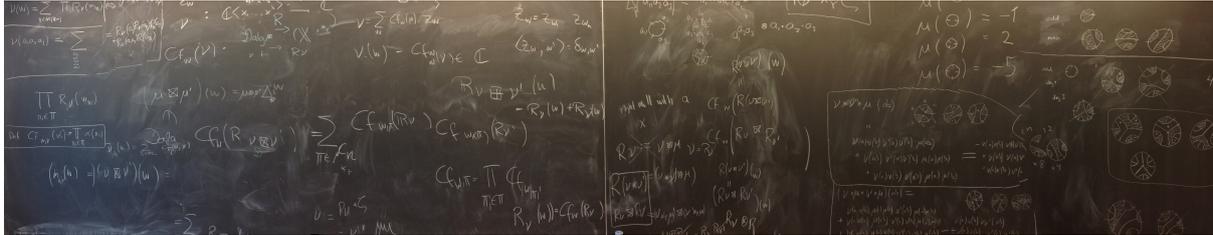


# NTNU visits — informal report

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I report on my BFS/TFS visiting professorship at the NTNU to work with Kurusch Ebrahimi-Fard. The visit was split into two parts of two weeks each, the first in June 2019, the second in February 2020.

Our collaboration is a bit uncommon, in that we have very different backgrounds: Kurusch comes from Lie theory, quantum field theory, numerical integration, stochastic calculus, and control theory; my own background is in algebraic geometry, homotopy theory, and category theory. This makes our interaction quite inspiring: many interesting ideas spring from the synergy of our different cultures.

Of course one cannot generally just put two disparate mathematicians together and ask them to interact. In our case, our interaction started ten years ago with some common interests in Feynman graphs. Over the years we learned from each other, in a not very target-oriented manner, building up some more overlap, in terms of language, topics, and stocks of examples. But for the past four years we have worked together more systematically on a project on operadic aspects of free probability, in a research team involving also Loïc Foissy (Calais) and Frédéric Patras (Nice).

A main task during my June visit was to finish a paper coauthored by the four of us. In principle the actual results were obtained in late 2018, when we all met in Barcelona, but as it is often the case, ironing out the details led to more work and new insights. It was lucky it was June, and the days were long enough for all our work. The paper was finished and submitted to a distinguished journal.

Briefly, this paper explains the relationship between two approaches to moment-cumulant relations in free probability: on one hand the main approach due to Speicher, based on Möbius inversion in the lattice of noncrossing partitions, and on the other hand the approach of Ebrahimi-Fard and Patras based on half-shuffles in the double tensor algebra. The relationship we found is expressed in terms of two different operad structures on noncrossing partitions, whose incidence bialgebras interact in a certain way (technically, they form together a comodule bialgebra).

The new insights, which are now developing into further papers, are that our machinery can lead to simplification and new results in the theory of multiplicative free convolution, through some more refined combinatorial language, more well-adapted to the phenomena modelled. We use the theory of decomposition spaces, a major theme in my own research, rather categorical of nature. It is not clear whether decomposition spaces will feature in the final form of our results, because we are keen on making the results useful for people actually working in the field (who might not be willing to invest the energy to learn about decomposition spaces). But it is clear that the theory has been essential for discovering the results. This situation is common in diverse research teams: while the full joint expertise is exploited in the investigations, general accessibility is a concern in actually presenting the results.

Between my two Trondheim visits, I met again with Foissy and Patras in Barcelona in January 2020 and did some ground work in the theory, establishing some basic results about a certain decomposition space of noncrossing multiary chord diagrams, which in our theory is the key combinatorial object. Now during my second stay in Trondheim, February 2020, Kurusch and I have had a very nice expansive phase, where we have outlined promising applications of the theory. We wanted to explore the scope of the theory before writing up the basics. If the theory were only a simplification of existing results, that would require one way of writing it up. Now we think that there is potential for much more than just simplification of existing results. Over the coming months we will discuss this further with our coauthors, and make more concrete plans.

In this way, while my first visit was mostly about closing one project, the second visit has been a starting point of one or more new projects. Both visits were very fruitful, the kind of intense mathematical experience we live for as mathematicians.

During my stays, I have annoyed as few people as possible. Actually I have mostly been annoying Kurusch, staying in his office from early morning to late afternoon, hardly giving him time to check his email. However, I have also given four talks: one specialised research talk in the Topology and Geometry Seminar, on ‘Infinity-operads as polynomial monads’. This was aimed at people with a background in homotopy theory. In February I gave two introductory talks targeted at a broader audience, on the theory of ‘Objective combinatorics and decomposition spaces’. Finally I gave a colloquium-style talk in the Mathematical Pearls seminar series, on ‘Möbius inversion — from number theory to renormalisation’. This topic is in natural continuation of my combinatorics talks, but upon Kurusch’s suggestion, I made this part a general-audience talk, and it seemed to be a good idea — many people said they liked it.

I was very well received at the math department, both by the staff and by the secretaries. During my stays I also had very interesting discussions with Claudia Scheimbauer and Nils Baas, both of whom I have known for a long time, and with Markus Szymik, Nicolas Gilliers, and Christian Skau, all of whom I had the pleasure to meet for the first time. I also had shorter chats with many other people, mostly in the lunch room, or in connection with my talks. I got a general impression of a nice relaxed atmosphere. I also felt welcome in Trondheim more generally. People were friendly and helpful, and patient with my not-always-understandable Danish.

The NTNU math department is a privileged place. While the mind is occupied in obscure mathematical thoughts, it is very nice for the eyes to contemplate the ever-changing colours (and grey-scales) of Trondheim fjord.

I feel very grateful for the privilege it was to stay in Trondheim, and I wish to thank Kurusch for thinking of me for this opportunity, and the Trond Mohn stiftelse and the Troms forskningsstiftelse for granting me.

Joachim Kock (March 2020).

