Report on V.Rybalko's visit to The Arctic University of Norway, UiT (campus Narvik) in the Fall 2018

During V.Rybalko's visit to The Arctic University of Norway in September-October 2018 we continued our joint researches of boundary value problems for singularly perturbed elliptic equations aiming at understanding the asymptotic behavior of solutions when the parameter in front of higher order derivatives becomes very small. Our previous studies of spectral Dirichlet problem for singularly perturbed operators with oscillating coefficients showed that the asymptotic behavior of solutions in the leading term is described by an ergodic problem for Hamilton-Jacobi equation with an effective Hamiltonian expressed via solutions of a cell problem. Moreover, under some natural structure assumptions on the effective Hamiltonian we were able to describe the next term in the asymptotic expansion of solutions via local analysis near so-called Aubry set of the effective Hamiltonian [1], [2]. We also treated singularly perturbed problems with the Neumann boundary condition in [3]), in the case when coefficients of equations are not oscillating. On the other hand, as shown in [4], oscillations in coefficients lead to boundary layers in the case of Neumann or Fourier boundary conditions, and require compatibility of the oscillations periods with boundary.

In our present study we consider spectral problems for singularly perturbed operators with oscillating coefficients in thin cylinder imposing the Neumann boundary condition on the lateral surface and the Fourier condition on the bases of the cylinder. The analysis of these problems leads to a dimension reduction due to small thickness of the cylinder and reduction to the Hamilton-Jacobi equation(due to singularly perturbed equation. The main difficulty comes from boundary layers near the bases of the cylinder. Analysis of these boundary layers required us to develop a qualitative theory of the Steklov spectral problems in the half-cylinder, their perturbations (due to the presence of slow variable in the coefficients) and Fredholm type theory in the case of inhomogeneous problems. These results allow us to describe the asymptotic behavior of the first eigenpair of spectral problem, and under some conditions describe behavior of other eigenvalues near the border of the spectrum; we also believe that these results will be helpful in the analysis of other homogenization problems.

V. Rybalko also delivered a mini course on free boundary problems which included a talk on his own results as well as lectures on some classical free boundary problems as the obstacle problem, mean curvature flow and the Stefan problem. The theoretical studies of mathematical problems were also supplemented by practical exploring beautiful nature of Norway which included mountain hiking, fishing, picking various berries and mushrooms. The latter was especially successful as seen from the photo.

References

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[4] Barles, G., Da Lio, F., Lions, P.-L., Souganidis, P.E., Ergodic Problems and Periodic Homogenization for Fully Nonlinear Equations in Half-space Type Domains with Neumann Boundary Conditions, *Indiana University Mathematics Journal* **57** (5), (2008), 2355-2375.