My visit to the Mathematics Department, University of Bergen

02–22 June, 2019

The rolling sphere, consisting of a 2-sphere rolling on the tangent plane at a point, without spinning and without sliding, is the most classical of all nonholonomic systems. In this simple rolling system, the sphere and the plane are always in contact along a spherical curve, called a rolling curve, and a curve in the tangent plane, called its development. In spite of the non-integrable constraints on velocities, the rolling sphere is completely controllable in the sense that given two admissible configurations of the sphere it is always possible to roll the sphere from one to the other without breaking the constraints of no-slip and no-twist. Knowing that, another interesting problem associated to this rolling system is the following: given two admissible configurations of the sphere, find the curve of shortest length that rolls the sphere from one configuration to the other. Interestingly enough, the solution of this problem is the curve whose development is an elastica.

Throughout the years, this simple rolling system has inspired a substantial amount of work about rolling a general Riemannian ou pseudo-Riemannian manifolds on another of the same dimension. Two main approaches have developed in parallel. One is a more direct generalisation of the rolling sphere, since it assumes that both manifolds are isometrically embedded in some Euclidean or pseudo-Euclidean manifold of bigger dimension. In this case, the rolling motion results from the action of a subgroup of the isometry group of the embedding manifold. The other approach studies rolling motions of manifolds in a more abstract setting that does not make use of the ambient space. The later approach in known in the literature as intrinsic rolling, while the former is extrinsic rolling.

Over the last years I have collaborated with Professor Irina Markina and some of her Ph.D. students to solve some problems using the intrinsic approach for rolling. During my stay, we worked primarily in establishing a bridge between the two approaches mentioned above, certain that the results obtained with one can be translated to the other, with benefits for both. We have been able to accomplish this objective and move to other important related issues, namely sub-Riemannian optimal control problems associated to rolling and interpolation problems on Riemannian manifolds that can be solved explicitly also using rolling motions.

Some particular Riemannian manifolds, such as Lie groups, Grassmannians and Stiefel manifolds, have been studied in more detail, motivated by the key role that they play in many engineering applications. For instance, Stiefel and Grassmann manifolds arise naturally in computer vision, machine learning and pattern recognition, since features and patterns that describe visual objects may be represented as elements in those spaces.

During my stay, I also acted as external examiner for two master thesis and one oral exam.

I would like to thank the University of Bergen for providing excellent working conditions and my host, Professor Irina Markina, for many interesting discussions in a very friendly environment.

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