Report on E. Ferapontov's visit to The Arctic University of Norway (Tromsø) in February 2019

Over his academic career, Evgeny Ferapontov has been investigating various links between the theory of (integrable) PDEs of mathematical physics and classical differential geometry. In the past 15 years, his research was primarily concentrated in the area of multi-dimensional dispersionless integrability. Based on the 'method of hydrodynamic reductions' [1], a variety of integrable systems within particularly interesting classes were studied, revealing non-standard geometric connections and bringing mathematical rigour into the area where only scattered results were previously available.

In particular, a geometric correspondence between dispersionless integrable systems and generalised conformal geometry was proposed in a joint paper with Boris Kruglikov [2] (unifying a whole body of previous work): it was shown that, for broad classes of dispersionless integrable PDEs, their characteristic varieties define conformal structures that must be Einstein-Weyl in 3D/self-dual in 4D. This demonstrated that Einstein-Weyl and conformal self-duality equations can be viewed as 'master-equations' underlying dispersionless integrability. These results suggested a fully contact-invariant approach to dispersionless integrability.

Developing this approach was the key objective of E. Ferapontov's recent visit to Tromsø: in collaboration with B. Kruglikov, a general class of scalar second-order PDEs in 4D was investigated. It was proved (jointly with S. Berjawi and V. Novikov) that the requirement of half-flatness of the characteristic conformal structure on every solution implies that the PDE must belong to the Monge-Ampère class. Some partial classification results of Monge-Ampère equations with half-flat conformal structure were also obtained.

As part of the research programme, E. Ferapontov gave a series of lectures on various existing approaches to integrability (which were attended by staff, postdocs and PhD students). This in particular included the following topics:

1. Existing approaches to integrability in 1+1D. Example of KdV.
2. Integrable systems of hydrodynamic type in 1+1D. The method of hydrodynamic reductions in 2+1D.
3. Dispersive deformations of 2+1D dispersionless integrable systems.

It it a great pleasure to thank the University of Tromso (and my host Prof B. Kruglikov) for providing me with excellent working conditions.

References

[1] E.V. Ferapontov, K.R. Khusnutdinova, On the integrability of (2+1)-dimensional quasilinear systems, Comm. Math. Phys. **248** (2004) 187-206.

[2] E.V. Ferapontov, B. Kruglikov, Dispersionless integrable systems in 3D and Einstein-Weyl geometry, J. Diff. Geom. **97** (2014) 215-254.