Derivatan av sin x

$$f(x) = \sin x$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$\frac{f(x+h) - f(x)}{h} = \frac{\sin(x+h) - \sin x}{h} =$$

$$= \frac{\sin x \cdot \cos h + \cos x \cdot \sin h - \sin x}{h} =$$

$$= \sin x \left(\frac{\cos h - 1}{h} \right) + \cos x \cdot \frac{\sin h}{h}$$

$$f'(x) = \lim_{h \to 0} \sin x \cdot \left(\frac{\cos h - 1}{h} \right) + \cos x \cdot \frac{\sin h}{h} =$$

$$= \sin x \cdot \lim_{h \to 0} \left(\frac{\cos h - 1}{h} \right) + \cos x \cdot \lim_{h \to 0} \frac{\sin h}{h}$$

L'Hôpitals regel =>

$$\begin{cases} \lim\limits_{h\to 0} \dfrac{\cos h - 1}{h} = \dfrac{\frac{d}{dh}(\cos h - 1)\big|_{h=0}}{\frac{d}{dh}(h)\big|_{h=0}} = \dfrac{-\sin h\,\big|_{h=0}}{1} = 0 \\[2em] \lim\limits_{h\to 0} \dfrac{\sin h}{h} = \dfrac{\frac{d}{dh}(\sin h)\big|_{h=0}}{\frac{d}{dh}(h)\big|_{h=0}} = \dfrac{\cos h\,\big|_{h=0}}{1} = 1 \end{cases}$$

=>

$$f'(x) = \sin x \cdot 0 + \cos x \cdot 1 = \cos x$$

$$\boxed{\frac{d}{dx}(\sin x) = \cos x}$$