Inre derivata

ex. 1 $\quad y(x) = e^{kx}$

Substitutionen:

$$p = kx$$

$$\frac{dp}{dx} = k$$

Kedjeregeln:

$$\frac{dy}{dx} = \frac{dy}{dp} \cdot \frac{dp}{dx} = e^{p} \cdot k = e^{kx} \cdot k$$

$$\frac{dy}{dx} = k \cdot e^{kx}$$

ex. 2

$$y = a^{kx}$$

$$y = (e^{\ln a})^{kx} = e^{k \cdot \ln a \cdot x}$$

Substitusjon:

$$p = k \cdot \ln a \cdot x$$

$$\frac{dp}{dx} = k \cdot \ln a$$

Kjederegelen:

$$\frac{dy}{dx} = \frac{dy}{dp} \cdot \frac{dp}{dx} = e^{p} \cdot k \cdot \ln a = e^{k \cdot \ln a \cdot x} \cdot k \cdot \ln a = a^{kx} \cdot k \cdot \ln a$$

$$\frac{dy}{dx} = k \cdot \ln a \cdot a^{kx}$$

ex. 3 $\quad y(x) = (e^{x^2} + x^2)^5$

Substitutieregel:

$p = x^2$, $q = e^p + p$

$\frac{dp}{dx} = 2x$, $\frac{dq}{dp} = e^p + 1$

Kedjeregeln:

$$\frac{dy}{dx} = \frac{dy}{dq} \cdot \frac{dq}{dp} \cdot \frac{dp}{dx} = 5q^4 \cdot (e^p + 1) \cdot 2x =$$

$$= 5(e^p + p)^4 (e^p + 1) \cdot 2x = 5(e^{x^2} + x^2)^4 (e^{x^2} + 1) \cdot 2x$$

$$\frac{dy}{dx} = 10x (e^{x^2} + x^2)^4 (e^{x^2} + 1)$$

ex. 4 $$y(x) = \frac{5}{(\cos x - x^3)^2}$$

Substitutionen:

$$p = \cos x - x^3$$

$$\frac{dp}{dx} = -(\sin x + 3x^2)$$

Kedjeregeln:

$$\frac{dy}{dx} = \frac{dy}{dp} \cdot \frac{dp}{dx} = \frac{-10}{p^3} \cdot -(\sin x + 3x^2) =$$

$$= \frac{-10}{(\cos x - x^3)^3} \cdot -(\sin x + 3x^2)$$

$$\frac{dy}{dx} = \frac{10(\sin x + 3x^2)}{(\cos x - x^3)^3}$$