

# Introduktion









DANMARKS FRIE  
FORSKNINGSFOND

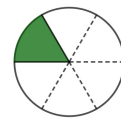
Pernille Ladegaard Pedersen

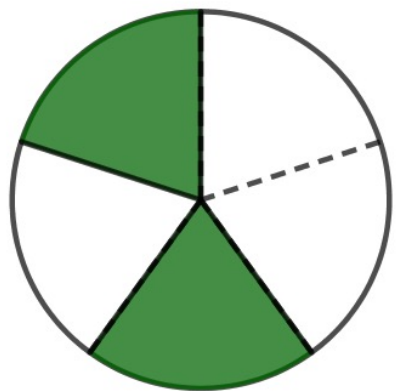
Ålborg Universitet/VIA

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# Program

-  Introduktion
-  Hvorfor er brøker vigtige?
-  Hvordan forstår vi en brøk?
-  Hvordan kan vi forklare forskellige heltalsdistraktorer?
-  Typer af ækvivalens
-  Opmærksomhedspunkter i undervisningen





Hvorfor er brøker vigtige?

# Hvorfor er brøker vigtige?

- Forståelse af brøk størrelse er en prædikter for senere brøk-aritmetik og samlet matematisk forståelse (Siegler et al., 2011).
- Sammenhæng til elevens algebra-forståelse (Booth and Newton, 2012)
- De lavest præsterende elever udvikler sig ikke (det samme i 6. og i 8. klasse) (Siegler and Pyke, 2013)





Hvordan forstår vi en brøk?



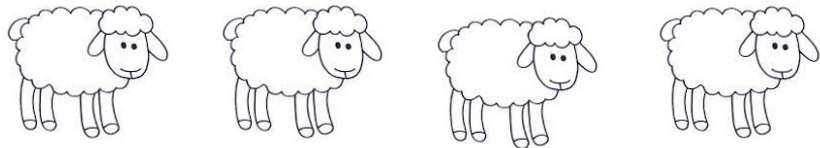
# Hvad er en brøk?

- Brøknotationen kan defineres som  $\frac{a}{b}$ ,  $b \neq 0$
- Brøker er et todelt symbol med en tæller og en nævner adskilt af en streg
- Alle rationale tal kan skrives som brøk, men ikke alle brøker er rationale tal fx  $\frac{3}{4}$  er et rationalt tal, men  $\frac{1}{\sqrt{2}}$  er ikke

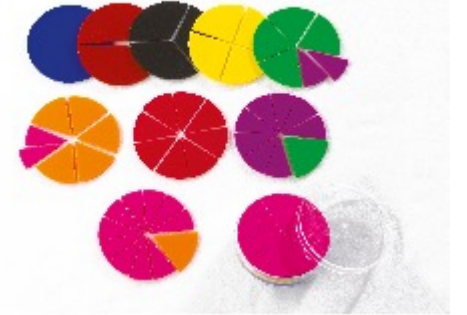
**I det her oplæg : rationale tal skrevet som en brøk**



# Naturlige tal



# Rationale tal



	<b>Naturlig tal (uden 0)</b>
Symbol/repræsentation	Et tal fx 8 eller 82
Orden/density	Rækkefølge –tæl fx 1, 2, 3 Efter et tal kommer et bestemt tal Der er ikke tal imellem
Addition - subtraktion	Man kan tælle frem eller tilbage i sekvensen
Multiplikation	Resultatet bliver altid større
Division	Resultat er altid mindre

Differences between natural numbers and fractions Stafylidou & Vosniadou (2004)

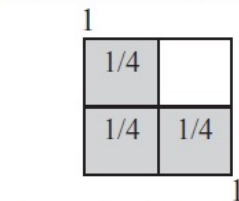
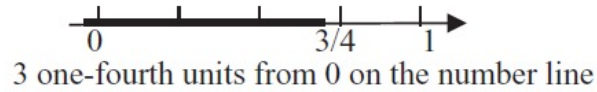
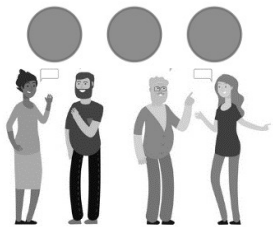
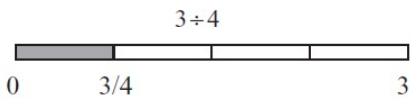
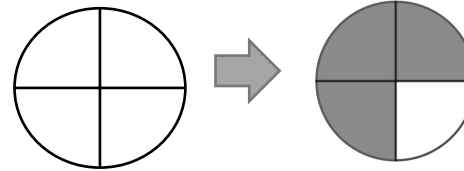
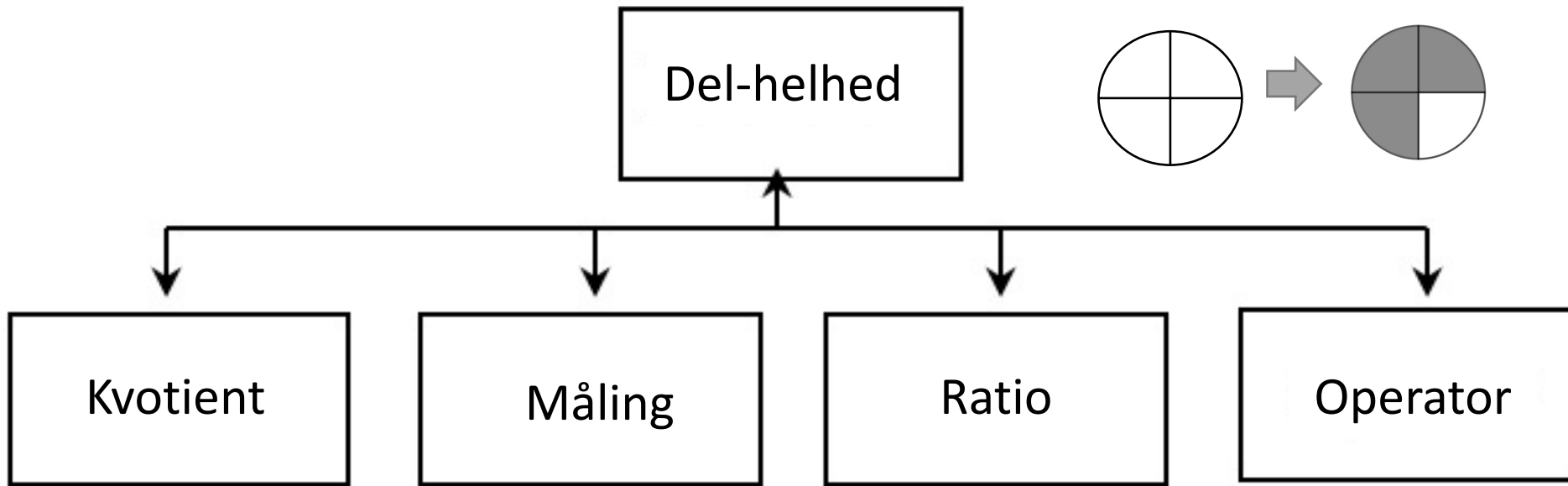




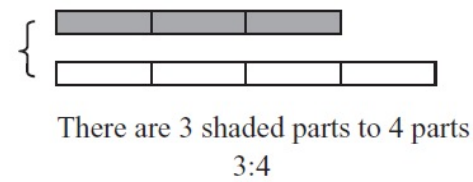
# Kieren's framework

$$\frac{3}{4}$$

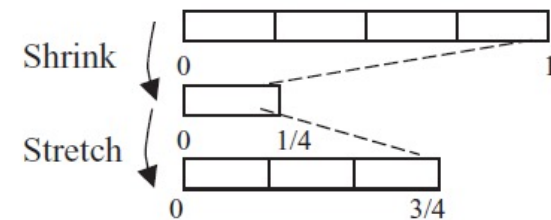
$$\frac{3}{4}$$



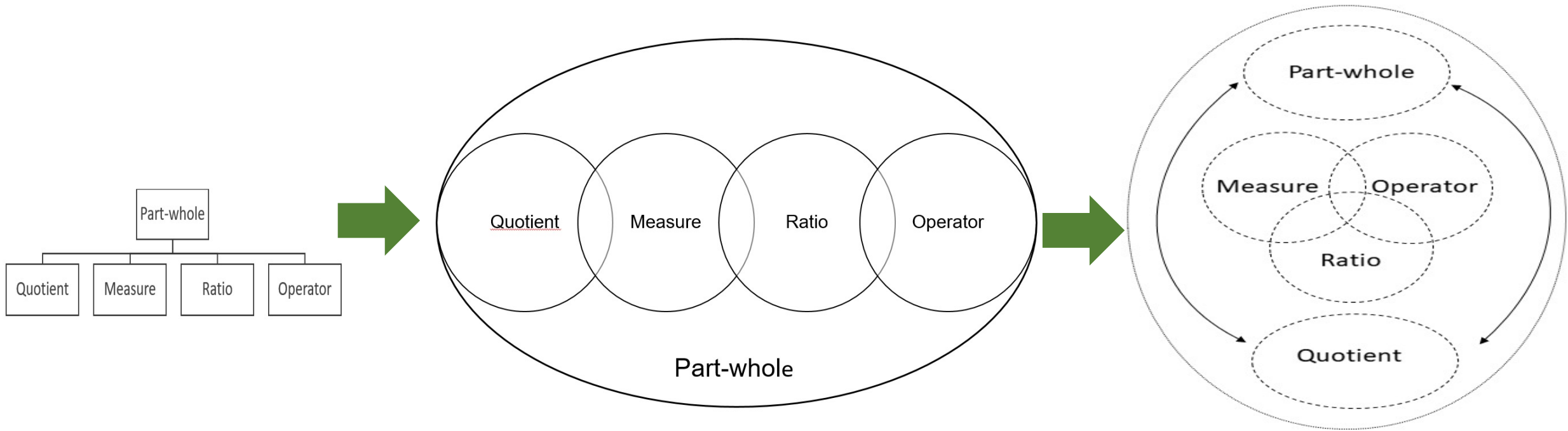
3 one-fourth units of a given area



## Rational tal



# Hvilken subkonstruktion er vigtigst?



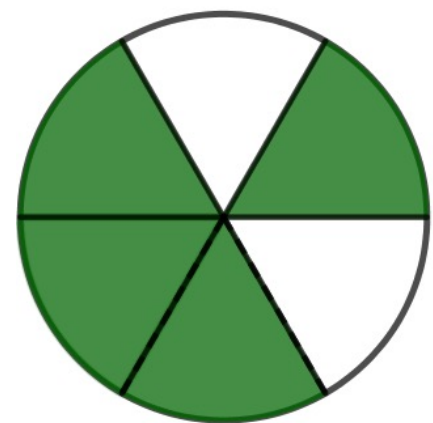
Mit udgangspunkt

Lidt klogere

I dag

**Hvorfor har vi brug for brøker?**





Hvordan kan vi forklare  
forskellige  
heltalsdistraktorer?

# Hvad vil du vurdere er sværest?

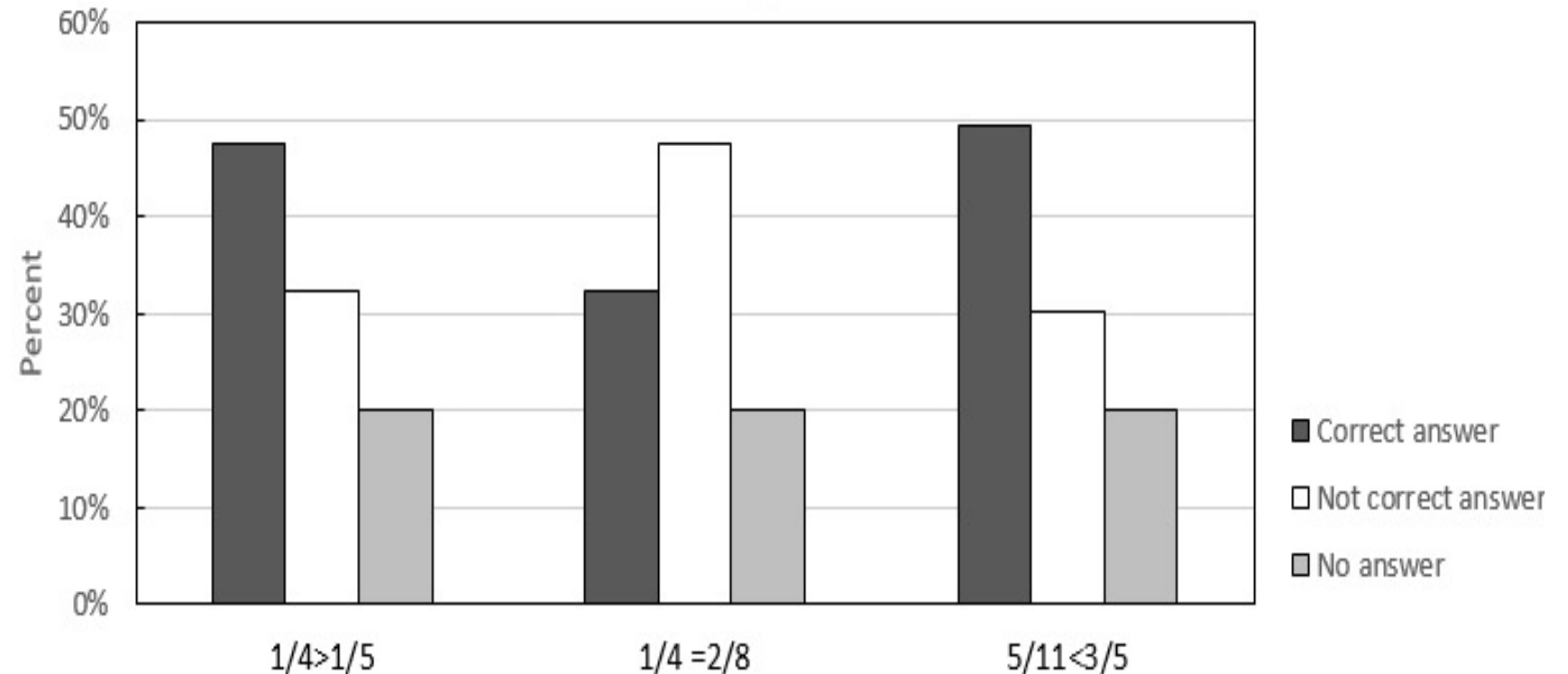
Indsæt det manglende tegn:  $>$ ,  $<$  eller  $=$ .

Tryk på pilen.

$$\frac{1}{4} \quad \boxed{>} \quad \frac{1}{5}$$

$$\frac{1}{4} \quad \boxed{=} \quad \frac{2}{8}$$

$$\frac{5}{11} \quad \boxed{<} \quad \frac{3}{5}$$



Hvorfor er ækvivalensforståelsen vigtig?

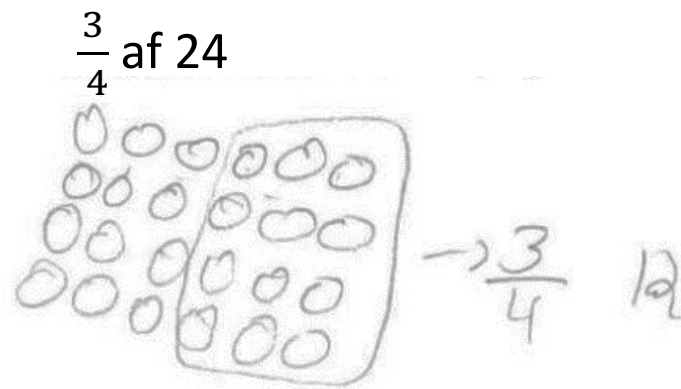
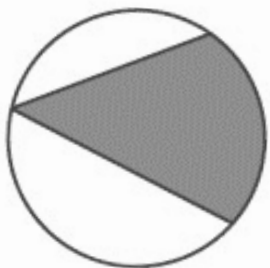


# Forskellige fejl

$$\frac{1}{4} > \frac{1}{3}$$

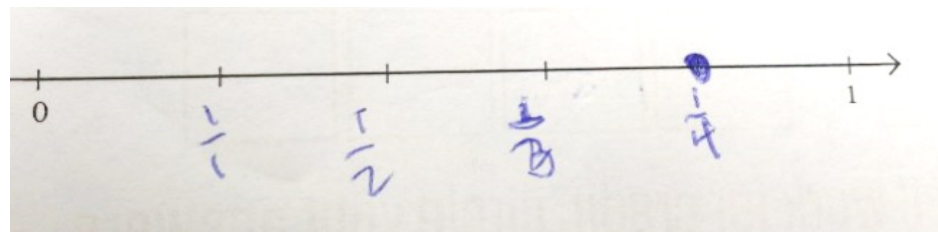
$$1\frac{3}{4} = \frac{13}{4}$$

Hvor er  $\frac{1}{3}$  farvet?

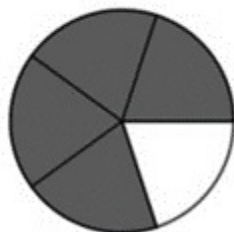


$$\frac{1}{4} \neq \frac{2}{8}$$

Skriv brøken



Hvor stor en brøkdel er farvet?

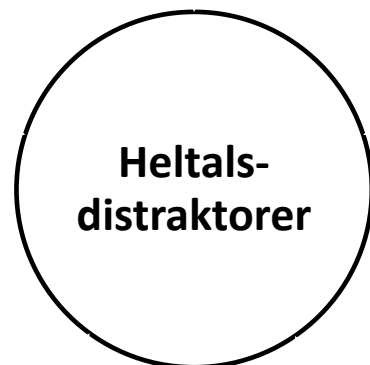


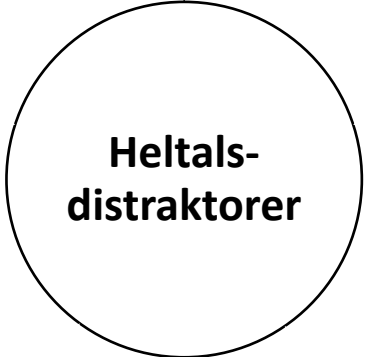
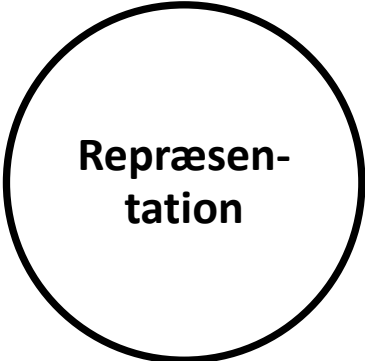
$$\frac{4}{1}$$

$$\frac{7}{4} < 1$$

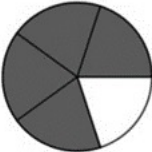


# Heltalsdistraktorer





Hvor stor en brøkdel er farvet?



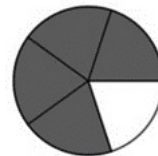
$$\frac{4}{1}$$

~~$$\frac{3}{4}$$~~

$$\frac{3}{4}$$



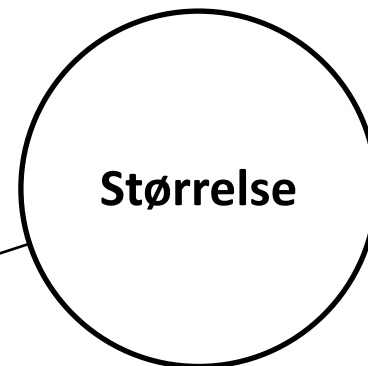
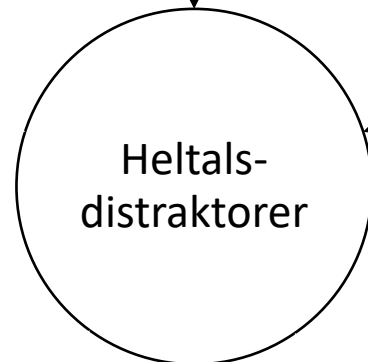
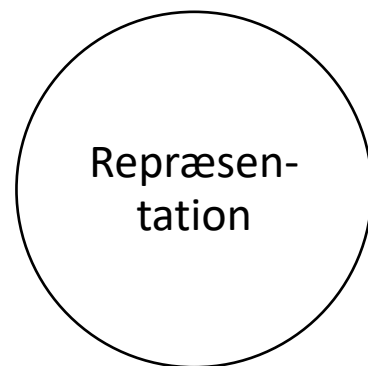
Hvor stor en brøkdel er farvet?



$$\frac{4}{1}$$

~~$$\frac{3}{4}$$~~

$$\frac{3}{4}$$



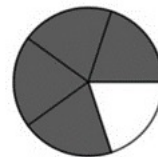
$$\frac{1}{4} > \frac{1}{3}$$

$$\frac{5}{11} < \frac{3}{5}$$





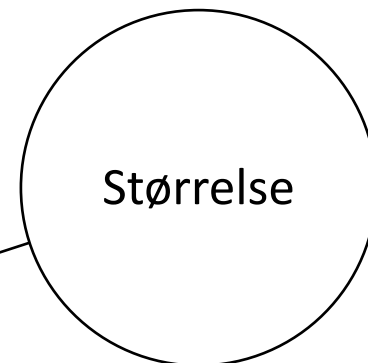
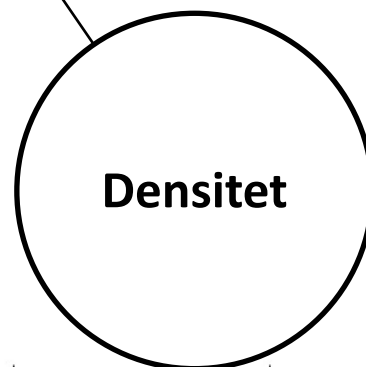
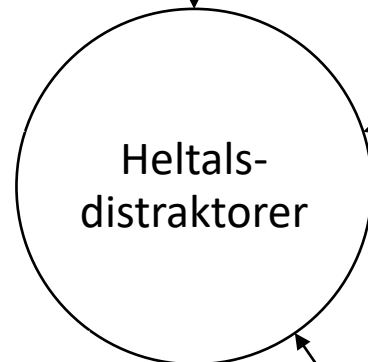
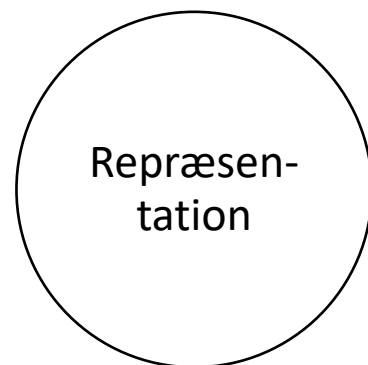
Hvor stor en brøkdel er farvet?



$$\frac{4}{1}$$

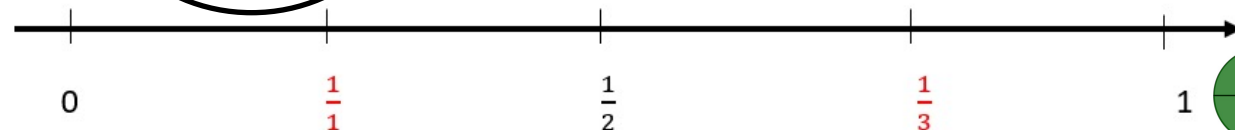
~~$$\frac{3}{4}$$~~

$$\frac{3}{4}$$

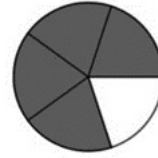


$$\frac{1}{4} > \frac{1}{3}$$

$$\frac{1}{5} \quad \frac{1}{4} \quad \frac{1}{3}$$



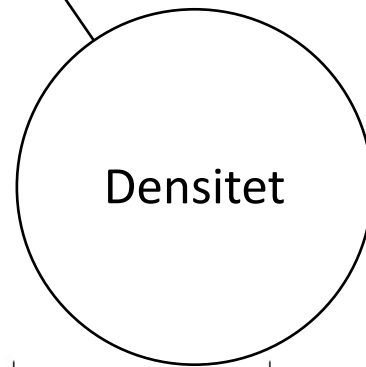
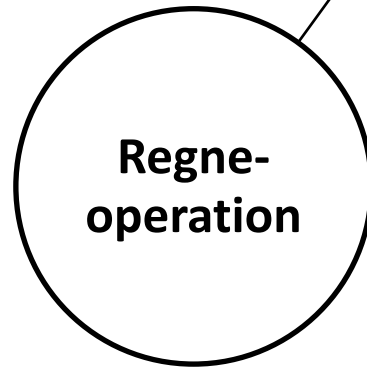
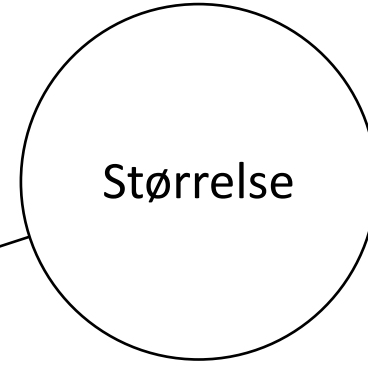
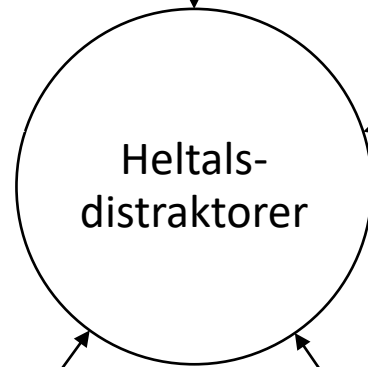
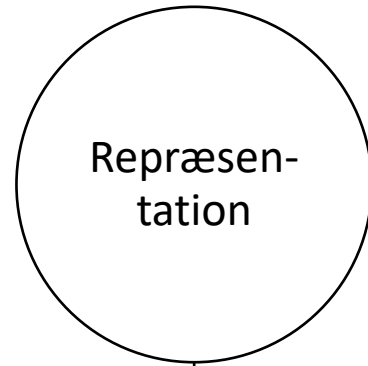
Hvor stor en brøkdel er farvet?



$$\frac{4}{1}$$

~~$$\frac{3}{4}$$~~

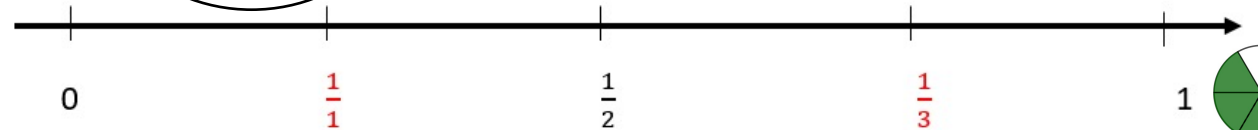
$$\frac{3}{4}$$



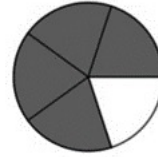
$$\frac{1}{4} > \frac{1}{3}$$

$$\frac{1}{5} \quad \frac{1}{4} \quad \frac{1}{3}$$

$$\frac{1}{5} + \frac{1}{3} = \frac{2}{8}$$



Hvor stor en brøkdel er farvet?

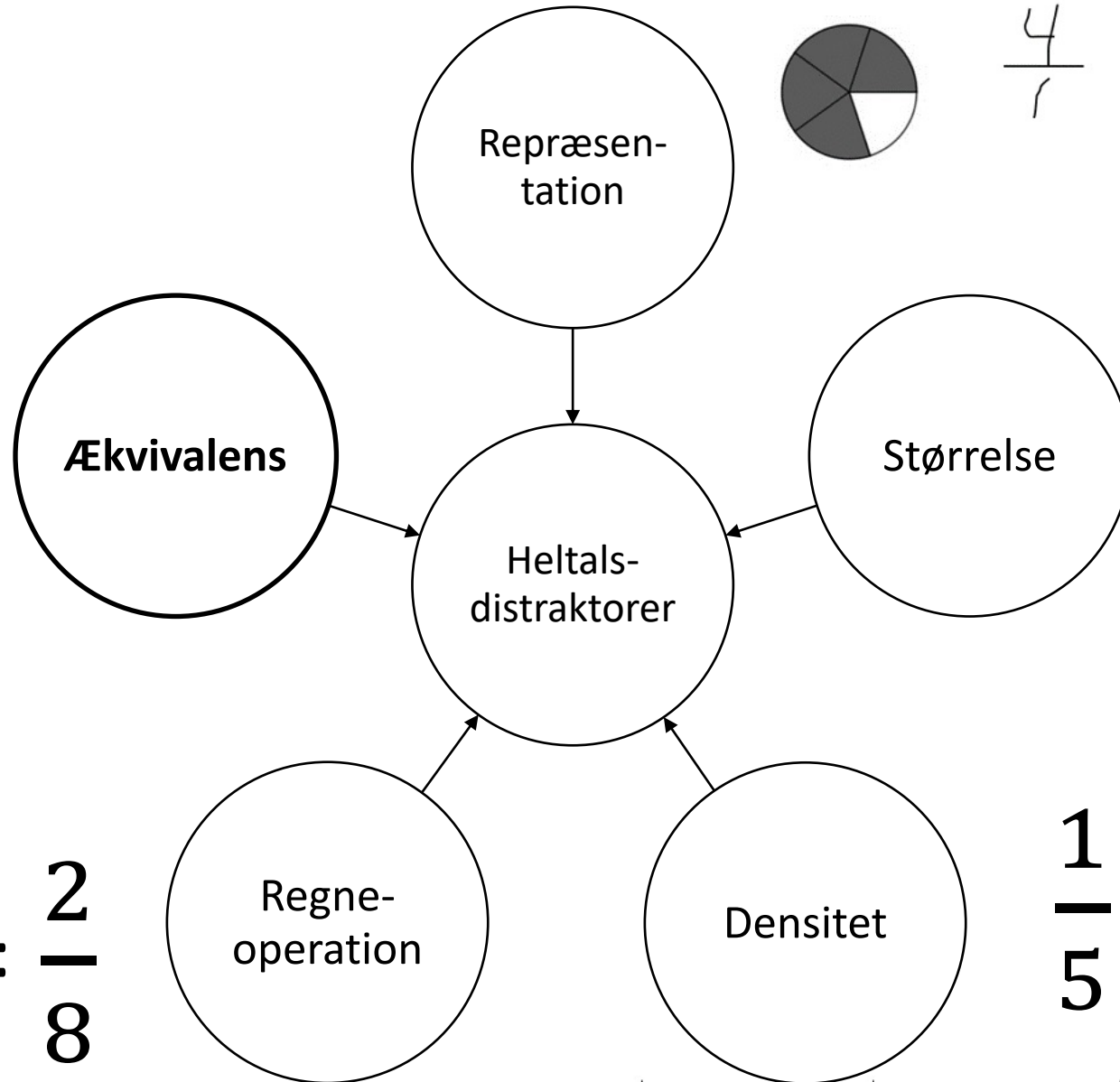


$$\frac{4}{1}$$

~~$$\frac{3}{4}$$~~

$$\frac{3}{4}$$

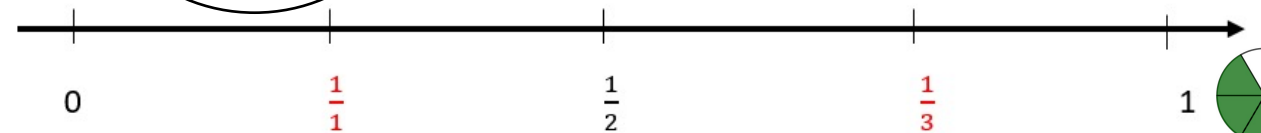
$$\frac{1}{4} \neq \frac{2}{8}$$

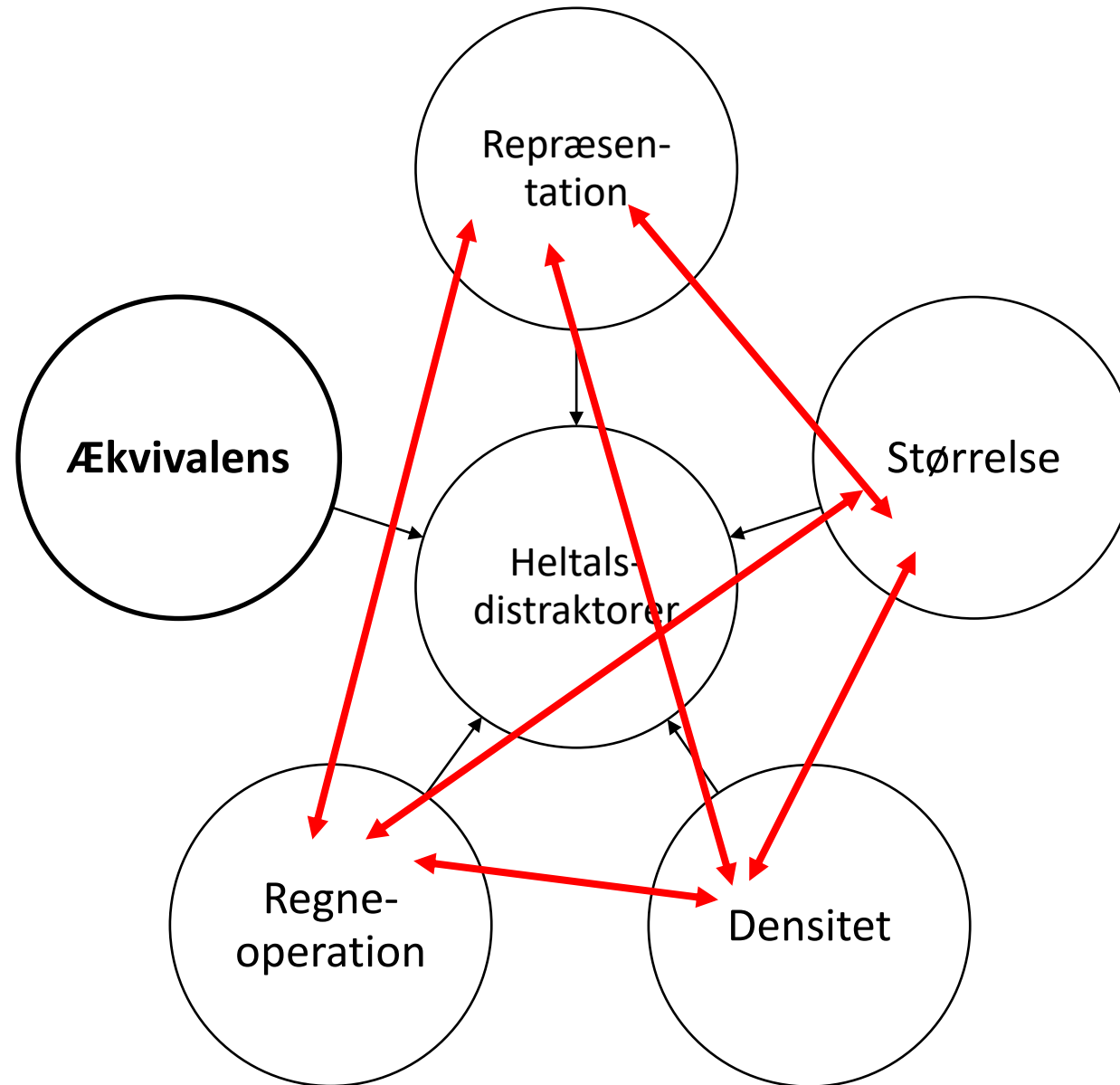


$$\frac{1}{4} > \frac{1}{3}$$

$$\frac{1}{5} + \frac{1}{3} = \frac{2}{8}$$

$$\frac{1}{5} \quad \frac{1}{4} \quad \frac{1}{3}$$

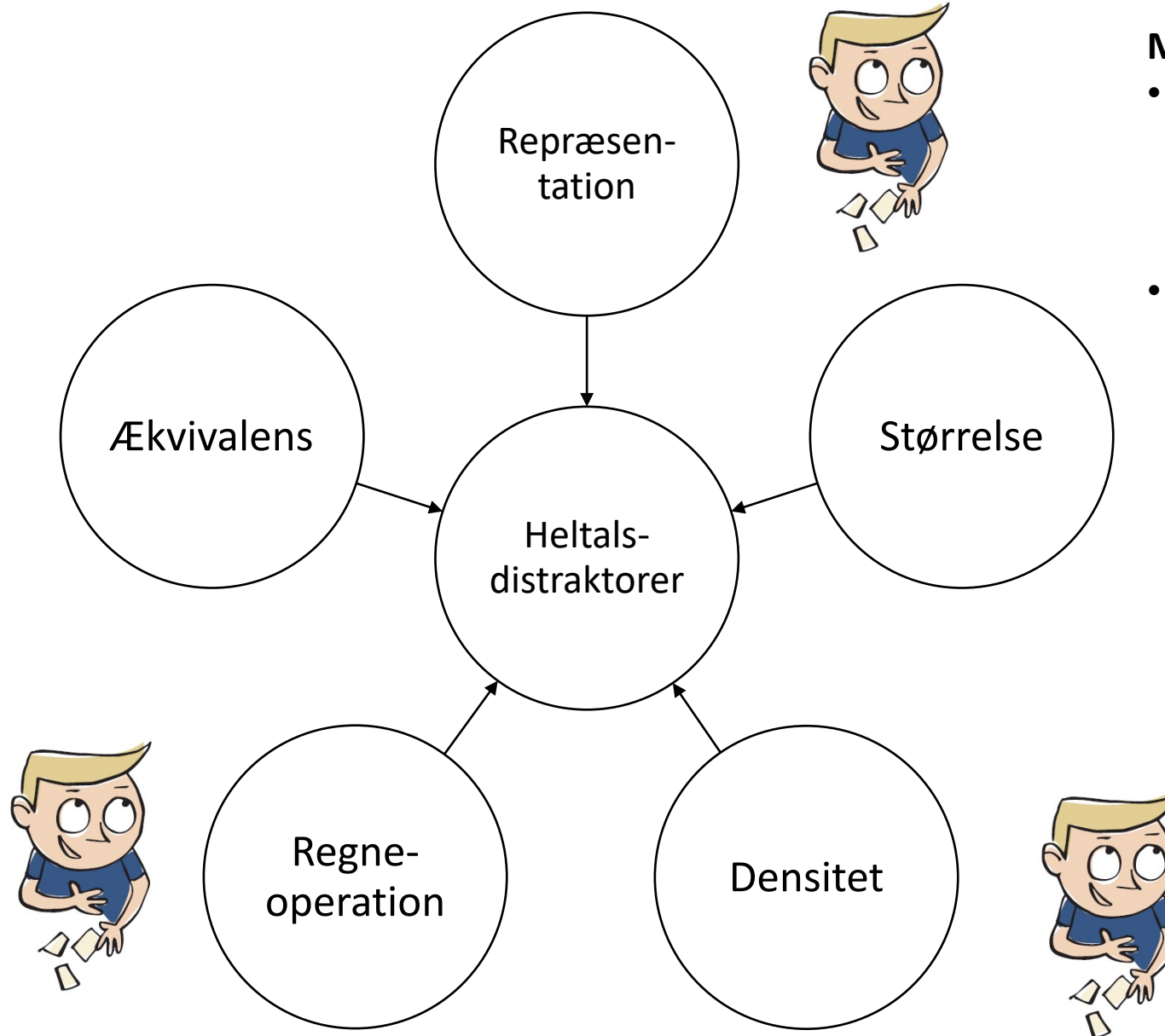




Min forskning:

- Tilføjelse af ækvivalens som en type heltalsdistraktor
- Ikke en sammenhæng mellem heltalsdistraktorerne (4.klasseelever)

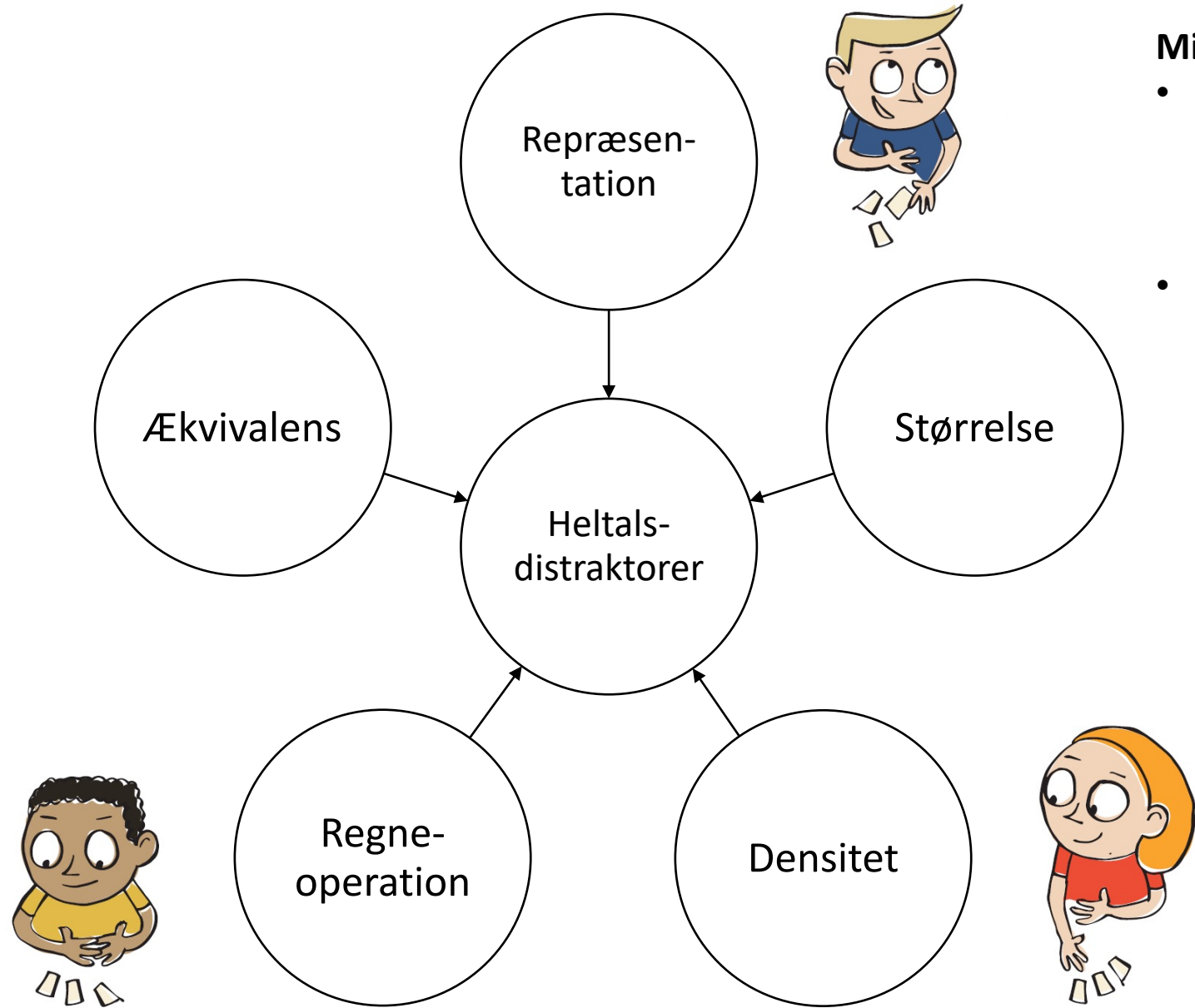




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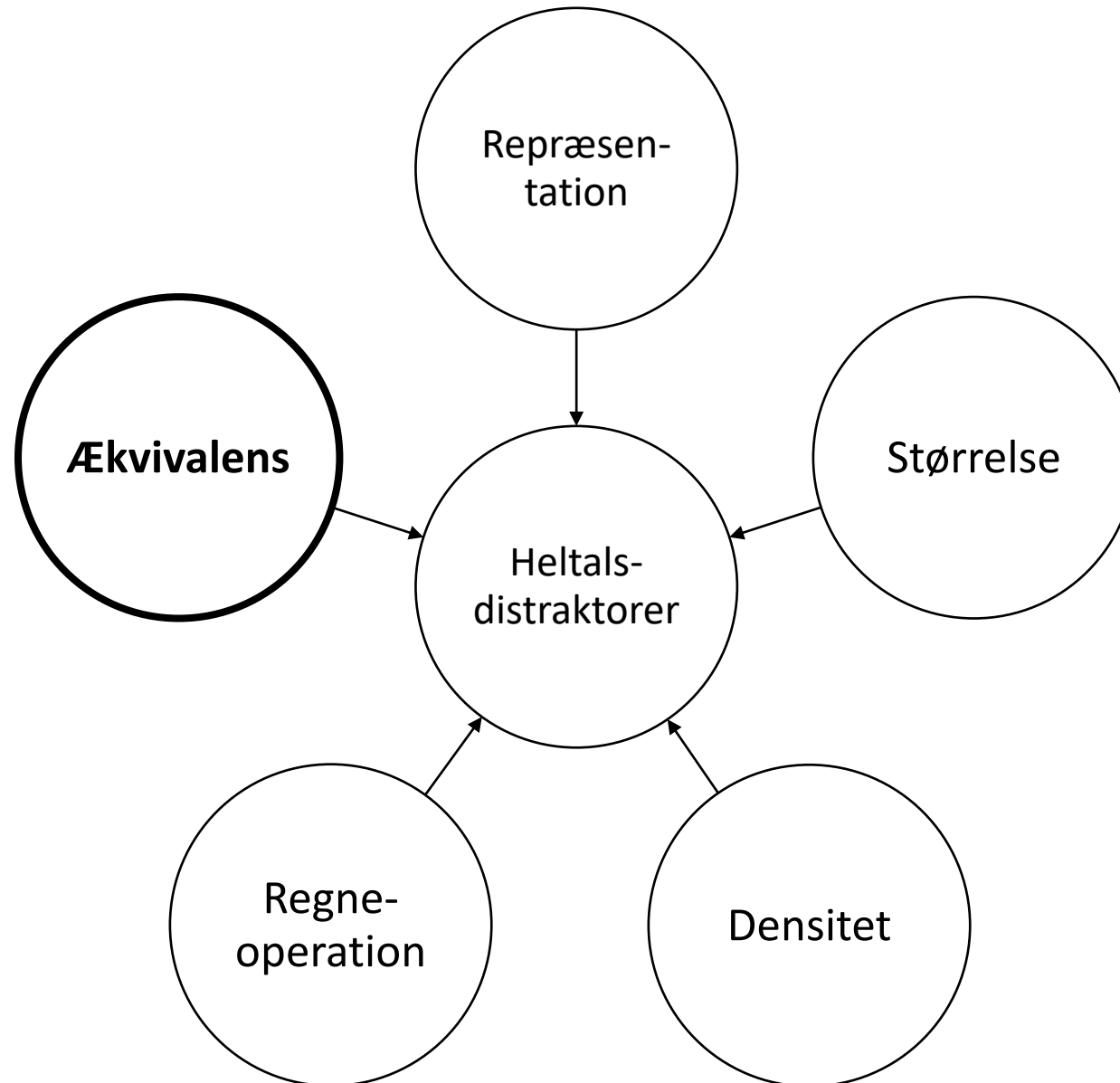




**Min forskning:**

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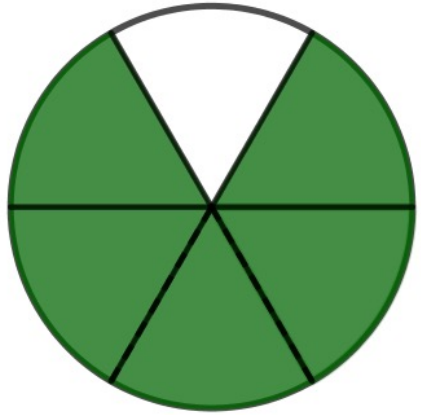




### Min forskning:

- Tilføjelse af ækvivalens som en type heltalsdistraktor
- Ikke en sammenhæng mellem heltalsdistraktorerne (4.klasseelever)





# Typer af ækvivalens

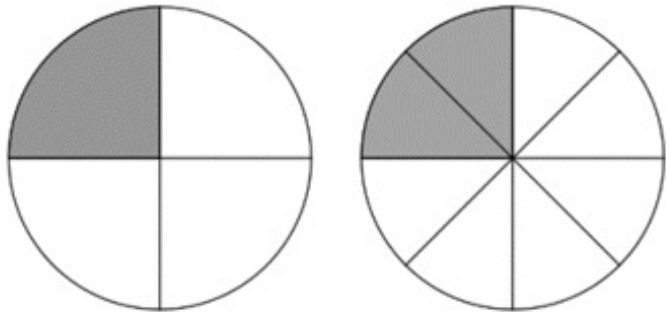
Mette Bjerre



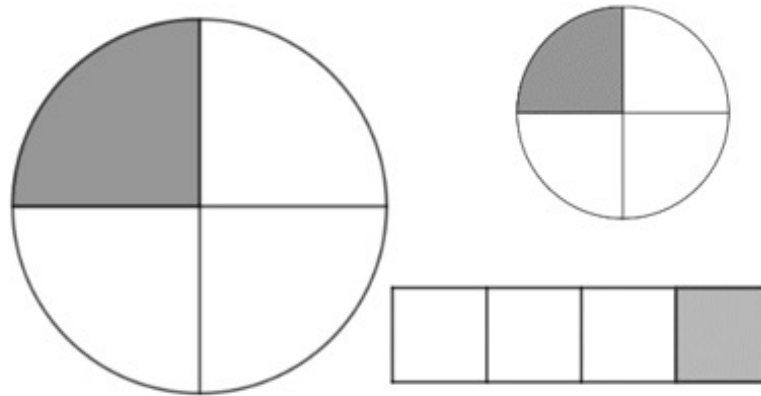


# To forskellige koncepter af ækvivalens

Enhedsækvivalens inkluderer  
proportionalækvivalens



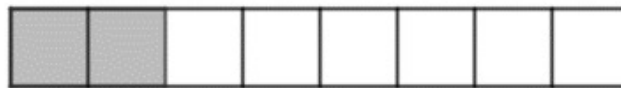
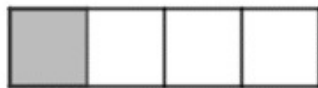
Proportionalækvivalens inkluderer  
ikke enhedsækvivalens

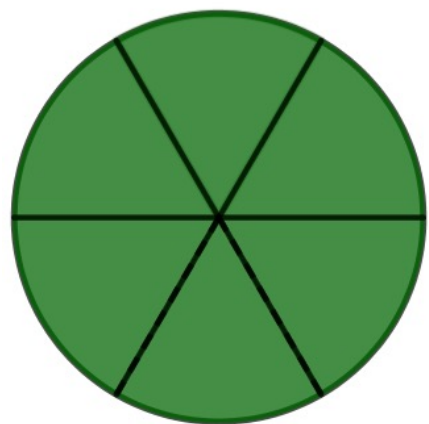


*Example of No Equal-Whole when Comparing  $\frac{1}{4}$  and  $\frac{2}{8}$ . This is Proportion Equivalence and Not*

*Unit Equivalence; and when Comparing the Size of a Fraction, it Must be Based on Unit*

*Equivalence*





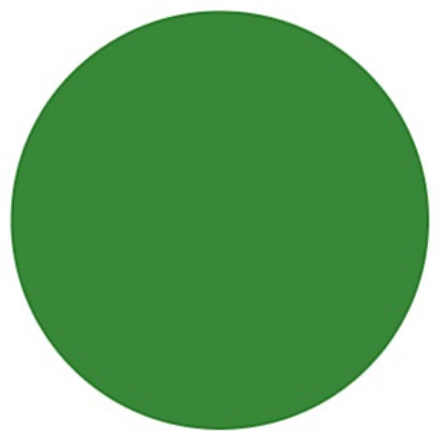
Opmærksomhedspunkter

# Opmærksomhedspunkter

a) Det er centralt, at eleverne får mulighed for at udvikle de to forståelser af ækvivalens – særligt fordi det hænger sammen med udviklingen af ækvivalens inden for fx algebra og procent. Ækvivalens kan dermed støtte en konceptuel forståelse af disse begreber, da det er med til at skabe sammenhæng mellem begreber.

b) Eleverne skal gives mulighed for at udvikle en forståelse af forskellene mellem naturlige tal og rationale tal i forskellige kontekster og dermed forstå forskellen mellem naturlige tal og brøker. Med andre ord skal de overkomme tendensen til distraktorerne fra de naturlige tal.





Tak for en brøkdel af jeres tid

# Referenceliste

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