Report of Oleg Morozov on the visit to the Arctic University of Norway (Tromsø) in March 2020

Investigation of integrable nonlinear partial differential equations (PDEs) has a long and distinguished history and to this day remains a subject of intense research. Since its discovery five decades ago the phenomenon of integrability of PDEs has led to great progress in various branches of modern physics including gravity, plasma physics, hydrodynamics and nonlinear optics, influencing the development of many areas of mathematics along the way.

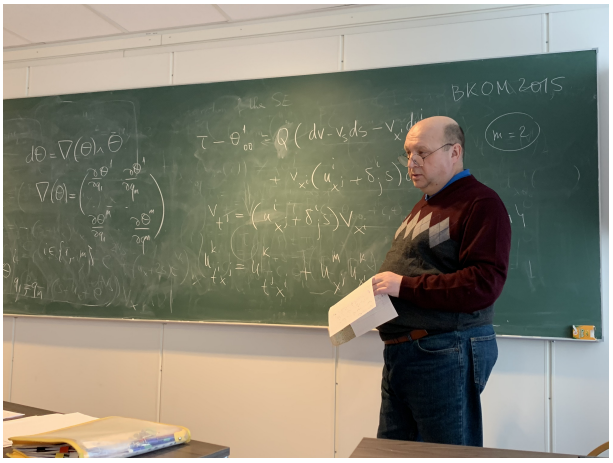
It is not surprising that such a widely used technique had rather uncertain boundaries, especially in the beginning stages, being actually a collection of intertwining concepts based on the existence of infinite dimensional symmetry algebras, hierarchies of conservation laws, and so-called Lax representation. The last notion, also known as an inverse scattering transform, the Wahlquist–Estabrook prolongation structure, or a zero-curvature representation, or an integrable extension, or a differential covering, is the starting point for such techniques of study of PDEs as Bäcklund transformations, Darboux transformations, recursion operators, etc.; it is now widely recognized as the characteristic feature of integrable PDEs. This lead naturally to the question: how can one determine whether a given PDE possesses a Lax representation? This difficult and challenging problem of tremendous importance in the theory of integrable PDEs is still far away from its complete solution. A number of techniques have been devised to handle this problem.

The aim of my research is to attack the problem by means of the methods based on Cartan's theory of Lie pseudogroups and the structure theory of infinite-dimensional Lie algebras. In particular, my recent papers [4] – [6] show that for some PDEs their Lax representations as well as the related integrable hierarchies can be inferred from the extensions of the Lie symmetry algebras of the PDEs generated by twisted (or deformed, or exotic) second cohomology groups of these algebras.

The main objective of my visit to UiT was to continue our collaboration with professor Boris Kruglikov in studying geometry of integrable PDEs, [1] – [3], with the focus on

- deformations of integrable PDEs,
- multi-component generalizations of integrable PDEs and related integrable hierarhcies,
- integrable PDEs with multi-dimesnional twisted cohomology groups of the symmetry algebras.

As part of the research program, I gave a series of lectures on integrable systems and extensions of symmetry algebras at the Sophus Lie seminar (http://www.math.uit.no/seminar/schedule.html), which were attended by staff, postdocs and PhD students.



It it my great pleasure to thank the University of Tromsø for the financial support and for providing me with excellent working conditions. My special thanks are to my host, professor Boris Kruglikov, for his kind help in organizing my visit and during the visit, and for very fruitful discussions.

References

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