Eulers tal, e

$$f(x) = e^x \Rightarrow$$

$$f'(x) = \lim_{h \to 0} \frac{e^{x+h} - e^x}{h}$$

$$\lim_{h \to 0} \frac{e^{x+h} - e^x}{h} = \lim_{h \to 0} \frac{e^x(e^h - 1)}{h} = e^x \cdot \lim_{h \to 0} \left(\frac{e^h - 1}{h}\right)$$

L'Hôpitals regel $\Rightarrow$

$$\lim_{h \to 0} \frac{e^h - 1}{h} = \frac{\frac{d}{dh}(e^h - 1)\big|_{h=0}}{\frac{d}{dh}(h)\big|_{h=0}} = \frac{e^0 - 0}{1} = 1$$

$$\lim_{h \to 0} \frac{e^h - 1}{h} \cdot h = 1 \cdot h \Rightarrow$$

$$\lim_{h \to 0} (e^h - 1) = h \Rightarrow$$

$$e = \lim_{h \to 0} (1+h)^{\frac{1}{h}}$$

om $\frac{1}{h} = n \Rightarrow n \to \infty$ då $h \to 0 \Rightarrow$

$$\boxed{e = \lim_{n \to \infty} \left(1 + \frac{1}{n}\right)^n \approx 2{,}718}$$

n	e
1	2
2	$1{,}5^2 = 2{,}25$
10	$1{,}1^{10} = 2{,}59$
100	$1{,}01^{100} = 2{,}705$
1000	$1{,}001^{1000} = 2{,}717$