An effective blackboard/whiteboard design (approximately 1 hour of teaching).

Kathrine Frey Frøslie, statistician, www.statistrikk.no

Use this to emphasize the difference between descriptive statistics, mean and SD on one hand, and estimation error and confidence interval on the other hand.
An additional point is that the mean can be used for two purposes: to summarize a set of observations AND to give an estimate for the population mean.

Start by drawing a line in the middle of the board to split it in two halves. The design with the divided board is ideal for handwriting of notes over a double-page notebook.

Headlines (to be written on the blackboard) are written on green background

The yellow outlined text denotes Oral information or (easy) questions for discussions in the entire class or in small groups (2-4 persons), depending on the group dynamics among the students and the room facilities

The rest of the text is supposed to be written on the blackboard. (Scetches of the histograms are also supposed to be handwritten)

Note that there are more text in Task 2 than in Task 1. The reason is that Task 2 is the stuff the students need to dwell on and to think about, to get a grasp on. Therefore, this handwritten (and somewhat imprecise) text serves as a supplement to the more rigorous text in text books (which of course are more precise and therefore can benefit from some softening) and it is important that the words are written in the note book, so that the students can repeat them as part of their homework.

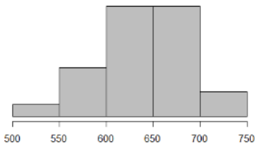
Imagine you work at a rehabilitation hospital and that you have developed a new test to measure people's walking abilities. You call it the 6 Minute Walking Test, 6MWT. The test is performed like this: People are supposed to walk as usual for 6 minutes, the distance walked is measured in meters. Which results can you expect among non-disabled persons? Discuss plausible walking distances.

Study 1: The pilot study

Objective: To study the 6MWD among males in the general population
Sample: n=25, employees at a rehabilitation hospital. Random sample? Representative?

Task 1: Summarize observed data (Descriptive statistics)

Which type of data? Categorical or continuous?
Continuous data ->
What must we do to inspect the distribution?
Must make histogram:



Note: "Like a first generation mobile phone picture; rough, imprecise"

Is the distribution symmetric or skewed?
-> Symmetric distribution:
Which numbers are good summary numbers?
mean=636 and SD=46 are good summary measures
What does this mean?

Properties of the Normal distribution imply:
The interval 636±2·46=(543, 728)* contains how much? most (approx. 95%) of the observations
The interval 636±3·46=(497, 774) contains how much? "all" (approx. 100%) of the observations

Task 2: Say something about the population, based on the data we have (inference)

Thoughts: If the mean in the sample is 636, then the mean in the population is most likely somewhere around that number too, provided we have a random sample.

The unknown mean in the population is called μ.
Hence, we assume that the observed mean, denoted $\bar{X}$, resembles μ; sloppily we can say $\bar{X} \approx \mu$
We say that $\bar{X}$ is an estimator for μ, and that the number 636 is an estimate for μ.

But how confident are we in the number 636? How far from 636 can we expect the unknown population mean μ to be?
Can it just as well be 640? 650? 680? 600? 700?

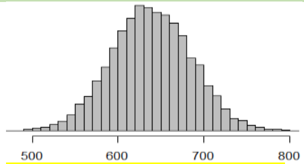
To answer this, we need to know the error inherent in $\bar{X}$: the estimation error (of $\bar{X}$).
The estimation error of an estimate is often called standard error, abbreviated S.E.
The estimation error of a mean ($\bar{X}$) is SE($\bar{X}$) = SD/√n = 46/√25 = 9.2

This number does not have a useful interpretation in itself, like SD has, but is rather used as an intermediate calculation to obtain a 95% confidence interval for μ, the population mean. In this specific situation, the 95% CI for μ is
$\bar{X}$±1.96·SD -> 636±1.96·46/5 -> (617, 654)
Note: This interval is built to cover the population mean, not the single observations as in Task 1*
This means we are 95% confident that the interval (617, 654) contains the true population mean.
I.e: Can it just as well be 640? 650? ~~680? 600? 700?~~

Study 2: The national research council is enthusiastic about the project, and gives you a grant to study this in a population-wide study.
Objective? Objective: Same.
Sample: n=10,000 males from the general population. Random sample? Representative?

Task 1: Still (and always) to summarize the observed data (Descriptive statistics)

Still: which data type?
Continuous data ->
Still, we must?
Must make histogram.
"Based on the pilot, what distribution do you expect?"



Like a modern mobile phone picture. More accurate & precise, more details

Symmetric distribution, i.e. mean and SD are good summary measures. Which numbers do you expect?
For mean? Larger, smaller, similar?
Mean=639
For SD? Larger, smaller, similar?
SD=47
Note: Almost the same as in the pilot

Properties of the Normal distribution still imply:
The interval 639±2·47=(545, 732)* contains most (approx. 95%) of the observations
The interval 639±3·47=(499, 779) contains "all" (approx. 100%) of the observations
Note: Almost the same as in the pilot, but these numbers are more trustworthy. Why?

Task 2: Say something about the population, based on the data we have (inference)

Thoughts: If the mean in the sample is 639, then the mean in the population is most likely somewhere around that number too, provided we have a random sample.

The unknown mean in the population is still called μ.

We say that $\bar{X}$ is an estimator for μ, and that the new number 639 is a new estimate for μ.

But how confident are we in the new number 639? How far from this new estimate, 639, can we expect the unknown population mean μ to be?
Can it just as well be 640? 650? 630?

The estimation error of this new estimate can be calculated by the same formula.
Would you expect it to be larger, smaller or similar to the pilot study estimate?
SE($\bar{X}$) = SD/√n = 47/√10,000 = 0.47

As in the pilot, this number does not have a useful interpretation in itself, like SD has, but is rather used as an intermediate calculation to obtain a 95% confidence interval for μ.
The 95% CI for μ is now
$\bar{X}$±1.96·SD -> 639±1.96·47/100 -> (637.7, 639.5)
Note: This interval is built to cover the population mean, not the single observations as in Task 1*
This means we are 95% confident that the interval (637.7, 639.5) contains the true μ.
I.e. Can it just as well be ~~640? 650? 630?~~