A GRAVITATIONAL WALK OVER THE LAST CENTURIES

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Newton's law of universal gravitation

$$F = G\frac{m_1 m_2}{r^2}$$

dates back to 1687 and states that every object of mass m_1 attracts any object of mass m_2 by a force F proportional to the product of their masses and inversely proportional to the square of their distance r. Newton's equations are invariant under the Galilean group, that is the group of transformations of the 4-dimensional affine space $\mathbb{R}^3 \times \mathbb{R}$ of absolute space and time.

Describing different phenomena within the framework of a unified theory was and has remained the leading principle of physics. For instance, Maxwell unified the electricity and magnetism in his theory of the electromagnetic field. Maxwell's equations are not invariant by the Galilean group but rather by the Poincaré group $P_{\bar{0}} = \mathrm{SO}(3,1) \ltimes \mathbb{R}^{3,1}$ of affine transformations of 4-dimensional spacetime $\mathbb{R}^{3,1}$. It consists of the translations in spacetime and the group $\mathrm{SO}(3,1)$ of 4×4 matrices with determinant $+1$ which preserve a scalar product of signature $(3,1)$.

Einstein was the first to seriously take into account the Poincaré invariance of Maxwell's equations, ultimately providing a unified description of gravity as a geometric property of spacetime. The equations of General Relativity of Einstein superseded Newton's law and were published in 1915:

$$\mathrm{Ric}(X,Y) - \tfrac{1}{2}sg(X,Y) = T(X,Y)\ .$$

Here Ric and s are the Ricci and scalar curvatures of the gravitational potential g and T is the stress/energy tensor describing the distribution of matter and energy in spacetime. In mathematical terms, it is a system of second-order PDE for a Lorentzian metric g (the curved version of a scalar product of signature (3,1)).

In this context, the irreducible unitary representations of the Poincaré group

$$\rho : P_{\bar{0}} \longrightarrow U(H)$$

corresponds to relativistic quantum particles. They are indexed by mass and spin. The question about the existence of a larger group whose representations contain the representations of $P_{\bar{0}}$ with same mass but different spins was positively solved in 1975 by Haag-Lopuszanski-Sohnius. They discovered supersymmetry, a special symmetry unifying matter particles of half-integer spin (called fermions) and force-carrying particles of integer spin (called bosons).

Supergravity theories are of special interest – this is Einstein's General Relativity put in harmony with supersymmetry – and of particular relevance is 11-dimensional supergravity. This is a theory invariant by the supergroup extension P of the Poincaré group in 11-dimensions and its equations were discovered by Cremmer-Julia-Scherk in 1978:

$$\begin{aligned}\mathrm{Ric}(X,Y) &= \frac{1}{2}g(\imath_X F, \imath_Y F) - \frac{1}{6}\|F\|^2 g(X,Y)\ ,\\ d\star F &= \frac{1}{2}F\wedge F\ .\end{aligned}$$

Here we have a Lorentzian metric g on an 11-dimensional spacetime satisfying a second-order PDE of Einstein type and a closed differential 4-form F subject to a first-order PDE of Maxwell type. This system of PDE is obtained when the gravitino (the unique fermion of the corresponding irreducible representation of the superPoincaré group P) is set to zero, otherwise the equations are more involved.

A prominent and long-standing problem is the classification of supergravity backgrounds that are supersymmetric, i.e. solutions of the equations of 11-dimensional supergravity which preserve some supersymmetry. In collaboration with José Figueroa-O'Farrill at the University of Edinburgh, I showed that the equations of Einstein and Maxwell type are automatically satisfied if enough supersymmetry is preserved and established a one-to-one correspondence of highly-supersymmetric backgrounds and a special kind of purely algebraic structures. We also showed that much of the geometry of the highly-supersymmetric background can be recovered by the 4-form F.

We believe that such an approach will lead to a systematic construction of new highly-supersymmetric supergravity backgrounds and at the same time shed light on their geometric features.