Aritmetisk summa

ex.   5,   10,   15,   20,   25,   30 $(d=5)$

a_1, a_2, a_3, a_4, a_5, a_6

a_1 , a_1+d , a_1+2d, a_1+3d , a_1+4d, a_1+5d

$$+$$
$$2a_1+5d$$

$$+$$
$$2a_1+5d$$

$$+$$
$$2a_1+5d$$
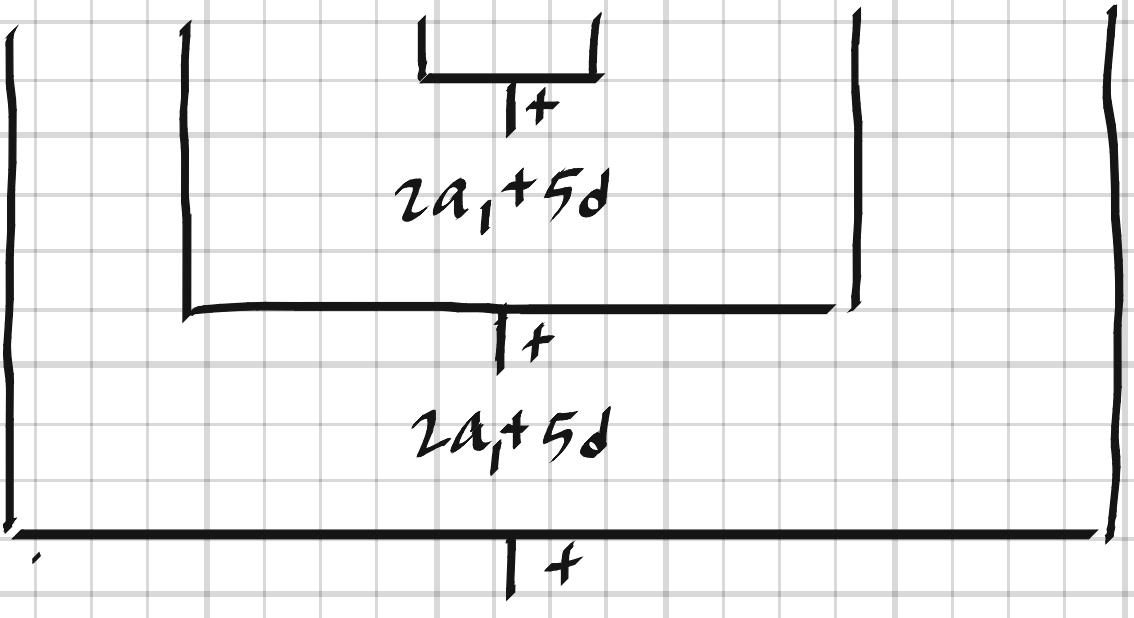
Summan, $S_6 = 3\cdot(2a_1+5d) = \dfrac{6}{2}(a_1+a_6)$

Ersätt 6 med n =>

$$S_n = \frac{n}{2}(a_1+a_n)$$

$$a = a_1, a_2, a_3, \ldots, a_n$$

$$d = a_2 - a_1 = a_3 - a_2 = \ldots = a_n - a_{n-1} = \frac{a_n - a_1}{n-1}$$

$$S_n = a_1 + a_2 + a_3 + \ldots + a_n =$$

$$= a_1 + (a_1 + d) + (a_1 + 2d) + \ldots + (a_1 + (n-1)d)$$

$$= n \cdot a_1 + (1 + 2 + 3 + \ldots + (n-1)) \cdot d$$

$$1 + 2 + 3 + \ldots + (n-1) = \frac{n(n-1)}{2} \implies$$

$$S_n = n \cdot a_1 + \frac{n(n-1)}{2} \cdot \frac{(a_n - a_1)}{n-1} =$$

$$= \frac{2n(n-1)a_1 + n(n-1)(a_n - a_1)}{2(n-1)} = \frac{2na_1 + n(a_n - a_1)}{2}$$

$$\boxed{S_n = \frac{n}{2}(a_1 + a_n)}$$