Report on A. Geynts's visit to The Arctic University of Norway, UiT (campus Narvik) in the Fall 2019

During A. Geynts's (Heintz in publications) visit to The Arctic University of Norway in September - October 2019 we continued our joint researches of asymptotic properties of boundary value problems for equations describing transport of solutes in porous materials aiming at understanding the asymptotic macroscopic behavior of solutions when periodic porosity becomes very small. Our previous studies led to a new understanding of the nature of transport of solutes in partially permeable materials and to new explicit formulas or osmotic pressure based on an asymptotic analysis of solutions to the corresponding system of Stokes and advection-diffusion equations. A special term describing forces acting on the solute molecules from the material boundaries is the source of the osmotic pressure observed in the macroscale. In our present study we consider large nonlinear effects due to possible higher velocity and derive corresponding macroscopic equations for the transport of the solute. It is natural to consider the boundary of a porous material as periodic Lipschiz surface with period tending to zero. The essential difficulty in this class of problems is the L-p regularity, necessary for the analysis of the nonlinear problem. The regularity of solutions to equations in Lipschitz porous domains was developed using a version of Calderon -Zigmund decomposition – the idea introduced by Caffarelli and Peral. These techniques are applied in our study to the derivation of a hierarchy of reverse Goelder inequalities for solutions to Stokes equations and to the advection-diffusion equation in perforated domains. The reverse Goelder inequalities in turn imply desired L-p estimates for solutions.
Another, new problem that we are working on is a problem of deriving a rigorous asymptotic theory giving macroscopic equations describing separation of solutes in chromatographic columns. A chromatographic column is a cylinder filled by porous grains having specially designed chemically active surface of pores. This device has three distinct scales: pores of the size of several nanometers in chromatographic grains, millimeter scale of flow and diffusion between and inside grains, and several decimeters scale of the whole column, that is interesting for applications. Pressing solutes through such columns shows a complicated interplay between flow, diffusion and adsorption, such that solutes with different chemical properties separate from each other. The system of equations describing processes of adsorption, diffusion and advection are strongly nonlinear. There is a great interest in rigorous derivation of equations describing processes on the largest "macroscale" based on the detailed information about the geometry and chemical processes on smaller scales. The main mathematical difficulty of this problem is that the advection is dominating in the intermediate and macro scales. It is therefore natural, as was suggested in earlier works by Piatnitsky, to consider solutions in the frame moving along with the macroscopic flow. In the problem of interest due to the non-linearity of chemical reactions, this advection is non-uniform in macro scale that makes the analysis of solutions to corresponding equations extra difficult.
A.Geynts delivered a course devoted to methods of harmonic analysis and regularity of solutions to elliptic equations in perforated Lipschitz domains with various applications to physics and chemistry including also a talk on his own research on mathematical theory for osmotic pressure and mathematical models in chromatography.

References

1. Osmosis for non-electrolyte solvents in permeable periodic porous media, by A. Heintz and A. Piatnitski, *Networks and Heterogeneous Media,* Volume 11, Issue 3, 2016, Pages 471-499.

2. A Pore Scale Model for Osmotic Flow: Homogenization and Lattice Boltzmann Simulations, by T. Gebäck and A. Heintz, *Transport in Porous Media,* 28 November 2017, Pages 1-16

The theoretical studies of mathematical problems were also supplemented by enthusiastic exploring beautiful nature of Norway which included mountain hiking, picking up berries and mushrooms and fishing. Fishing was especially successful as seen from the photo.