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## List of integrals of trigonometric functions

The following is a list of integrals (antiderivative functions) of trigonometric functions. For antiderivatives involving both exponential and trigonometric functions, see List of integrals of exponential functions. For a complete list of antiderivative functions, see Lists of integrals. For the special antiderivatives involving trigonometric functions, see Trigonometric integral.

Generally, if the function $\sin x$ is any trigonometric function, and $\cos x$ is its derivative,

$$
\int a \cos n x d x=\frac{a}{n} \sin n x+C
$$

In all formulas the constant $a$ is assumed to be nonzero, and $C$ denotes the constant of integration.

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## Integrands involving only sine

$\int \sin a x d x=-\frac{1}{a} \cos a x+C$
$\int \sin ^{2} a x d x=\frac{x}{2}-\frac{1}{4 a} \sin 2 a x+C=\frac{x}{2}-\frac{1}{2 a} \sin a x \cos a x+C$
$\int \sin ^{3} a x d x=\frac{\cos 3 a x}{12 a}-\frac{3 \cos a x}{4 a}+C$
$\int x \sin ^{2} a x d x=\frac{x^{2}}{4}-\frac{x}{4 a} \sin 2 a x-\frac{1}{8 a^{2}} \cos 2 a x+C$
$\int x^{2} \sin ^{2} a x d x=\frac{x^{3}}{6}-\left(\frac{x^{2}}{4 a}-\frac{1}{8 a^{3}}\right) \sin 2 a x-\frac{x}{4 a^{2}} \cos 2 a x+C$
$\int x \sin a x d x=\frac{\sin a x}{a^{2}}-\frac{x \cos a x}{a}+C$
$\int\left(\sin b_{1} x\right)\left(\sin b_{2} x\right) d x=\frac{\sin \left(\left(b_{2}-b_{1}\right) x\right)}{2\left(b_{2}-b_{1}\right)}-\frac{\sin \left(\left(b_{1}+b_{2}\right) x\right)}{2\left(b_{1}+b_{2}\right)}+C \quad\left(\right.$ for $\left.\left|b_{1}\right| \neq\left|b_{2}\right|\right)$
$\int \sin ^{n} a x d x=-\frac{\sin ^{n-1} a x \cos a x}{n a}+\frac{n-1}{n} \int \sin ^{n-2} a x d x \quad($ for $n>0)$
$\int \frac{d x}{\sin a x}=-\frac{1}{a} \ln |\csc a x+\cot a x|+C$
$\int \frac{d x}{\sin ^{n} a x}=\frac{\cos a x}{a(1-n) \sin ^{n-1} a x}+\frac{n-2}{n-1} \int \frac{d x}{\sin ^{n-2} a x} \quad($ for $n>1)$

$$
\begin{aligned}
& \int x^{n} \sin a x d x=-\frac{x^{n}}{a} \cos a x+\frac{n}{a} \int x^{n-1} \cos a x d x \\
& =\sum_{k=0}^{2 k \leq n}(-1)^{k+1} \frac{x^{n-2 k}}{a^{1+2 k}} \frac{n!}{(n-2 k)!} \cos a x+\sum_{k=0}^{2 k+1 \leq n}(-1)^{k} \frac{x^{n-1-2 k}}{a^{2+2 k}} \frac{n!}{(n-2 k-1)!} \sin a x \\
& =-\sum_{k=0}^{n} \frac{x^{n-k}}{a^{1+k}} \frac{n!}{(n-k)!} \cos \left(a x+k \frac{\pi}{2}\right) \quad(\text { for } n>0) \\
& \int \frac{\sin a x}{x} d x=\sum_{n=0}^{\infty}(-1)^{n} \frac{(a x)^{2 n+1}}{(2 n+1) \cdot(2 n+1)!}+C \\
& \int \frac{\sin a x}{x^{n}} d x=-\frac{\sin a x}{(n-1) x^{n-1}}+\frac{a}{n-1} \int \frac{\cos a x}{x^{n-1}} d x \\
& \int \sin \left(a x^{2}+b x+c\right) d x=\left\{\begin{array}{l}
\sqrt{a} \sqrt{\frac{\pi}{2}} \cos \left(\frac{b^{2}-4 a c}{4 a}\right) S\left(\frac{2 a x+b}{\sqrt{2 a \pi}}\right)+\sqrt{a} \sqrt{\frac{\pi}{2}} \sin \left(\frac{b^{2}-4 a c}{4 a}\right) C\left(\frac{2 a x+b}{\sqrt{2 a \pi}}\right) \text { to } b^{2}-4 a c>0 \\
\sqrt{a} \sqrt{\frac{\pi}{2}} \cos \left(\frac{b^{2}-4 a c}{4 a}\right) S\left(\frac{2 a x+b}{\sqrt{2 a \pi}}\right)-\sqrt{a} \sqrt{\frac{\pi}{2}} \sin \left(\frac{b^{2}-4 a c}{4 a}\right) C\left(\frac{2 a x+b}{\sqrt{2 a \pi}}\right) \text { to } b^{2}-4 a c<0
\end{array} \text { for } a \neq 0, a>0\right. \\
& \int \frac{d x}{1 \pm \sin a x}=\frac{1}{a} \tan \left(\frac{a x}{2} \mp \frac{\pi}{4}\right)+C \\
& \int \frac{x d x}{1+\sin a x}=\frac{x}{a} \tan \left(\frac{a x}{2}-\frac{\pi}{4}\right)+\frac{2}{a^{2}} \ln \left|\cos \left(\frac{a x}{2}-\frac{\pi}{4}\right)\right|+C \\
& \int \frac{x d x}{1-\sin a x}=\frac{x}{a} \cot \left(\frac{\pi}{4}-\frac{a x}{2}\right)+\frac{2}{a^{2}} \ln \left|\sin \left(\frac{\pi}{4}-\frac{a x}{2}\right)\right|+C \\
& \int \frac{\sin a x d x}{1 \pm \sin a x}= \pm x+\frac{1}{a} \tan \left(\frac{\pi}{4} \mp \frac{a x}{2}\right)+C
\end{aligned}
$$

## Integrands involving only cosine

$$
\begin{aligned}
& \int \cos a x d x=\frac{1}{a} \sin a x+C \\
& \int \cos ^{2} a x d x=\frac{x}{2}+\frac{1}{4 a} \sin 2 a x+C=\frac{x}{2}+\frac{1}{2 a} \sin a x \cos a x+C \\
& \int \cos ^{n} a x d x=\frac{\cos ^{n-1} a x \sin a x}{n a}+\frac{n-1}{n} \int \cos ^{n-2} a x d x \quad(\text { for } n>0) \\
& \int x \cos a x d x=\frac{\cos a x}{a^{2}}+\frac{x \sin a x}{a}+C \\
& \int x^{2} \cos ^{2} a x d x=\frac{x^{3}}{6}+\left(\frac{x^{2}}{4 a}-\frac{1}{8 a^{3}}\right) \sin 2 a x+\frac{x}{4 a^{2}} \cos 2 a x+C \\
& \int x^{n} \cos a x d x=\frac{x^{n} \sin a x}{a}-\frac{n}{a} \int x^{n-1} \sin a x d x \\
& =\sum_{k=0}^{2 k+1 \leq n}(-1)^{k} \frac{x^{n-2 k-1}}{a^{2+2 k}} \frac{n!}{(n-2 k-1)!} \cos a x+\sum_{k=0}^{2 k \leq n}(-1)^{k} \frac{x^{n-2 k}}{a^{1+2 k}} \frac{n!}{(n-2 k)!} \sin a x \\
& =\sum_{k=0}^{n}(-1)^{\lfloor k / 2\rfloor} \frac{x^{n-k}}{a^{1+k}} \frac{n!}{(n-k)!} \cos \left(a x-\frac{(-1)^{k}+1}{2} \frac{\pi}{2}\right) \\
& =\sum_{k=0}^{n} \frac{x^{n-k}}{a^{1+k}} \frac{n!}{(n-k)!} \sin \left(a x+k \frac{\pi}{2}\right) \quad(\text { for } n>0) \\
& \int \frac{\cos a x}{x} d x=\ln |a x|+\sum_{k=1}^{\infty}(-1)^{k} \frac{(a x)^{2 k}}{2 k \cdot(2 k)!}+C \\
& \int \frac{\cos a x}{x^{n}} d x=-\frac{\cos a x}{(n-1) x^{n-1}}-\frac{a}{n-1} \int \frac{\sin a x}{x^{n-1}} d x \quad(\text { for } n \neq 1) \\
& \int \frac{d x}{\cos a x}=\frac{1}{a} \ln \left|\tan \left(\frac{a x}{2}+\frac{\pi}{4}\right)\right|+C \\
& \int \frac{d x}{\cos ^{n} a x}=\frac{\sin a x}{a(n-1) \cos ^{n-1} a x}+\frac{n-2}{n-1} \int \frac{d x}{\cos ^{n-2} a x} \quad(\text { for } n>1) \\
& \int \frac{d x}{1+\cos a x}=\frac{1}{a} \tan \frac{a x}{2}+C \\
& \int \frac{d x}{1-\cos a x}=-\frac{1}{a} \cot \frac{a x}{2}+C
\end{aligned}
$$

$\int \frac{x d x}{1+\cos a x}=\frac{x}{a} \tan \frac{a x}{2}+\frac{2}{a^{2}} \ln \left|\cos \frac{a x}{2}\right|+C$
$\int \frac{x d x}{1-\cos a x}=-\frac{x}{a} \cot \frac{a x}{2}+\frac{2}{a^{2}} \ln \left|\sin \frac{a x}{2}\right|+C$
$\int \frac{\cos a x d x}{1+\cos a x}=x-\frac{1}{a} \tan \frac{a x}{2}+C$
$\int \frac{\cos a x d x}{1-\cos a x}=-x-\frac{1}{a} \cot \frac{a x}{2}+C$
$\int\left(\cos a_{1} x\right)\left(\cos a_{2} x\right) d x=\frac{\sin \left(\left(a_{2}-a_{1}\right) x\right)}{2\left(a_{2}-a_{1}\right)}+\frac{\sin \left(\left(a_{2}+a_{1}\right) x\right)}{2\left(a_{2}+a_{1}\right)}+C \quad\left(\right.$ for $\left.\left|a_{1}\right| \neq\left|a_{2}\right|\right)$

## Integrands involving only tangent

$\int \tan a x d x=-\frac{1}{a} \ln |\cos a x|+C=\frac{1}{a} \ln |\sec a x|+C$
$\int \tan ^{2} x d x=\tan x-x+C$
$\int \tan ^{n} a x d x=\frac{1}{a(n-1)} \tan ^{n-1} a x-\int \tan ^{n-2} a x d x \quad($ for $n \neq 1)$
$\int \frac{d x}{q \tan a x+p}=\frac{1}{p^{2}+q^{2}}\left(p x+\frac{q}{a} \ln |q \sin a x+p \cos a x|\right)+C \quad\left(\right.$ for $\left.p^{2}+q^{2} \neq 0\right)$
$\int \frac{d x}{\tan a x \pm 1}= \pm \frac{x}{2}+\frac{1}{2 a} \ln |\sin a x \pm \cos a x|+C$
$\int \frac{\tan a x d x}{\tan a x \pm 1}=\frac{x}{2} \mp \frac{1}{2 a} \ln |\sin a x \pm \cos a x|+C$

## Integrands involving only secant

See Integral of the secant function.
$\int \sec a x d x=\frac{1}{a} \ln |\sec a x+\tan a x|+C=\frac{1}{a} \ln \left|\tan \left(\frac{a x}{2}+\frac{\pi}{4}\right)\right|+C=\frac{1}{a} \operatorname{artanh}(\sin a x)+C$
$\int \sec ^{2} x d x=\tan x+C$
$\int \sec ^{3} x d x=\frac{1}{2} \sec x \tan x+\frac{1}{2} \ln |\sec x+\tan x|+C$.
$\int \sec ^{n} a x d x=\frac{\sec ^{n-2} a x \tan a x}{a(n-1)}+\frac{n-2}{n-1} \int \sec ^{n-2} a x d x \quad($ for $n \neq 1)$
$\int \frac{d x}{\sec x+1}=x-\tan \frac{x}{2}+C$
$\int \frac{d x}{\sec x-1}=-x-\cot \frac{x}{2}+C$

## Integrands involving only cosecant

$\int \csc a x d x=-\frac{1}{a} \ln |\csc a x+\cot a x|+C=\frac{1}{a} \ln |\csc a x-\cot a x|+C=\frac{1}{a} \ln \left|\tan \left(\frac{a x}{2}\right)\right|+C$
$\int \csc ^{2} x d x=-\cot x+C$
$\int \csc ^{3} x d x=-\frac{1}{2} \csc x \cot x-\frac{1}{2} \ln |\csc x+\cot x|+C=-\frac{1}{2} \csc x \cot x+\frac{1}{2} \ln |\csc x-\cot x|+C$
$\int \csc ^{n} a x d x=-\frac{\csc ^{n-2} a x \cot a x}{a(n-1)}+\frac{n-2}{n-1} \int \csc ^{n-2} a x d x \quad($ for $n \neq 1)$
$\int \frac{d x}{\csc x+1}=x-\frac{2}{\cot \frac{x}{2}+1}+C$
$\int \frac{d x}{\csc x-1}=-x+\frac{2}{\cot \frac{x}{2}-1}+C$

## Integrands involving only cotangent

$\int \cot a x d x=\frac{1}{a} \ln |\sin a x|+C$
$\int \cot ^{2} x d x=-\cot x-x+C$
$\int \cot ^{n} a x d x=-\frac{1}{a(n-1)} \cot ^{n-1} a x-\int \cot ^{n-2} a x d x \quad($ for $n \neq 1)$
$\int \frac{d x}{1+\cot a x}=\int \frac{\tan a x d x}{\tan a x+1}=\frac{x}{2}-\frac{1}{2 a} \ln |\sin a x+\cos a x|+C$
$\int \frac{d x}{1-\cot a x}=\int \frac{\tan a x d x}{\tan a x-1}=\frac{x}{2}+\frac{1}{2 a} \ln |\sin a x-\cos a x|+C$

## Integrands involving both sine and cosine

An integral that is a rational function of the sine and cosine can be evaluated using Bioche's rules.
$\int \frac{d x}{\cos a x \pm \sin a x}=\frac{1}{a \sqrt{2}} \ln \left|\tan \left(\frac{a x}{2} \pm \frac{\pi}{8}\right)\right|+C$
$\int \frac{d x}{(\cos a x \pm \sin a x)^{2}}=\frac{1}{2 a} \tan \left(a x \mp \frac{\pi}{4}\right)+C$
$\int \frac{d x}{(\cos x+\sin x)^{n}}=\frac{1}{2(n-1)}\left(\frac{\sin x-\cos x}{(\cos x+\sin x)^{n-1}}+(n-2) \int \frac{d x}{(\cos x+\sin x)^{n-2}}\right)$
$\int \frac{\cos a x d x}{\cos a x+\sin a x}=\frac{x}{2}+\frac{1}{2 a} \ln |\sin a x+\cos a x|+C$
$\int \frac{\cos a x d x}{\cos a x-\sin a x}=\frac{x}{2}-\frac{1}{2 a} \ln |\sin a x-\cos a x|+C$
$\int \frac{\sin a x d x}{\cos a x+\sin a x}=\frac{x}{2}-\frac{1}{2 a} \ln |\sin a x+\cos a x|+C$
$\int \frac{\sin a x d x}{\cos a x-\sin a x}=-\frac{x}{2}-\frac{1}{2 a} \ln |\sin a x-\cos a x|+C$
$\int \frac{\cos a x d x}{(\sin a x)(1+\cos a x)}=-\frac{1}{4 a} \tan ^{2} \frac{a x}{2}+\frac{1}{2 a} \ln \left|\tan \frac{a x}{2}\right|+C$
$\int \frac{\cos a x d x}{(\sin a x)(1-\cos a x)}=-\frac{1}{4 a} \cot ^{2} \frac{a x}{2}-\frac{1}{2 a} \ln \left|\tan \frac{a x}{2}\right|+C$
$\int \frac{\sin a x d x}{(\cos a x)(1+\sin a x)}=\frac{1}{4 a} \cot ^{2}\left(\frac{a x}{2}+\frac{\pi}{4}\right)+\frac{1}{2 a} \ln \left|\tan \left(\frac{a x}{2}+\frac{\pi}{4}\right)\right|+C$
$\int \frac{\sin a x d x}{(\cos a x)(1-\sin a x)}=\frac{1}{4 a} \tan ^{2}\left(\frac{a x}{2}+\frac{\pi}{4}\right)-\frac{1}{2 a} \ln \left|\tan \left(\frac{a x}{2}+\frac{\pi}{4}\right)\right|+C$
$\int(\sin a x)(\cos a x) d x=\frac{1}{2 a} \sin ^{2} a x+C$
$\int\left(\sin a_{1} x\right)\left(\cos a_{2} x\right) d x=-\frac{\cos \left(\left(a_{1}-a_{2}\right) x\right)}{2\left(a_{1}-a_{2}\right)}-\frac{\cos \left(\left(a_{1}+a_{2}\right) x\right)}{2\left(a_{1}+a_{2}\right)}+C \quad\left(\right.$ for $\left.\left|a_{1}\right| \neq\left|a_{2}\right|\right)$
$\int\left(\sin ^{n} a x\right)(\cos a x) d x=\frac{1}{a(n+1)} \sin ^{n+1} a x+C \quad($ for $n \neq-1)$
$\int(\sin a x)\left(\cos ^{n} a x\right) d x=-\frac{1}{a(n+1)} \cos ^{n+1} a x+C \quad($ for $n \neq-1)$

$$
\begin{aligned}
& \int\left(\sin ^{n} a x\right)\left(\cos ^{m} a x\right) d x=-\frac{\left(\sin ^{n-1} a x\right)\left(\cos ^{m+1} a x\right)}{a(n+m)}+\frac{n-1}{n+m} \int\left(\sin ^{n-2} a x\right)\left(\cos ^{m} a x\right) d x \quad(\text { for } m, n>0) \\
& =\frac{\left(\sin ^{n+1} a x\right)\left(\cos ^{m-1} a x\right)}{a(n+m)}+\frac{m-1}{n+m} \int\left(\sin ^{n} a x\right)\left(\cos ^{m-2} a x\right) d x \quad(\text { for } m, n>0) \\
& \int \frac{d x}{(\sin a x)(\cos a x)}=\frac{1}{a} \ln |\tan a x|+C \\
& \int \frac{d x}{(\sin a x)\left(\cos ^{n} a x\right)}=\frac{1}{a(n-1) \cos ^{n-1} a x}+\int \frac{d x}{(\sin a x)\left(\cos ^{n-2} a x\right)} \quad(\text { for } n \neq 1) \\
& \int \frac{d x}{\left(\sin ^{n} a x\right)(\cos a x)}=-\frac{1}{a(n-1) \sin ^{n-1} a x}+\int \frac{d x}{\left(\sin ^{n-2} a x\right)(\cos a x)} \quad(\text { for } n \neq 1) \\
& \int \frac{\sin a x d x}{\cos ^{n} a x}=\frac{1}{a(n-1) \cos ^{n-1} a x}+C \quad(\text { for } n \neq 1) \\
& \int \frac{\sin ^{2} a x d x}{\cos a x}=-\frac{1}{a} \sin a x+\frac{1}{a} \ln \left|\tan \left(\frac{\pi}{4}+\frac{a x}{2}\right)\right|+C \\
& \int \frac{\sin ^{2} a x d x}{\cos ^{n} a x}=\frac{\sin a x}{a(n-1) \cos ^{n-1} a x}-\frac{1}{n-1} \int \frac{d x}{\cos ^{n-2} a x} \quad(\text { for } n \neq 1) \\
& \int \frac{\sin ^{2} x}{1+\cos ^{2} x} d x=\sqrt{2} \operatorname{arctangant}\left(\frac{\tan x}{\sqrt{2}}\right)-x \quad(\text { for } \mathrm{x} \text { in }]-\frac{\pi}{2} ;+\frac{\pi}{2}[) \\
& \left.=\sqrt{2} \operatorname{arctangant}\left(\frac{\tan x}{\sqrt{2}}\right)-\operatorname{arctangant}(\tan x) \quad \text { (this time } \mathrm{x} \text { being any real number }\right) \\
& \int \frac{\sin ^{n} a x d x}{\cos a x}=-\frac{\sin ^{n-1} a x}{a(n-1)}+\int \frac{\sin ^{n-2} a x d x}{\cos a x} \quad(\text { for } n \neq 1) \\
& \int \frac{\sin ^{n} a x d x}{\cos ^{m} a x}= \begin{cases}\frac{\sin ^{n+1} a x}{a(m-1) \cos ^{m-1} a x}-\frac{n-m+2}{m-1} \int \frac{\sin ^{n} a x d x}{\cos ^{m-2} a x} & (\text { for } m \neq 1) \\
\frac{\sin ^{n-1} a x}{a(m-1) \cos ^{m-1} a x}-\frac{n-1}{m-1} \int \frac{\sin ^{n-2} a x d x}{\cos ^{m-2} a x} & (\text { for } m \neq 1) \\
-\frac{\sin ^{n-1} a x}{a(n-m) \cos ^{m-1} a x}+\frac{n-1}{n-m} \int \frac{\sin ^{n-2} a x d x}{\cos ^{m} a x} & (\text { for } m \neq n)\end{cases} \\
& \int \frac{\cos a x d x}{\sin ^{n} a x}=-\frac{1}{a(n-1) \sin ^{n-1} a x}+C \quad(\text { for } n \neq 1) \\
& \int \frac{\cos ^{2} a x d x}{\sin a x}=\frac{1}{a}\left(\cos a x+\ln \left|\tan \frac{a x}{2}\right|\right)+C \\
& \int \frac{\cos ^{2} a x d x}{\sin ^{n} a x}=-\frac{1}{n-1}\left(\frac{\cos a x}{a \sin ^{n-1} a x}+\int \frac{d x}{\sin ^{n-2} a x}\right) \quad(\text { for } n \neq 1) \\
& \int \frac{\cos ^{n} a x d x}{\sin ^{m} a x}= \begin{cases}-\frac{\cos ^{n+1} a x}{a(m-1) \sin ^{m-1} a x}-\frac{n-m+2}{m-1} \int \frac{\cos ^{n} a x d x}{\sin ^{m-2} a x} & (\text { for } m \neq 1) \\
-\frac{\cos ^{n-1} a x}{a(m-1) \sin ^{m-1} a x}-\frac{n-1}{m-1} \int \frac{\cos ^{n-2} a x d x}{\sin ^{m-2} a x} & (\text { for } m \neq 1) \\
\frac{\cos ^{n-1} a x}{a(n-m) \sin ^{m-1} a x}+\frac{n-1}{n-m} \int \frac{\cos ^{n-2} a x d x}{\sin ^{m} a x} & (\text { for } m \neq n)\end{cases}
\end{aligned}
$$

## Integrands involving both sine and tangent

$$
\begin{aligned}
& \int(\sin a x)(\tan a x) d x=\frac{1}{a}(\ln |\sec a x+\tan a x|-\sin a x)+C \\
& \int \frac{\tan ^{n} a x d x}{\sin ^{2} a x}=\frac{1}{a(n-1)} \tan ^{n-1}(a x)+C \quad(\text { for } n \neq 1)
\end{aligned}
$$

## Integrand involving both cosine and tangent

$$
\int \frac{\tan ^{n} a x d x}{\cos ^{2} a x}=\frac{1}{a(n+1)} \tan ^{n+1} a x+C \quad(\text { for } n \neq-1)
$$

## Integrand involving both sine and cotangent

$\int \frac{\cot ^{n} a x d x}{\sin ^{2} a x}=-\frac{1}{a(n+1)} \cot ^{n+1} a x+C \quad($ for $n \neq-1)$

## Integrand involving both cosine and cotangent

$$
\int \frac{\cot ^{n} a x d x}{\cos ^{2} a x}=\frac{1}{a(1-n)} \tan ^{1-n} a x+C \quad(\text { for } n \neq 1)
$$

## Integrand involving both secant and tangent

$\int(\sec x)(\tan x) d x=\sec x+C$

## Integrand involving both cosecant and cotangent

$\int(\csc x)(\cot x) d x=-\csc x+C$

## Integrals in a quarter period

$\int_{0}^{\frac{\pi}{2}} \sin ^{n} x d x=\int_{0}^{\frac{\pi}{2}} \cos ^{n} x d x= \begin{cases}\frac{n-1}{n} \cdot \frac{n-3}{n-2} \cdots \frac{3}{4} \cdot \frac{1}{2} \cdot \frac{\pi}{2}, & \text { if } n \text { is even } \\ \frac{n-1}{n} \cdot \frac{n-3}{n-2} \cdots \frac{4}{5} \cdot \frac{2}{3}, & \text { if } n \text { is odd and more than } 1 \\ 1, & \text { if } n=1\end{cases}$

## Integrals with symmetric limits

$\int_{-c}^{c} \sin x d x=0$
$\int_{-c}^{c} \cos x d x=2 \int_{0}^{c} \cos x d x=2 \int_{-c}^{0} \cos x d x=2 \sin c$
$\int_{-c}^{c} \tan x d x=0$
$\int_{-\frac{a}{2}}^{\frac{a}{2}} x^{2} \cos ^{2} \frac{n \pi x}{a} d x=\frac{a^{3}\left(n^{2} \pi^{2}-6\right)}{24 n^{2} \pi^{2}} \quad($ for $n=1,3,5 \ldots)$
$\int_{\frac{-a}{2}}^{\frac{a}{2}} x^{2} \sin ^{2} \frac{n \pi x}{a} d x=\frac{a^{3}\left(n^{2} \pi^{2}-6(-1)^{n}\right)}{24 n^{2} \pi^{2}}=\frac{a^{3}}{24}\left(1-6 \frac{(-1)^{n}}{n^{2} \pi^{2}}\right) \quad($ for $n=1,2,3, \ldots)$

## Integral over a full circle

$$
\begin{array}{ll}
\int_{0}^{2 \pi} \sin ^{2 m+1} x \cos ^{n} x d x=0 & n, m \in \mathbb{Z} \\
\int_{0}^{2 \pi} \sin ^{m} x \cos ^{2 n+1} x d x=0 & n, m \in \mathbb{Z}
\end{array}
$$

## See also

- Trigonometric integral

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