accommodated with relatively small changes of cushion pressure.

2.2 Assumptions

The analysis is based on the following assumptions and conditions:-

(a) The flow of air throughout the cushion system satisfies the condition of continuity of volume flow at each instant in time.

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* As the design leakage rate of air from the cushion is reduced and the heave forcing frequency is increased, the flow process of air through the cushion will tend to be increasingly governed by continuity of mass flow considerations which infer density changes.

b) Perturbations of pure heave motion are small such that the variation of pressures and volume flow with displacement and rate of displacement can be assumed to be linear. Hence the equation of motion in heave for small disturbances can be expressed in terms of the static values of the main parameters.

c) Pressure is uniform throughout the cushion at each instant in time.

d) Cushion planform is rectangular.

e) Craft is forced by regular waves in head or following seas.

f) Motion of flexible skirts relative to the craft is negligible.

g) The effects of forward speed on cushion characteristics can be ignored for a plenum system which is almost fully contained by peripheral skirts or hard structure.

2.3 Cushion System Configuration

Two variations of the plenum chamber are shown diagrammatically in Fig. 1. Current design practice for amphibious craft corresponds to configuration (a) but some craft have rear skirts which are shortened to allow a majority of cushion airflow to escape rearwards in static conditions. Configuration 1b) is an extreme case of this type of design.

The following physical processes are assumed as a basis for this preliminary analysis (Fig. 1):-

a) Air is delivered from a fan at a total head (H) to a secondary plenum or expansion chamber. For simplicity, recovery of dynamic head due to forward speed at the fan intake is neglected.

b) Air is fed to the cushion at a velocity (V_F) through a feed area (S_F) to sustain a static cushion pressure (p_c). Dynamic pressure (q_c) within the cushion is neglected, (i.e. $q_c << p_c$).

c) Air is finally discharged from the cushion to atmosphere at a velocity (V_E) through an exit area (S_E) by total conversion of static cushion pressure to dynamic head.

2.4 Derivation of Equations

Equations will be derived for continuity of volume flow through the cushion system, assuming two typical fan operating conditions i.e. at constant fan RPM and at constant fan power and efficiency.

These equations will be used to derive the suspension characteristics for small motions in heave using the definitions of stiffness and damping appropriate to a second order linear system. A volume flow and cushion power requirement will then be derived in terms of craft heave response in given operating conditions.

2.4.1 Cushion Equations

The instantaneous volume flow demanded by the air cushion (Q_D) is the sum of the flow leaving the cushion and the rate of change of cushion volume:-

Then $$Q_D = C_E S_E V_E + Q_c \qquad (1)$$

The volume flow supplied to the cushion is:-

$$Q_F = C_F S_F V_F \qquad (2)$$

Hence for continuity of volume flow (Q) through the cushion system

$$Q = Q_F = Q_D \qquad (3)$$

or $$C_F S_F V_F = C_E S_E V_E + Q_c \qquad (4)$$

where $$S_E = p(h_E + z) \qquad \text{(See Fig. 1)} \qquad (5)$$

and $$Q_c = S_c \dot{z} \qquad (6)$$

It is now assumed that the fan total head is converted to cushion pressure with a loss of dynamic head due to the feed velocity, and that cushion pressure is then fully converted to dynamic head at the exit condition:-

Then $$H = p_c + \tfrac{1}{2}\rho V_F^2 \qquad (7)$$

and $$p_c = \tfrac{1}{2}\rho V_E^2 \qquad (8)$$

The independent variables are the relative heave displacement (z) and rate of displacement ($\dot{z}$) between craft and surface, taken as positive upwards from the equilibrium heave configuration (Fig. 1):-

Then $$z = (z_o - z_i) \qquad (9)$$

$$\dot{z} = (\dot{z}_o - \dot{z}_i) \qquad \text{(See Fig. 1)} \qquad (10)$$

2.4.2 Fan Characteristic

A typical fan characteristic is shown in Fig.2
For small changes about the fan design point:-

$$H = \alpha + \beta Q \quad \text{at constant fan RPM} \qquad (11)$$

$$H = \frac{m}{Q} \quad \text{at constant fan power and efficiency}$$

where $$m = 550\eta_F \text{Fan H.P (assumed constant)} \qquad (12)$$

2.4.3 Equation of Motion in Heave

The equation of motion in heave relates the accelerations on the craft to the changes in cushion force (ΔZ) about the equilibrium value (W), expressed in terms of the independent variables (z) and ($\dot{z}$).

The basic equation for heave motion is:-

$$\frac{W}{g}\ddot{z}_o = \Delta Z \qquad (13)$$

The cushion force is the product of the cushion area (assumed constant) and the cushion pressure:-

$$\text{Hence } \Delta Z = S_c \, \Delta p_c \qquad (14)$$

$$\text{where } \Delta p_c = \frac{dp_c}{dz} z + \frac{dp_c}{d\dot{z}} \dot{z} \qquad (15)$$

The equation of heave motion (13) can now be written:-

$$\frac{W}{g}\ddot{z}_o = - kz - c\dot{z} \qquad (16)$$

where the negative signs indicate a stable system for positive values of stiffness (k) and damping (c), where:-

$$\text{Hence } k = S_c \frac{dp_c}{dz} \qquad (17)$$

$$\text{and } c = S_c \frac{dp_c}{d\dot{z}} \qquad (18)$$

The suspension system described by equation (16) is shown diagrammatically in Fig. 3 and consists of a spring and damper in parallel which are exercised by relative motion (z) and ($\dot{z}$) between craft and surface.

Equation (16) can be re-written in the usual form of second order system using equations (9) and (10):-

$$\frac{W}{g}\ddot{z}_o + c\dot{z}_o + kz_o = c\dot{z}_i + kz_i \qquad (19)$$

or in the more general form:-

$$\ddot{z}_o + 2\zeta\omega_n\dot{z}_o + \omega_n^2 z_o = 2\zeta\omega_n\dot{z}_i + \omega_n^2 z_i \qquad (20)$$

where (ω_n) is the undamped natural frequency defined by:-

$$\omega_n^2 = \frac{kg}{W} \qquad (21)$$

and (ζ) is the damping ratio, defined by:-

$$\zeta = \frac{c}{2\left(\frac{Wk}{g}\right)^{\frac{1}{2}}} \qquad (22)$$

Damping ratio has a value of unity for critical or "dead beat" response to a step input

3. Suspension Characteristics

The suspension characteristics are dependent primarily on the parameters $\frac{dp_c}{dz}$ and $\frac{dp_c}{d\dot{z}}$ (See equations 17, 18, 21 and 22).

Appendix A derives the following relationships for small motions about an equilibrium configuration, such that:-

$$p_c S_c \rightarrow W$$

TABLE 1

PARAMETER	CONSTANT FAN RPM	CONSTANT FAN POWER AND EFFICIENCY	
$\frac{dp_c}{dz}$ (Appendix A)	$-2\frac{p_c}{h_E} f_1$	$-2\frac{p_c}{h_E} f_2$	(23)
Heave Stiffness (k) (See Equation 17)	$2\frac{W}{h_E} f_1$	$2\frac{W}{h_E} f_2$	(24)
Natural frequency (ω_n) (See equation 21)	$\left(2\frac{g}{h_E} f_1\right)^{\frac{1}{2}}$	$\left(2\frac{g}{h_E} f_2\right)^{\frac{1}{2}}$	(25)
$\frac{dp_c}{d\dot{z}}$ (Appendix A)	$-2\frac{W}{Q} f_1$	$-2\frac{W}{Q} f_2$	(26)
Heave damping (c) (See Equation 18)	$2\frac{WS_c}{Q} f_1$	$2\frac{WS_c}{Q} f_2$	(27)
Damping Coefficient (ζ) (See Equation 22)	$\frac{g\,S_c}{\omega_n Q} f_1$	$\frac{g\,S_c}{\omega_n Q} f_2$	(28)

where $f_1 = \frac{1 - \bar{p}_c - \frac{1}{2}F}{1 - \frac{1}{2}F}$ (Fig. 4)

$f_2 = 1 - \frac{2}{3}\bar{p}_c$ (Fig. 4) (29)

and $\bar{p}_c = \frac{p_c}{H}$; $F = \beta\frac{Q}{H}$ (30)

($\bar{p}_c$) is a pressure ratio between the cushion and fan.
Typical values are 0.8 to 0.5 or even lower depending upon duct design.

The value of $\bar{p}_c$ in static conditions is given by equations (7) (8) and (4) when $Q_c = 0$:-

Then $$\bar{p}_c = \frac{1}{1 + \left(\frac{C_E\,S_E}{C_F\,S_F}\right)^2} \qquad (31)$$

Fig. 5 shows the variation of $\bar{p}_c$ with area ratio in static conditions assuming that the fan total head (H) is applied directly at the cushion feed area without intermediate duct loss.

(F) is a fan characteristic coefficient where the fan slope ($\beta = \frac{dH}{dQ}$) is usually negative for conventional fan designs.

From Fig. 2, (F) can be written as:-

$$F = - \frac{Q}{Q_o - Q} \qquad (32)$$

Typical values of (F) are -2 to -4.

Hence when $\bar{p}_c$ = 0.5 and F = -3, for example:-

f_1 = 0.8

f_2 = 0.667

At constant fan RPM:-

a) $f_1 \rightarrow 0$ as $\bar{p}_c \rightarrow 1.0$ and $F \rightarrow 0$

and b) $f_1 \rightarrow 1$ as $\bar{p}_c \rightarrow 0$ or $F \rightarrow \infty$

Condition a) corresponds to zero pressure loss between fan and cushion and constant total head over the working range of volume flow delivery from the fan. Hence pressure changes throughout the cushion system tend towards zero in dynamic conditions,

Condition b) represents a high loss and hence power system as $\bar{p}_c \rightarrow 0$ or a constant volume flow system as $F \rightarrow \infty$

The fan slope (β) and hence (F) can be positive over a local part of the fan characteristic especially in the case of axial fans. The value of (f_1) may become zero or even negative depending upon the relative magnitude of ($\bar{p}_c$) and (+F), indicating a neutral or unstable system respectively.

At constant fan power and efficiency:-

$f_2 \rightarrow \frac{1}{3}$ as $\bar{p}_c \rightarrow 1.0$
and some cushion pressure change will always occur in dynamic conditions.

In general, an increase in negative slope of the fan characteristic (F) and an increase in pressure loss ($\bar{p}_c$) both increase the undamped natural frequency and the damping coefficient of the cushion system.

It can be shown from Table 1, that in general:-

$$\zeta = \frac{S_c}{2p\, C_E\, V_E}\, \omega_n \qquad (33)$$

using equations (1) (3) and (5) as (z) and ($\dot{z}$)$\rightarrow$ 0:-

$$\zeta = \frac{S_c\, h_E}{2Q}\, \omega_n \qquad (34)$$

$$\text{or} \quad \zeta = \frac{T}{2}\, \omega_n \qquad (35)$$

where (T) is a constant representing the time for a volume of air ($S_c\ h_E$) to exit from the cushion in static conditions.

Typical values of (T) are 0.05 to 0.125 seconds from an air cushion with a full peripheral air gap.

4. Heave Response and Acceleration

The maximum instantaneous heave acceleration occurring during each cycle of motion for a linear system subjected to a sinusoidal input, is given by:-

$$(ng)\ max = \pm \bar{z}_o \omega_e^2 \qquad (36)$$

$$\text{where } \omega_e = \frac{2\pi V_R}{\lambda} \qquad (37)$$

and (V_R) is the speed of wave encounter relative to the craft.

$$\text{Hence } V_R = V \pm V_W \text{ in head or following seas} \qquad (38)$$

Where (V_w) is the wave propagation speed

Equation (36) can be written:-

$$(ng)\ max = \pm (M \bar{z}_i)\ \omega_e^2 \qquad (39)$$

where (M) is the amplitude response factor $= \dfrac{\bar{z}_o}{\bar{z}_i}$

and is defined in terms of the constants of equation (20) and frequency of wave encounter (ω_e) by standard analysis, as:-

$$M = \left[\frac{1 + 4\zeta^2 \left(\frac{\omega_e}{\omega_n}\right)^2}{\left\{1 - \left\{\frac{\omega_e}{\omega_n}\right\}^2\right\}^2 + 4\zeta^2 \left(\frac{\omega_e}{\omega_n}\right)^2}\right]^{\frac{1}{2}} \qquad (40)$$

The variation of (M) with frequency ratio $\frac{\omega_e}{\omega_n}$) is shown in Fig. 6 for typical values of damping coefficient (ζ). In general, the craft heave response and hence, heave acceleration resulting from surface forcing is reduced with:-

a) High values of damping when $\omega_e < 1.414 \times \omega_n$

b) Low values of damping when $\omega_e > 1.414 \times \omega_n$

The choice of damping coefficient is, therefore, a compromise between the requirements for operating in these two different parts of the frequency band and should also take account of the preferred craft response in heave following a sudden input from the surface or due to direct forcing of the craft by water impact.

4.1 Wave Forcing

It is convenient for purposes of analysis to treat sinusoidal surfaces in terms of an equivalent "flat plate" surface which forces the cushion in heave by sweeping out the same cushion volume change per cycle as the waves themselves.

The amplitude of this equivalent "flat plate" can be written generally as:-

$$\bar{z}_i = K \frac{h_w}{2} \tag{41}$$

where (h_w) is the wave height (crest to trough) and (K) is the wave form factor in heave.

Appendix B derives the following expression for a rectangular cushion planform only.

$$K = \frac{\lambda}{\pi \ell} \sin \frac{\pi \ell}{\lambda} \tag{42}$$

Then equation (39) becomes:-

$$(ng)\ max = \pm M(K \frac{h_w}{2})\ \omega_e^2 \tag{43}$$

4.2 Short Waves

When $(\frac{\lambda}{\ell}) < 1.0$, the function (K) exhibits a series of decreasing maxima as $(\frac{\lambda}{\ell}) \rightarrow 0$ (Fig. 7.)

These maxima are bounded by a linear envelope:-

$$K_{max} = \frac{1}{\pi} \cdot \frac{\lambda}{\ell} \qquad \text{where } (\frac{\lambda}{\ell} = \frac{2}{a}) \tag{44}$$

and (a) is a positive odd integer greater than unity. The physical derivation of this boundary is explained in Appendix B in terms of a typical wave form.

Hence the maximum heave acceleration in short waves is given by equations (43) and (44):-

$$(ng)_{max} = \pm \frac{h_w}{2\pi} \frac{\lambda}{\ell} M \omega_e^2 \tag{45}$$

A further simplification can be made by assuming that operation over short waves corresponds to the condition:-

$$\frac{\omega_e}{\omega_n} > 3.0 \tag{46}$$

It can then be shown from equation (40) that:-

$$M \doteqdot 2 \frac{\zeta \omega_n}{\omega_e} \tag{47}$$

$$(\pm 10\% \text{ reducing as } \frac{\omega_e}{\omega_n} >> 3 \quad \text{and } 1.0 > \zeta > 0.5) \tag{48}$$

Using equation (37) and (47), equation (45) can be re-written as:-

$$n_{max} \doteqdot \pm 2 \frac{V_R}{g} \frac{h_w}{\lambda} \frac{\lambda}{\ell} (\zeta \omega_n) \tag{49}$$

NOTE: Conditions of (44), (46), (48) apply to equation (49)

Equation (49) shows that heave acceleration over short waves is:-

a) directly proportional to wave encounter speed (V_R) or frequency (ω_e) and not to the square of these parameters as might be expected.

b) directly proportional to the suspension parameter ($\zeta\omega_n$)

c) directly proportional to the wave slope ($\frac{h_w}{\lambda}$)

The parameter ($\zeta\omega_n$) varies inversely as the square root of the linear scale factor of craft and must be used with caution when comparing suspension characteristics of different craft. However, it is convenient parameter in the present analysis.

Combining equations (28) and (49):-

$$n_{max} = 2 \frac{S_c h_w}{Q} \frac{V_R}{\ell} (f_1 \text{ or } f_2) \qquad (50)$$

Now the maximum rate of change of cushion volume due to short waves such that $\frac{\lambda}{\ell} = \frac{2}{a}$, is given by:-

$$Q_w = b\, h_w\, V_R \qquad \text{(Appendix B)} \qquad (51)$$

also $S_c = b\ell$ (52)

Then equation (50) can be written in the form of a volume flow requirement:-

$$\frac{Q}{Q_w} = \frac{2}{n_{max}} (f_1 \text{ or } f_2) \qquad (53)$$

For a typical case, when the cushion characteristics f_1 or f_2 = 0.7, the volume flow requirement (Q) <u>in short waves</u> for a maximum instantaneous heave acceleration = 0.2 g is given by:-

$$Q = 7 \times Q_w \qquad (54)$$

4.3 Long Waves

A craft will contour very long waves at low frequency with virtually no motion relative to the surface. Close contouring of the surface i.e. (M) $\rightarrow$ 1.0 can be maintained at higher encounter frequencies provided that the following conditions are simultaneously satisfied:-

a) $\frac{\omega_e}{\omega_n} < 1.414$

b) $(\frac{\lambda}{\ell})$ is large such that (K) $\rightarrow$ 1 (Fig. 8)

c) ζ >1.0 such that (M) $\rightarrow$ 1. (Fig. 6)

Then equation (43) tends to the simple form:-

$$(ng)_{max} = \frac{h_w}{2} \quad \omega_e^2 \tag{55}$$

or

$$\frac{n_{max}}{h_w} = 0.614 \quad f_e^2 \tag{56}$$

(f_e) is the encounter frequency in cycles/second, defined by:-

$$f_e = \frac{(V \pm V_W)}{\lambda} \tag{57}$$

where $V_w = 2.265 \sqrt{\lambda}$ in deep water. (Ref. 1). (58)

(f_e) is plotted in Fig. 9 as a function of craft speed (V knots) for a range of wavelengths (λ) in head and following seas.

5. CUSHION POWER REQUIREMENT

The power requirement of an air cushion, excluding propulsion power, is:-

$$\eta_m \text{ BHP} = \frac{HQ}{550\eta_F} \tag{59}$$

5.1 Static or Calm water Conditions

A power loading requirement for a plenum air cushion can be derived from equations (1) and (59) when $Q_c = 0$:-

$$\eta_m \frac{\text{BHP}}{\text{TON}} = 4.07 \frac{C_E}{\bar{p}_c \eta_F} \quad \frac{S_E}{S_c} \quad V_E \tag{60}$$

For a rectangular planform with a full peripheral air gap of mean height (h_E):-

$$\frac{S_E}{S_c} = 2 \frac{h_E}{\ell} \quad (1 + \frac{\ell}{b}) \tag{61}$$

Using equations (8) and (61), the power loading requirement becomes

$$\eta_m \frac{\text{BHP}}{\text{TON}} = 8.14 \frac{C_E}{\bar{p}_c \eta_F} \quad \frac{h_E}{\ell} \left(1 + \frac{\ell}{b}\right) \left(\frac{2}{\rho} \quad p_c\right)^{\frac{1}{2}} \tag{62}$$

Alternatively, it is sometimes convenient to use a volume flow coefficient (C_Q) defined in static conditions, by:-

$$C_Q = \frac{Q}{S_c V_E} = C_E \frac{S_E}{S_c} \tag{63}$$

$$\text{Then } \eta_m \frac{\text{BHP}}{\text{TON}} = 4.07 \frac{C_Q}{\bar{p}_c \eta_F} \left(\frac{2}{\rho} \quad p_c\right)^{\frac{1}{2}} \tag{64}$$

Typical values of $\frac{h_E}{\ell}$ are 0.002 to 0.005

and $\frac{S_E}{S_c}$ are 0.005 to 0.030

For example, the following data is typical of amphibious craft:-

$C_Q = 0.015; \quad \eta_F = 0.8; \quad p_c = 50 \text{ lb/ft}^2 \quad \bar{p}_c = 0.5$

Then equation (64) gives:-

$$\eta_m \frac{BHP}{TON} = 31.3 .$$

5.2 Dynamic or Over Wave Conditions

The specific power loading for a plenum chamber air cushion can be expressed as:-

$$\eta_m \frac{BHP}{TON\ KNOT} = \frac{6.88}{\bar{p}_c \eta_F} \quad \frac{Q}{S_c\ V} \tag{65}$$

5.2.1 Short Waves

Using equations (28) and (49), equation (65) becomes:-

$$\eta_m \frac{BHP}{TON\ KNOT} = \frac{13.76}{\eta_{max}} \frac{h_w}{\lambda} \cdot \frac{\lambda}{\ell} \cdot \frac{(f_1 \text{ or } f_2)}{\bar{p}_c \ \eta_F} \cdot \frac{1 \pm V_w}{V} \tag{66}$$

Using typical wave and craft data:-

$\frac{h_w}{\lambda} = \frac{1}{20}; \quad \frac{\lambda}{\ell_c} = 0.4; \quad f_1 \text{ or } f_2 = 0.667$

$\bar{p}_c = 0.5; \quad \eta_F = 0.8; \quad \eta_{max} = 0.2; \quad \frac{V_w}{V} << 1.0$

Then $\eta_m \frac{BHP}{TON\ KNOT} = 2.3$

5.2.2 Long Waves

A high value of damping coefficient is required for wave contouring (See Section 4.3) and equation (28) indicates that this is achieved by low volume flow. This condition does not give rise to a critical power requirement.

5.2.3 Intermediate Wavelengths

Craft response over intermediate wavelengths will involve significant pitch and heave motion relative to the surface. Some preliminary analysis of combined longitudinal motion has been carried out at NPL and is reported in References 2, 3 and 4.

6. Discussion

6.1 Heave Stiffness and Damping

Non-dimensional heave stiffness and damping can be derived from Table 1 (page 5) as follows:-

$$k\left(\frac{h}{W}\right) \text{ or } c\left(\frac{Q}{S_c W}\right) = 2 f_1 \quad \text{at constant fan r.p.m.}$$

$$= 2 f_2 \quad \text{at constant fan power and efficiency}$$

where (f_1) and (f_2) are cushion system coefficients (See page 6):-

$$f_1 = \frac{1 - \bar{p}_c - \frac{1}{2}F}{1 - \frac{1}{2}F}$$

$$f_2 = 1 - \tfrac{2}{3}\bar{p}_c$$

In practice, typical values of non-dimensional suspension characteristics are 1.6 at constant fan r.p.m. in the normal range of 0 to 2.0 or 1.333 at constant fan power and efficiency in the range 0.667 to 2.0

when $\bar{p}_c = 0.5$ and $F = -3$.

The heave suspension is "softened" by increasing the cushion/fan pressure ratio ($\bar{p}_c$) towards unity and/or reducing the negative value of fan characteristic coefficient (F) towards zero.

The analysis has been simplified by making the usual assumption of small motions as z and $\dot{z} \rightarrow 0$, to justify a comparison of the suspension characteristics with a linear system.

Table 1 expressions can be written more generally using Appendix A as:-

$$k = \frac{2W}{(h_E+z)} \times (f_1 \text{ or } f_2)$$

$$c = \frac{2WS_c}{(Q-S_c\dot{z})} \times (f_1 \text{ or } f_2)$$

Hence the stiffness and damping are non-linear in terms of their independent variable (z or $\dot{z}$). The validity of these expressions could be tested by forcing a fixed model in heave. Measurement of cushion pressure and surface displacement for a range of frequencies could be compared with the output of a computer programme representing the simple equations and input conditions discussed in this report.

6.2 Limitations of Theory

Theory has been based on the assumption of continuity of volume flow through the cushion system. This assumption is convenient and simplifies the analysis but may not be representative of a cushion system which is subjected to large and rapid volume changes. In these conditions, continuity of mass flow may be more representative.

In this analysis, cushion shape has been restricted for convenience of deriving the forcing function of sinusoidal waves, to rectangular plan forms. The significance of a bow shape on wave form factor (K) and its variation with wave length ratio $\left(\frac{\lambda}{\ell}\right)$ has yet to be established in terms of power requirement.

The effect of skirt motion, relative to the craft, has been ignored in this simple analysis but in practice such motion may have a significant effect on suspension characteristics due to consequential changes of cushion volume and exit area with time.

6.3. Cushion Power Requirement

The power requirement given by equation (66) for critical short waves varies inversely as the maximum instantaneous heave acceleration chosen to represent an acceptable comfort level for regular inputs. There is some evidence (Fig. 10) and (Ref. 5) to suggest that R.M.S. acceleration is the representative measurement of human response to regular motion.

The designer must also consider whether he can justify the choice of a high level of acceleration for design purposes on the grounds that regular waves are rarely experienced for any appreciable time, in practice, and that passengers are probably more tolerant of irregular motion.

The power requirement in short waves refers to the steady state value which is necessary to restrict the maximum instantaneous heave acceleration in each cycle of regular motion, to a given value. Power requirement has been derived for conditions of small motions about an equilibrium condition.

7. CONCLUSIONS

1) For small motions the suspension characteristics of a plenum air cushion in heave correspond to a spring and damper in parallel which are exercised by relative motion between craft and surface.

2) The heave suspension characteristics can be defined for small motions in terms of cushion geometry and fan/duct parameters.

3) For small motions, minimum heave response requires high values of damping at low frequency ratios, and vice versa.

4) For typical motions the stiffness and damping are non-linear and the significance of this on craft response and cushion power requirement requires further analysis.

5) A specific power requirement for the plenum air cushion can be defined in short regular waves for critical wavelengths and peak heave accelerations. This criterion suggests that a specific cushion power of 2.0 or more is required for conventional cushions to provide even a limiting level of comfort (0.2 g) over regular waves. This power level corresponds to the total installed specific power of typical amphibious hovercraft in service conditions.

6) However, it is not possible to formulate a representative cushion power requirement at the present time, as the criterion derived in this report for short wave operation could be significantly reduced by operation in irregular waves, use of a non-rectangular or shaped cushion planform, and by relative motion between flexible structure and hard structure.

7) Further theoretical work on these aspects and information on comfort levels for irregular inputs are required before firm recommendations can be made for cushion power requirements.

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8.5 J.C. GUIGNARD, "Human Response to Intense Low-Frequency Noise and Vibration". Proceedings of The Institution of Mechanical Engineers 1967-68, Vol. 182, Part 1.

9. NOTATION

a	Constant defining short waves (See equation 44)
b	Width of rectangular cushion planform
C_E	Coefficient of discharge at cushion exit
C_F	Coefficient of discharge at cushion entry
C_Q	Volume flow coefficient (See equation 63)
c	Heave damping of air cushion due to pressure change
f_1, f_2	Cushion system coefficients (See Appendix A and equations 29 and 30)
F	Fan coefficient (See Figure 2)
g	Acceleration due to gravity
H	Total pressure head delivered by fan relative to atmosphere
h_E	Mean height of cushion exit area in static conditions
h_w	Wave height (crest to trough)
k	Heave stiffness of air cushion due to pressure change
K	Wave form factor in heave (Appendix B)
ℓ	Length of rectangular cushion planform
m	Constant of fan characteristic at constant fan power (See equation 12)
M	Amplitude ratio of linear second order system
n	Incremental 'g' acceleration about the '1g' datum
p	Perimeter of cushion exit area
p_c	Cushion pressure relative to atmosphere
Δp_c	Change in cushion pressure from equilibrium value
$\bar{p}_c$	Cushion pressure/fan pressure ratio (See equation 31)
Q	Volume flow through cushion system
Q_o	Maximum volume flow delivery of fan (See Figure 2)
Q_c	Rate of change of cushion volume (See equation 6)
Q_f	Maximum rate of change of cushion volume due to a "flat plate" heaving surface (Appendix B)
Q_w	Maximum rate of change of cushion volume due to short critical wavelengths.
q_c	Dynamic head in cushion due to volume flow
S_c	Cushion area
S_E	Cushion exit area
S_F	Cushion feed area
V	Speed of craft
V_R	Speed of craft relative to waves
V_w	Propagation speed of waves in deep water
W	Weight of craft
Z	Heave load on craft due to air cushion
ΔZ	Change of heave load on craft from equilibrium value (W)
z	Relative heave displacement between craft and surface

NOTATION (Contined)

$\dot{z}$	Rate of change of heave displacement between craft and surface
$\bar{z}_i$	Amplitude of surface heave motion
z_i	Displacement of surface heave motion
$\dot{z}_i$	Rate of change of diśplacement of surface heave motion
$\bar{z}_o$	Amplitude of craft heave motion
z_o	Displacement of craft heave motion
$\dot{z}_o$	Rate of change of displacement of craft heave motion

α	Constant of linear fan characteristic at constant fan R.P.M.
β	Slope of linear fan characteristic at constant fan R.P.M.
ζ	Damping coefficient of air cushion in heave
η_F	Fan efficiency
η_M	Mechanical efficiency between engine output and fan input
λ	Wave length
ρ	Air density
ω_e	Frequency of encounter of waves
ω_n	Undamped natural frequency of air cushion in heave

APPENDIX A

Derivation of Cushion Pressure Change Parameters for Small Motions.

1. Cushion Pressure Change $\left(\frac{dp_c}{dz}\right)$

For small displacements $\frac{dp_c}{dz}$ can be determined by differentiating equations (1) to (12) with respect to (z) when ($\dot{z}$) is zero.

From equations (2) and (7) after some simplification:-

$$\frac{dQ}{dz} = \frac{Q}{2(H-p_c)}\left(\frac{dH}{dz} - \frac{dp_c}{dz}\right) \qquad (1a)$$

and again from equations (1), (5) and (8)

$$\frac{dQ}{dz} = \frac{Q}{2p_c}\frac{dp_c}{dz} + \frac{Q}{(h_E+z)} \qquad (2a)$$

Hence, equating equations (1a) and (2a) as (z) → o:-

$$\frac{dH}{dz} - \frac{H}{p_c}\frac{dp_c}{dz} = \frac{2}{h_E}(H - p_c) \qquad (3a)$$

1.1 Constant fan RPM

Equations (2) and (11) give:-

$$\frac{dH}{dz} = \frac{\beta Q}{2(H-p_c)}\left(\frac{dH}{dz} - \frac{dp_c}{dz}\right) \qquad (4a)$$

or re-arranging:-

$$\frac{dH}{dz} = -\frac{F}{2(1 - \bar{p}_c - \frac{1}{2}F)}\frac{dp_c}{dz} \qquad (5a)$$

where $F = \frac{\beta Q}{H}$ and $\bar{p}_c = \frac{p_c}{H}$

Substituting (5a) in (3a) and re-arranging:-

$$\frac{dp_c}{dz} = \frac{-2p_c f_1}{h_E} \qquad (6a)$$

where $$f_1 = \frac{1 - \bar{p}_c - \frac{1}{2}F}{1 - \frac{1}{2}F} \qquad (7a)$$

1.2 Constant fan power and efficiency

Equation (12) gives:-

$$\frac{dH}{dz} = -\frac{m}{Q^2}\frac{dQ}{dz} \qquad (8a)$$

Using equation (1a) and re-arranging:-

$$\frac{dH}{dz} = \frac{1}{1 + 2(1 - \bar{p}_c)}\frac{dp_c}{dz} = \frac{1}{3 - 2\bar{p}_c}\frac{dp_c}{dz} \qquad (9a)$$

Substituting (9a) in (3a):-

$$\frac{dp_c}{dz} = \frac{2p_c}{h_E} f_2 \qquad (10a)$$

where $f_2 = 1 - \frac{2}{3}\ \bar{p}_c$ (11a)

2. Cushion Pressure Change $\left(\frac{dp_c}{d\dot{z}}\right)$

For small motions, equations (1) to (12) can be differentiated with respect to the relative heave rate of displacement ($\dot{z}$) when the relative heave displacement (z) is zero. This procedure is valid for linear systems and is adopted for convenience in this simple analysis of small motions.

From equations (2) and (7) after some simplification

$$\frac{dQ}{d\dot{z}} = \frac{Q}{2(H - p_c)}\left(\frac{dH}{d\dot{z}} - \frac{dp_c}{dz}\right) \qquad (12a)$$

And, again, from equations (1) (6) and (8), after some simplification:-

$$\frac{dQ}{d\dot{z}} = \frac{Q}{2p_c}\left(1 - \frac{Q_c}{Q}\right)\frac{dp_c}{d\dot{z}} + S_c \qquad (13a)$$

Where the Q_c is the rate of change of cushion volume due to the relative heave rate (z)

or $Q_c = S_c\dot{z}$ (14a)

Then equating (12a) and (13a) as $Q_c \to 0$:-

$$\frac{dH}{d\dot{z}} - \frac{dp_c}{d\dot{z}} = \left(\frac{1 - \bar{p}_c}{\bar{p}_c}\right)\frac{dp_c}{d\dot{z}} + \frac{2\ S_cH}{Q}(1 - \bar{p}_c) \qquad (15a)$$

where $\bar{p}_c = \frac{p_c}{H}$

2.1 Constant Fan RPM

Equations (2) and (11) give:-

$$\frac{dH}{d\dot{z}} = \frac{\beta Q}{2(H - p_c)}\left(\frac{dH}{d\dot{z}} - \frac{dp_c}{d\dot{z}}\right) \qquad (16a)$$

or re-arranging:-

$$\frac{dH}{d\dot{z}} = -\frac{F}{2(1 - p_c - \frac{1}{2}F)}\frac{dp_c}{d\dot{z}} \qquad (17a)$$

where $F = \beta\frac{Q}{H}$ and $\bar{p}_c = \frac{p_c}{H}$

Substituting (17a) in (15a) and simplifying:-

$$\frac{dp_c}{d\dot{z}} = -2\frac{W}{Q}f_1 \quad \text{where } p_c S_c \to W \qquad (18a)$$

where $f_1 = \frac{1 - p_c - \frac{1}{2}F}{1 - \frac{1}{2}F}$ (19a)

2.2 Constant Fan Power and Efficiency

Equation (12) gives:-

$$\frac{dH}{d\dot{z}} = -\frac{m}{Q^2}\frac{dQ}{d\dot{z}} \tag{20a}$$

Using (1a) and re-arranging:-

$$\frac{dH}{d\dot{z}} = -\frac{1}{2(1-\bar{p}_c)}\left(\frac{dH}{d\dot{z}} - \frac{dp_c}{d\dot{z}}\right) \tag{21a}$$

or $$\frac{dH}{d\dot{z}} = \frac{1}{3-2\bar{p}_c}\left(\frac{dp_c}{d\dot{z}}\right) \tag{22a}$$

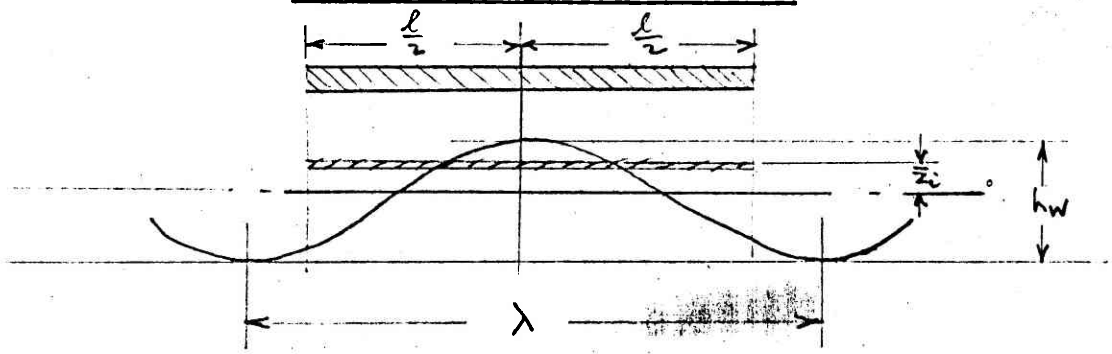
Then substituting (22a) in (15a) and simplifying:-

$$\frac{dp_c}{d\dot{z}} = -\frac{2W}{Q}f_2 \tag{23a}$$

where $f_2 = 1 - \frac{2}{3}\bar{p}_c$

APPENDIX B

Surface Forcing Factor in Heave



Area of sinusoidal wave of length (λ) within a cushion length (ℓ) is given by:-

$$A = \int_{-\frac{\ell}{2}}^{\frac{\ell}{2}} \frac{h_w}{2} \cos \frac{2\pi x}{\lambda} \, dx \tag{1b}$$

$$\text{or} \quad A = \frac{h_w \lambda}{2\pi} \sin \frac{\pi \ell}{\lambda} \tag{2b}$$

Then mean amplitude of a flat plate surface within the cushion:-

$$\bar{z}_i = \frac{h_w}{2} \frac{\lambda}{\pi \ell} \sin \frac{\pi \ell}{\lambda} \tag{3b}$$

$$\text{or} \quad \bar{z}_i = K \frac{h_w}{2} \tag{4b}$$

Where (k) is a wave form factor in Heave

$$K = \frac{\lambda}{\pi \ell} \sin \frac{\pi \ell}{\lambda} \tag{5b}$$

The "flat plate" surface must sweep out the same rate of change of cushion volume as the waves which pass through the cushion.

For short wave lengths such that:-

$$\frac{\lambda}{\ell} = \frac{2}{a}$$

where (a) is an odd positive integer greater than unity, the maximum rate of change of cushion volume due to these wavelengths is deduced typically from Fig. 11.

$$Q_w = b \, h_w \, V_R \tag{6b}$$

The maximum rate of change of cushion volume due to a heaving "flat plate" is given by:

$$Q_f = K \frac{h_w}{2} S_c \, \omega_e \tag{7b}$$

Using equation (37), equation (7b) becomes:-

$$Q_f = \pi K h_w \, S_c \frac{V_R}{\lambda} \tag{8b}$$

APPENDIX B continued

But $S_c = \ell b$.

$$\therefore \quad Q_f = \pi \frac{\ell}{\lambda} K (h_w b V_R) \qquad (9b)$$

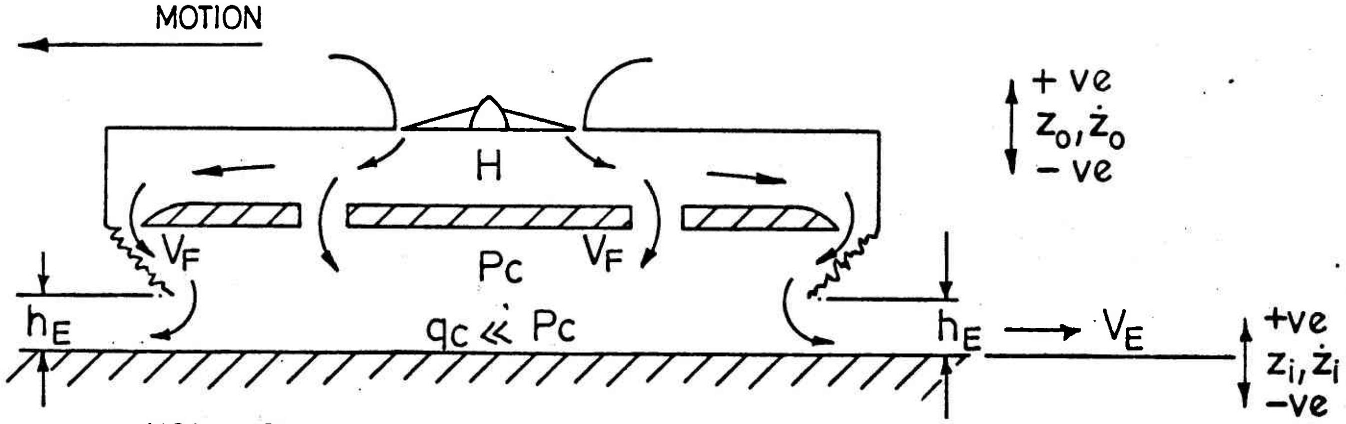
When $Q_f = Q_w$

Then for short waves only, such $\frac{\lambda}{\ell} = \frac{2}{3}, \frac{2}{5}, \frac{2}{7}$ etc:-

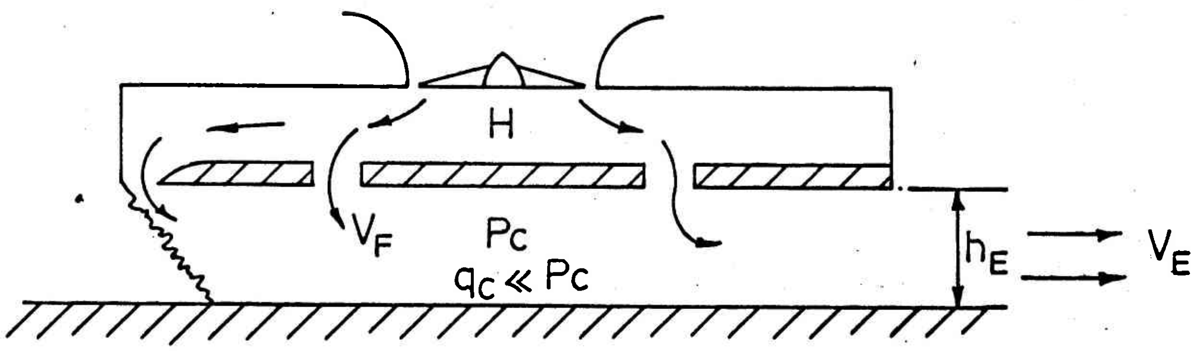
$$K = \frac{\lambda}{\pi \ell} \qquad (10b)$$

PLENUM AIR CUSHION VARIANTS

P = PERIMETER OF AIR GAP



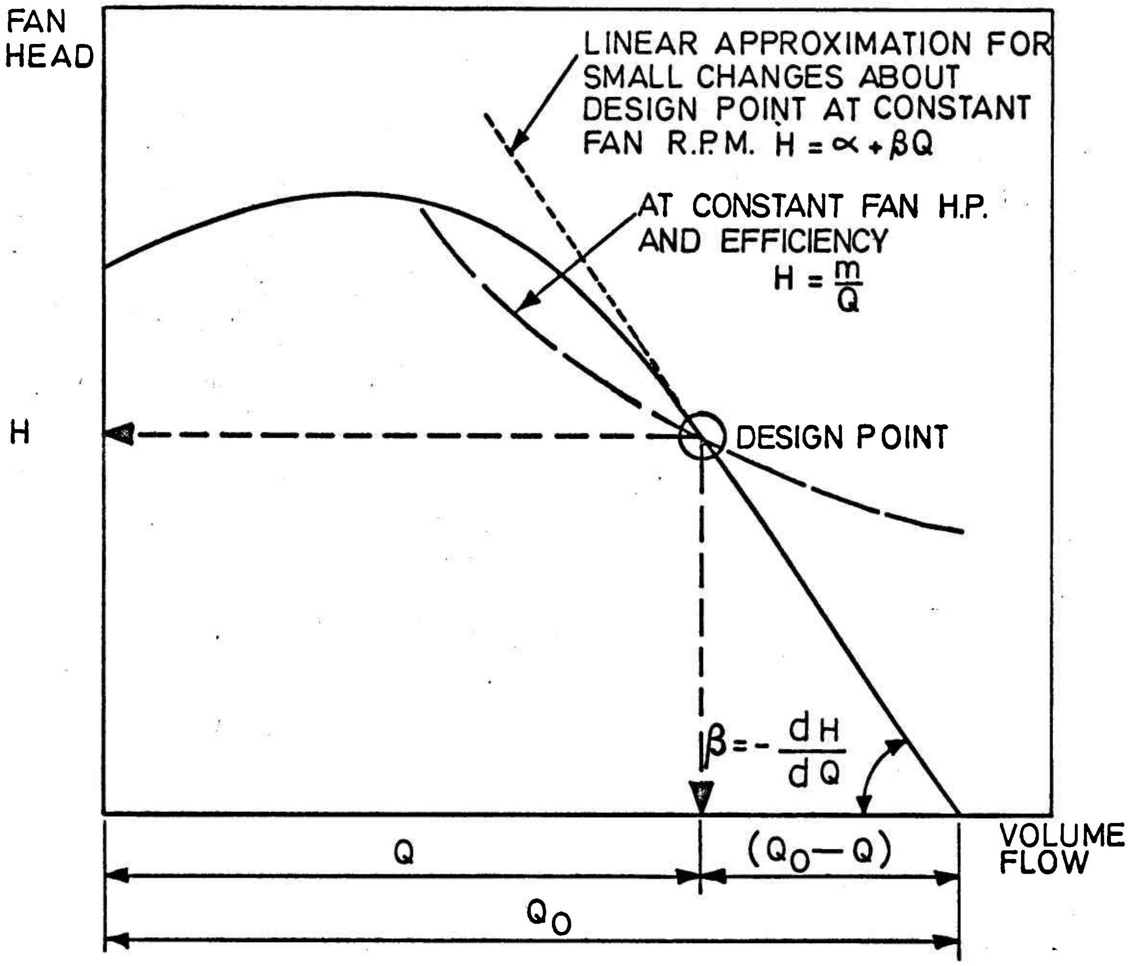
a. NON-CONTACTING - PERIPHERAL EXIT AREA VARIES WITH RELATIVE HEAVE MOTION



b. CONTACTING - REAR EXIT AREA VARIES WITH RELATIVE HEAVE MOTION

FIG. 1

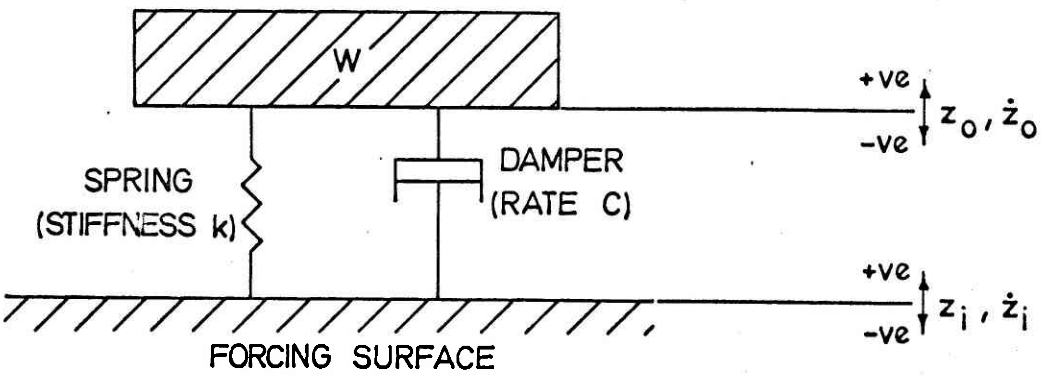
DEFINITION OF FAN PARAMETER (F)



$$F = \frac{Q}{H}\,\frac{dH}{dQ} = \left(\frac{Q}{Q_0 - Q}\right)$$

FIG. 2

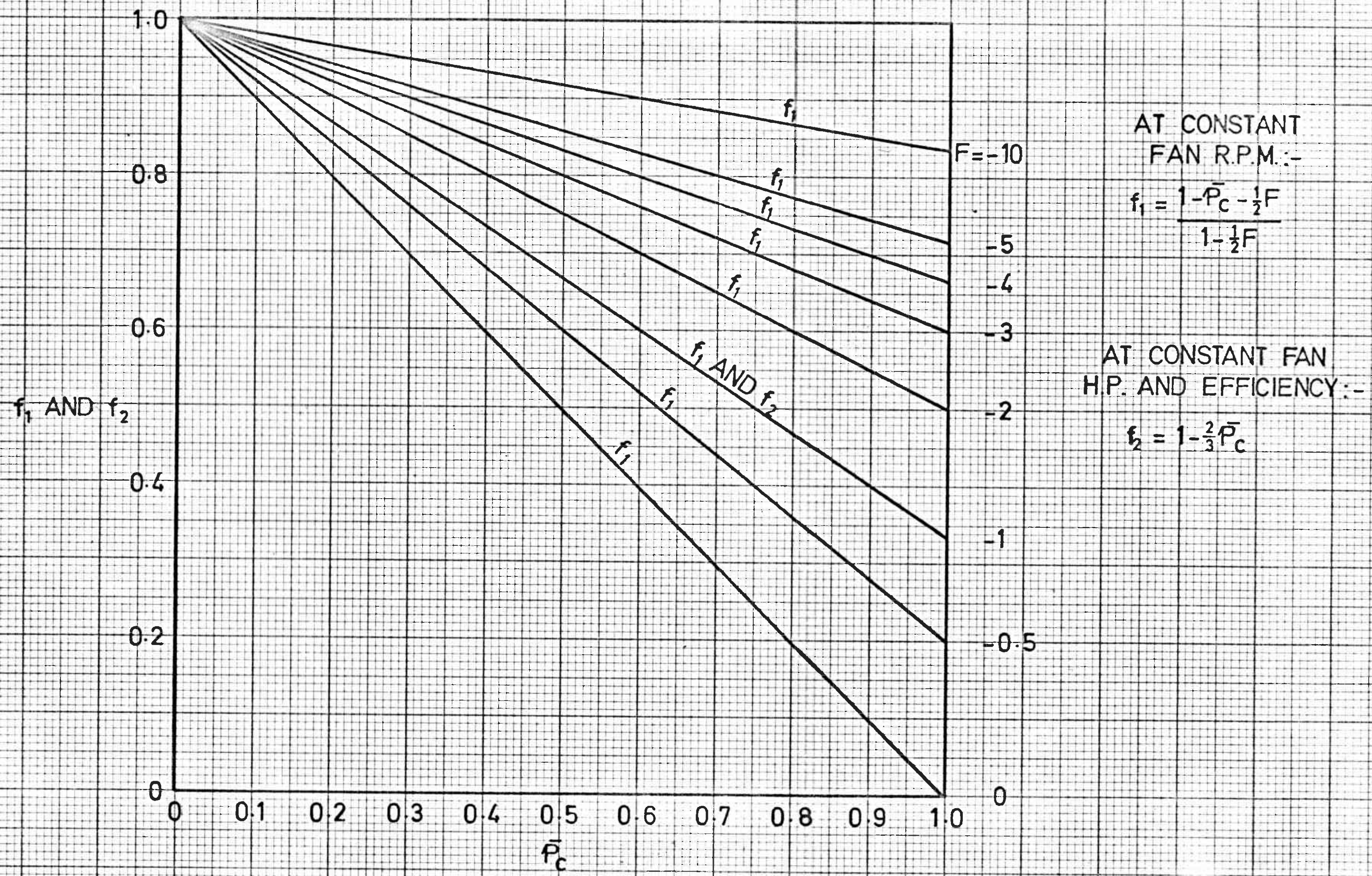
SIMPLE SUSPENSION SYSTEM IN HEAVE



RELATIVE DISPLACEMENT $z = (z_o - z_i)$

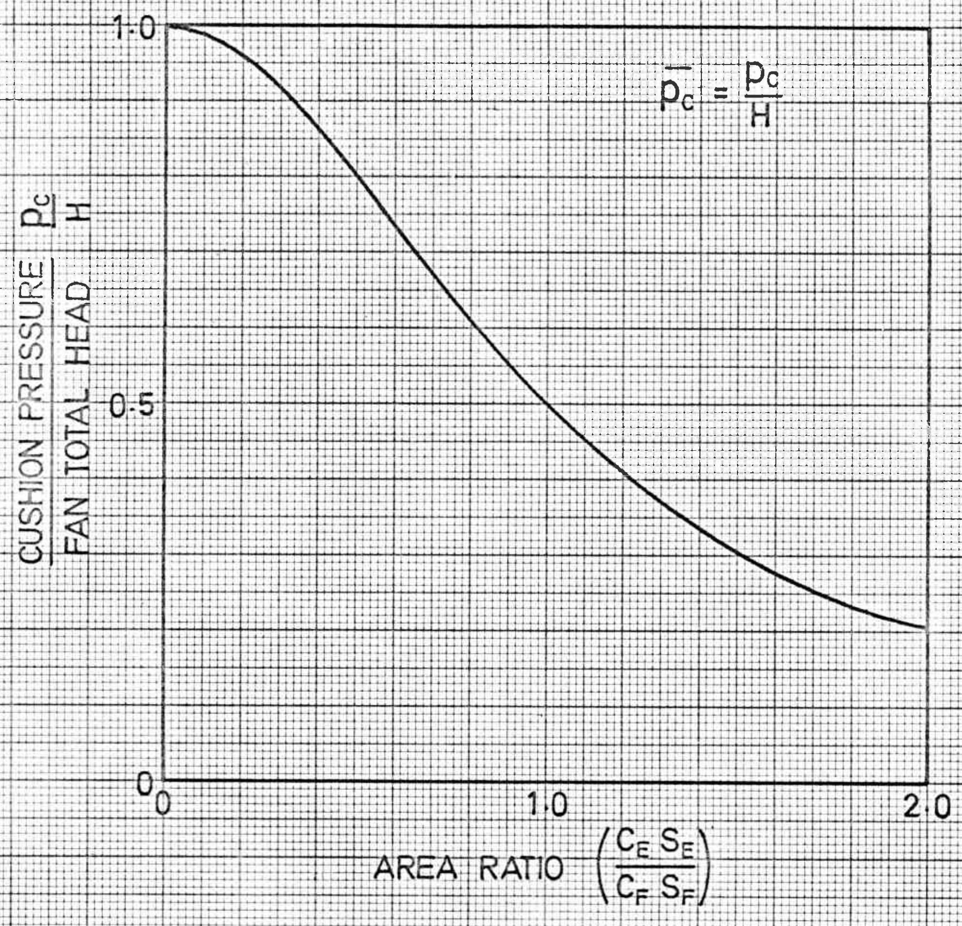
RELATIVE RATE OF DISPLACEMENT $\dot{z} = (\dot{z}_o - \dot{z}_i)$

FIG. 3



CUSHION SYSTEM COEFFICIENTS

FIG. 4



VARIATION OF PRESSURE RATIO
WITH AREA RATIO IN STATIC CONDITIONS

FIG. 5

FREQUENCY RESPONSE OF SIMPLE HEAVE SYSTEM

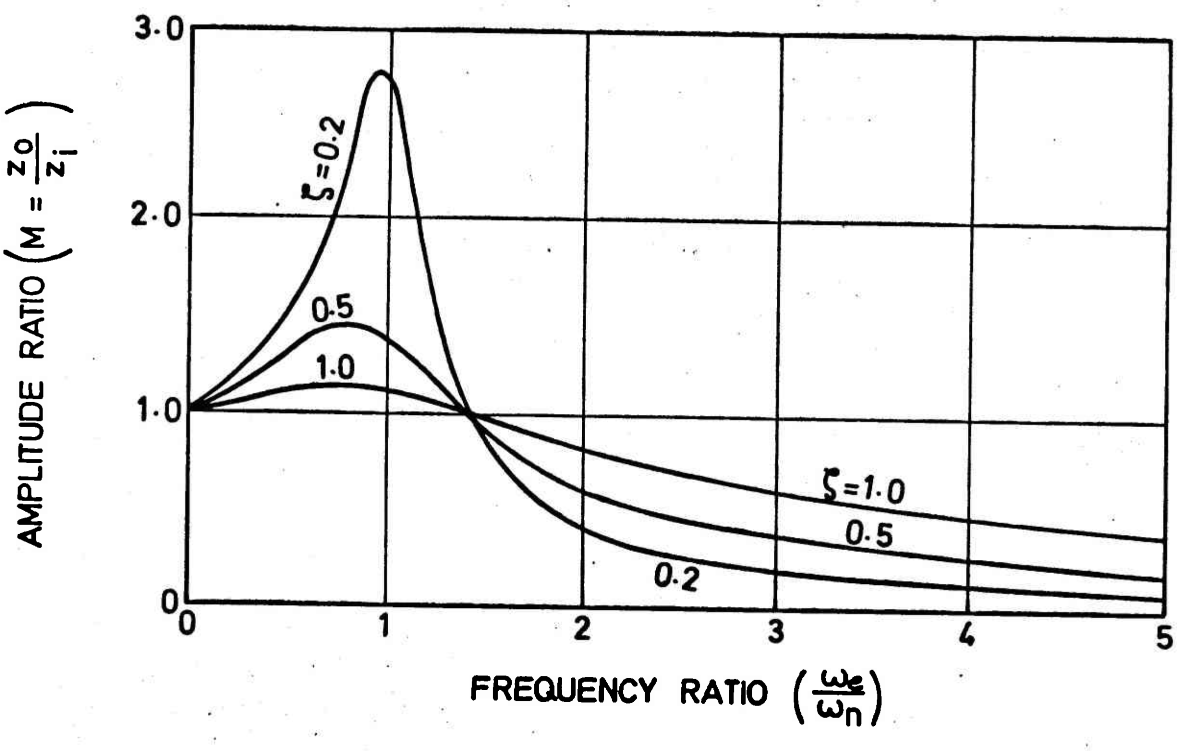


FIG. 6

"FLAT PLATE" FORCING FACTOR IN HEAVE DUE TO SINUSOIDAL WAVES IN A RECTANGULAR CUSHION

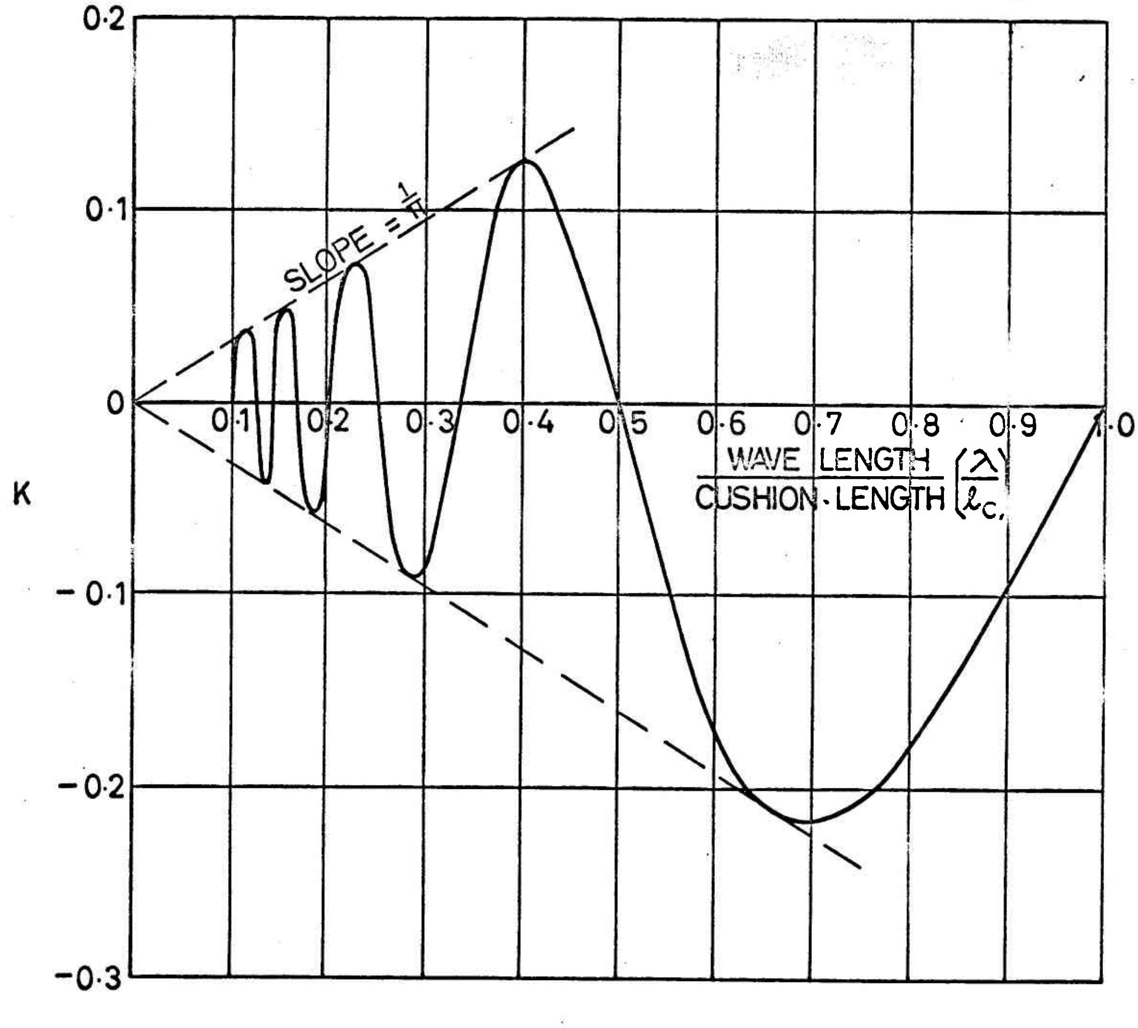
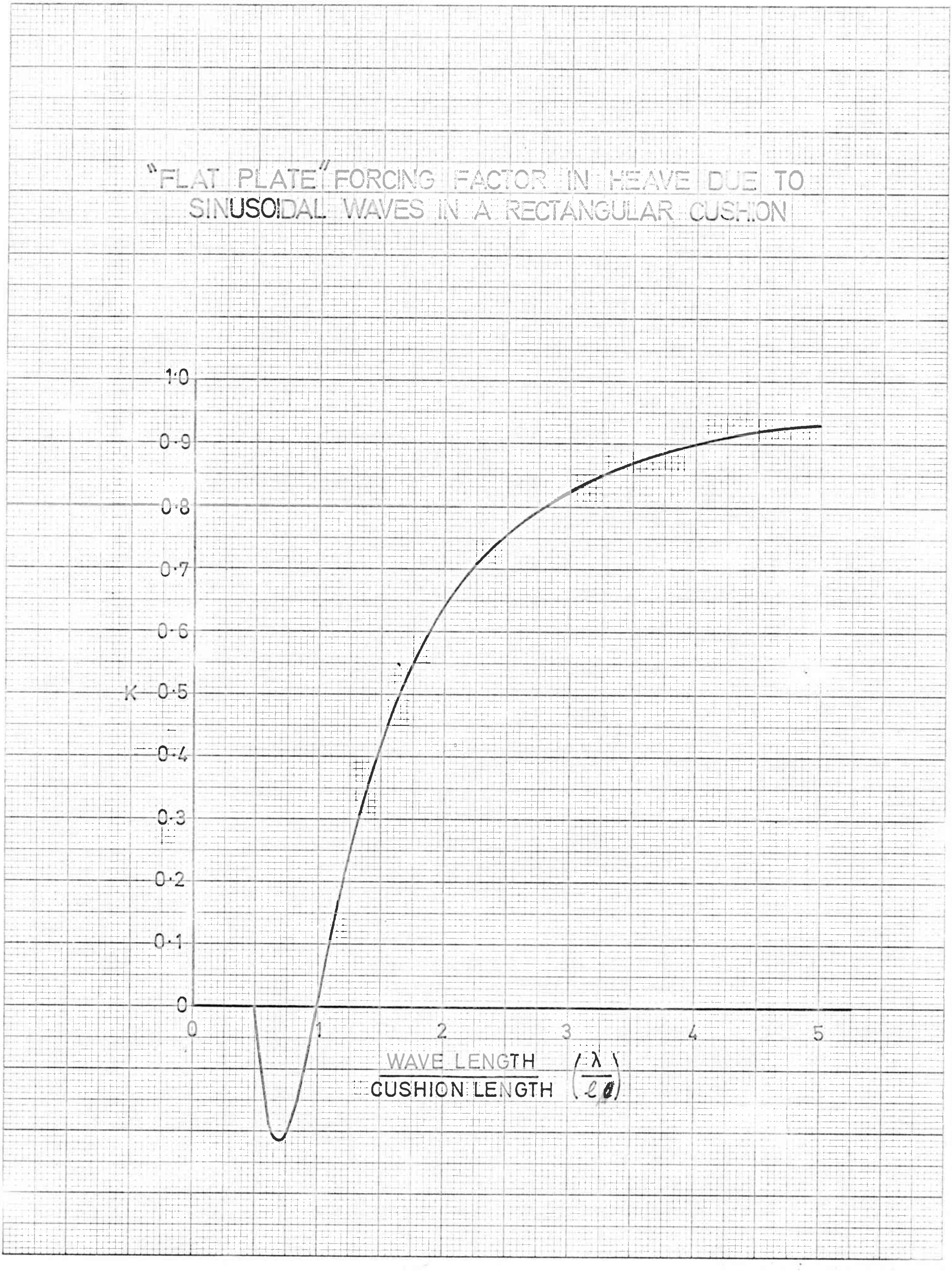


FIG. 7



"FLAT PLATE" FORCING FACTOR IN HEAVE DUE TO SINUSOIDAL WAVES IN A RECTANGULAR CUSHION
K
1·0
0·9
0·8
0·7
0·6
0·5
0·4
0·3
0·2
0·1
0
0
1
2
3
4
5
WAVE LENGTH / CUSHION LENGTH (λ / ℓ)

FIG. 8

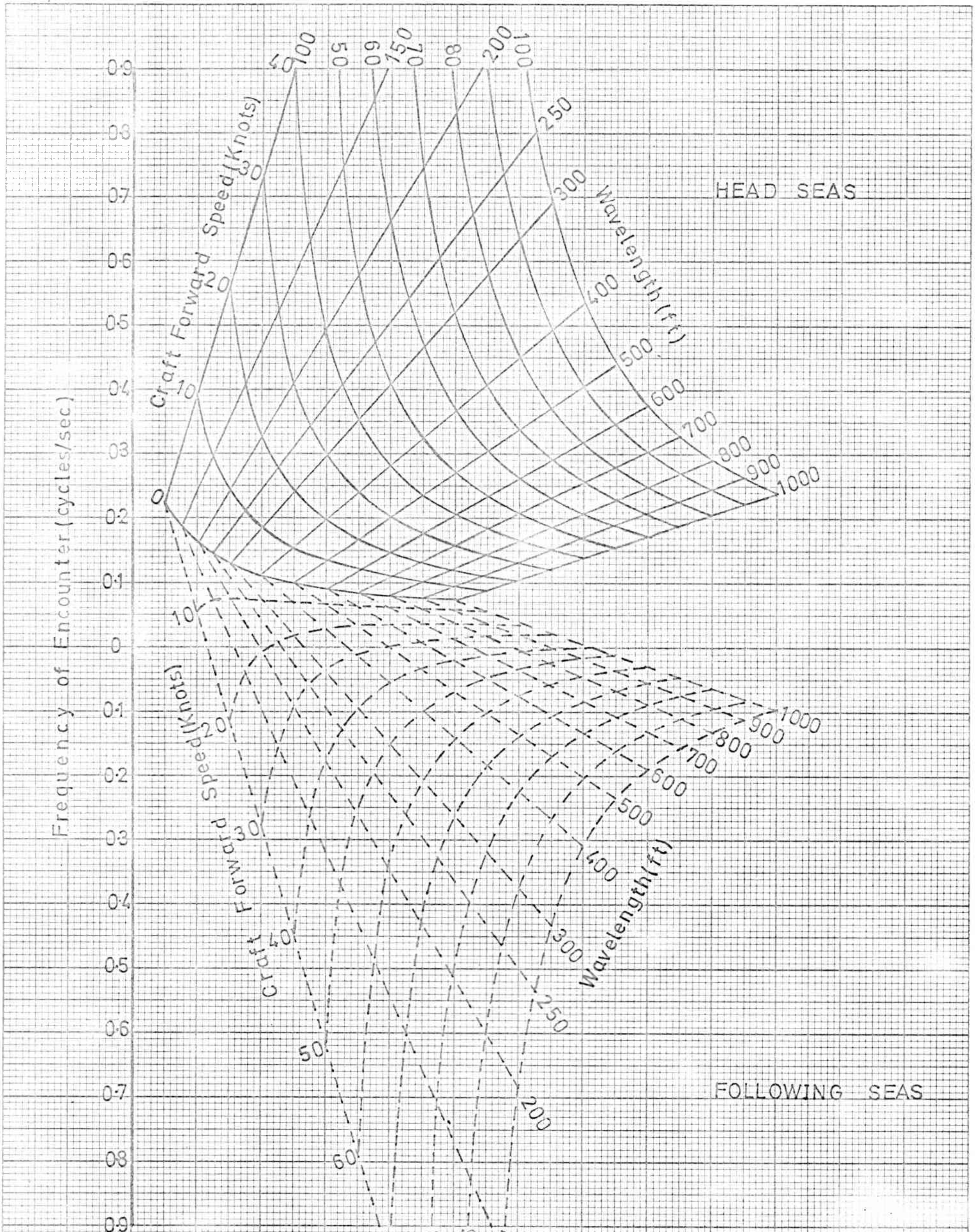
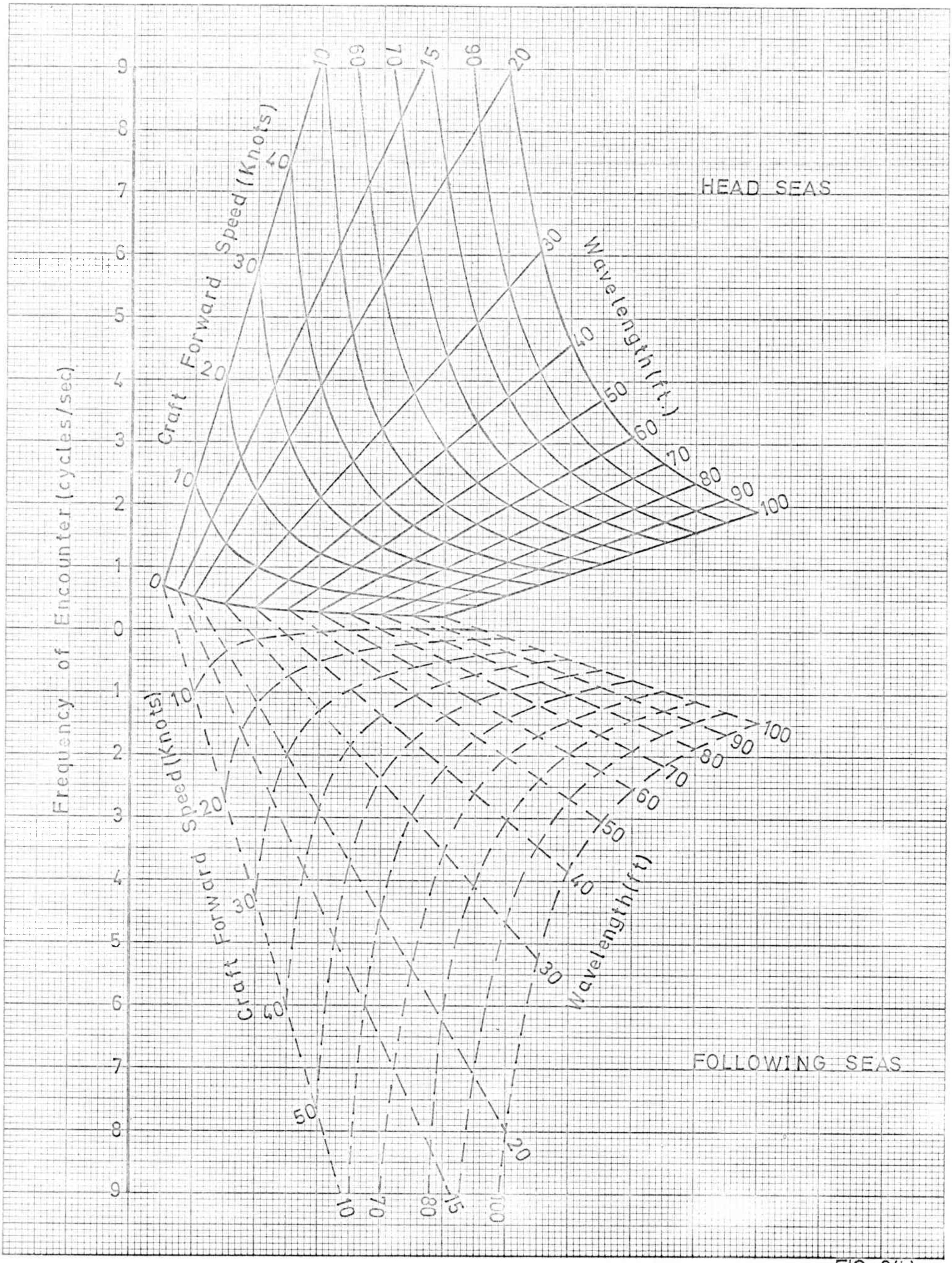

Frequency of Encounter (cycles/sec)
0·9
0·8
0·7
0·6
0·5
0·4
0·3
0·2
0·1
0
HEAD SEAS
FOLLOWING SEAS
Craft Forward Speed (Knots)
Wavelength (ft)

FIG. 9(a)



HEAD SEAS
FOLLOWING SEAS
Frequency of Encounter (cycles/sec)
Craft Forward Speed (Knots)
Wavelength (ft.)
Craft Forward Speed (Knots)
Wavelength (ft)

FIG. 9(b)

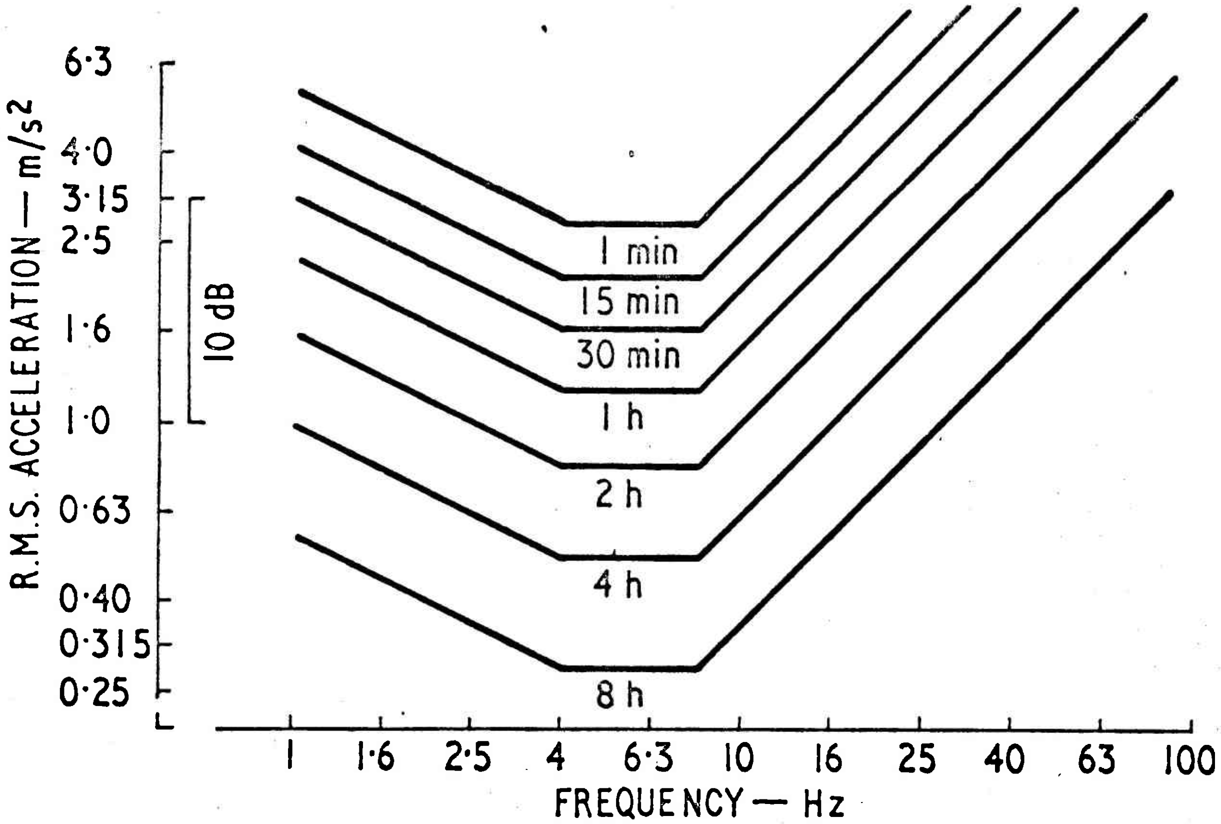


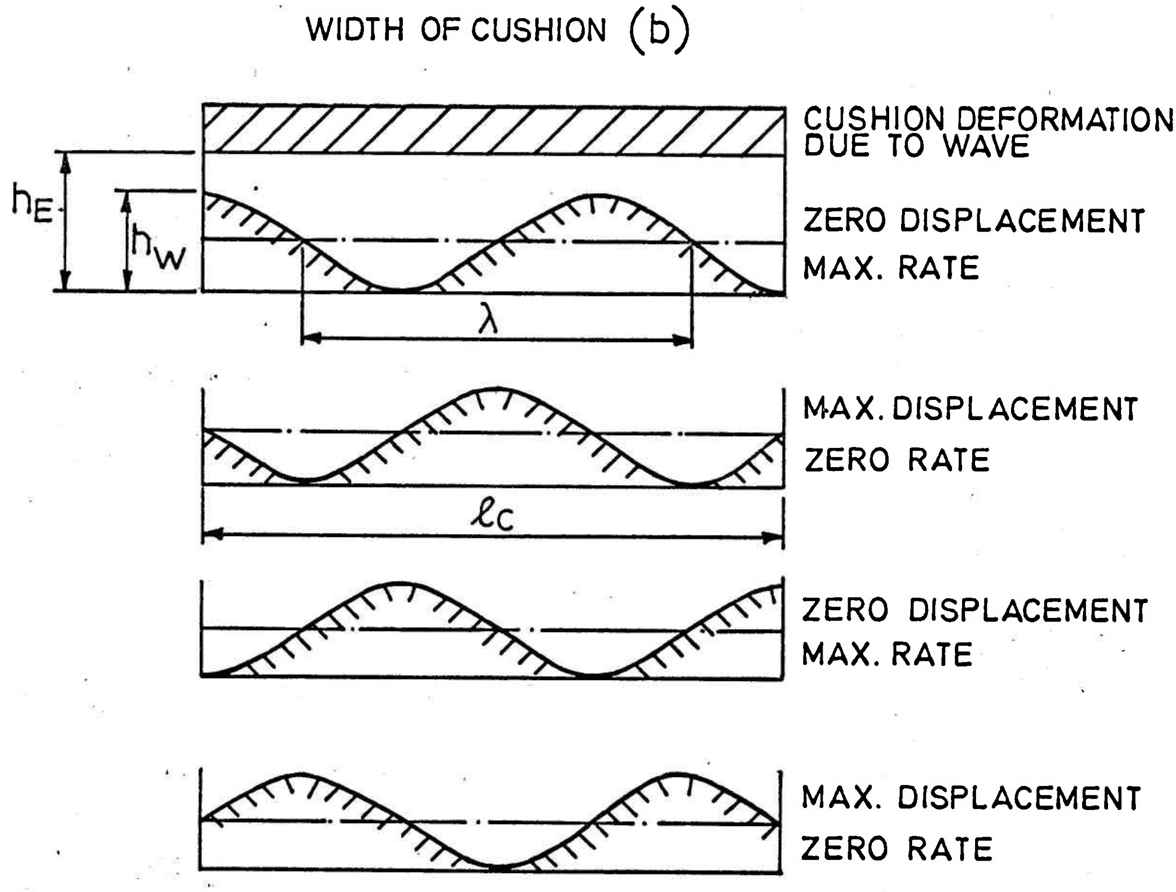
Fig. 2. Proposed 'fatigue-decreased proficiency' limits for vertical vibration in the range 1 to 90 Hz: weighting factors are given in Table 1

(Information reproduced by courtesy of the Secretariat, Working Group 7, ISO/TC 108)

Table 1. Proposed weighting factors for adaptation of the limits shown in Fig. 2

Transverse (horizontal) vibration	subtract	3 dB
Reduced comfort boundaries .	subtract	10 dB
Safe exposure limits . . .	add	6 dB

CUSHION VOLUME CHANGE
DUE TO A CRITICAL SINUSOIDAL WAVE ($\lambda < \ell_c$)



$Q_w = h_W \times b \times V_R$ CUSECS.

FIG. 11

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