

United Kingdom  
Mathematics Trust

# INTERMEDIATE MATHEMATICAL CHALLENGE

1 – 4 February 2021

Organised by the United Kingdom Mathematics Trust

supported by  

*England & Wales: Year 11 or below*  
*Scotland: S4 or below*  
*Northern Ireland: Year 12 or below*

## INSTRUCTIONS

1. Do not open the paper until the invigilator tells you to do so.
2. Time allowed: **60 minutes**.  
No answers, or personal details, may be entered after the allowed time is over.
3. The use of blank or lined paper for rough working is allowed; **squared paper, calculators and measuring instruments are forbidden**.
4. **Use a B or an HB non-propelling pencil**. Mark at most one of the options A, B, C, D, E on the Answer Sheet for each question. Do not mark more than one option.
5. **Do not expect to finish the whole paper in the time allowed**. The questions in this paper have been arranged in approximate order of difficulty with the harder questions towards the end. You are not expected to complete all the questions during the time. You should bear this in mind when deciding which questions to tackle.
6. **Scoring rules:**  
5 marks are awarded for each correct answer to Questions 1-15;  
6 marks are awarded for each correct answer to Questions 16-25;  
Each incorrect answer to Questions 16-20 loses 1 mark;  
Each incorrect answer to Questions 21-25 loses 2 marks.
7. Your Answer Sheet will be read by a machine. **Do not write or doodle on the sheet except to mark your chosen options**. The machine will read all black pencil markings even if they are in the wrong places. If you mark the sheet in the wrong place, or leave bits of eraser stuck to the page, the machine will interpret the mark in its own way.
8. **The questions on this paper are designed to challenge you to think, not to guess**. You will gain more marks, and more satisfaction, by doing one question carefully than by guessing lots of answers. This paper is about solving interesting problems, not about lucky guessing.

Enquiries about the Intermediate Mathematical Challenge should be sent to:

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1. What is the value of  $2021 - 2223 + 2425$ ?

- A 2122                      B 2223                      C 2324                      D 2425                      E 2526

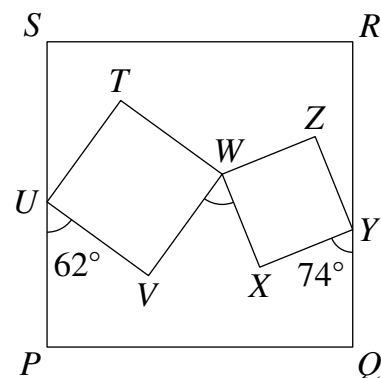
2. The day before the day before yesterday was two days after the day before my birthday. Today is Thursday. On what day was my birthday?

- A Sunday                      B Monday                      C Tuesday                      D Wednesday                      E Friday

3. What is the value of  $2 - (-2 - 2) - (-2 - (-2 - 2))$ ?

- A 0                      B 2                      C 4                      D 6                      E 8

4. The diagram shows three squares,  $PQRS$ ,  $TUVW$  and  $WXYZ$ . Angles  $PUV$  and  $QYX$  are  $62^\circ$  and  $74^\circ$  respectively. What is angle  $VWX$ ?



- A  $44^\circ$                       B  $48^\circ$                       C  $60^\circ$                       D  $64^\circ$                       E  $68^\circ$

5. April, May and June have 90 sweets between them. May has three-quarters of the number of sweets that June has. April has two-thirds of the number of sweets that May has. How many sweets does June have?

- A 60                      B 52                      C 48                      D 40                      E 36

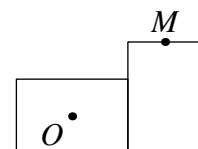
6. Kai has begun to list, in ascending order, the positive integers which are *not* factors of 240. What is the sixth number on Kai's list?

- A 11                      B 13                      C 14                      D 15                      E 17

7. What is the value of  $(4 - \frac{1}{4}) \div (2 - \frac{1}{2})$ ?

- A  $1\frac{1}{2}$                       B 2                      C  $2\frac{1}{2}$                       D 3                      E  $4\frac{1}{4}$

8. The diagram shows two 10 by 14 rectangles which are edge-to-edge and share a common vertex. It also shows the centre  $O$  of one rectangle and the midpoint  $M$  of one edge of the other.



What is the distance  $OM$ ?

- A 12                      B 15                      C 18                      D 21                      E 24

9. How many of the following statements are true?

- A prime multiplied by a prime is always a prime.
- A square multiplied by a square is always a square.
- An odd number multiplied by an odd number is always an odd number.
- An even number multiplied by an even number is always an even number.

- A 0                      B 1                      C 2                      D 3                      E 4

10. The prime factor decomposition of 2021 is  $43 \times 47$ .

What is the value of  $53 \times 57$ ?

- A 2221                      B 2521                      C 2921                      D 3021                      E 3031

11. The line with equation  $y = 2x + 3$  is reflected in the  $x$ -axis.

Which of the following is the equation of the new line?

- A  $y = 2x - 3$               B  $y = -2x + 3$               C  $x = 2y + 3$               D  $y = \frac{1}{2}x + 3$               E  $y = -2x - 3$

12. Andrew calculates that 40% of 50% of  $x$  is equal to 20% of 30% of  $y$ , where  $x \neq 0$ .

Which of the following is true?

- A  $y = \frac{2x}{3}$                       B  $y = \frac{4x}{3}$                       C  $y = 2x$                       D  $y = \frac{8x}{3}$                       E  $y = \frac{10x}{3}$

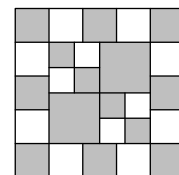
13. What is the remainder when  $12\,345 \times 54\,321$  is divided by 9?

- A 0                              B 1                              C 2                              D 3                              E 4

14. The diagram shows a large square divided into squares of three different sizes.

What percentage of the large square is shaded?

- A 61%                      B 59%                      C 57%                      D 55%                      E 53%



15. Patrick drives from P to Q at an average speed of 40 mph. His drive back from Q to P is at an average speed of 45 mph and takes two minutes less.

How far, in miles, is it from P to Q?

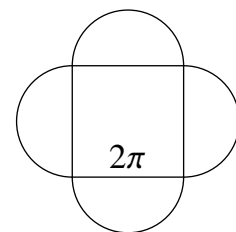
- A 1.5                      B 6                              C 9                              D 12                              E 15

16. A semicircle is drawn on each side of a square, as shown.

The square has sides of length  $2\pi$ .

What is the area of the resulting shape?

- A  $2\pi^2(\pi + 1)$               B  $\pi^2(\pi + 2)$               C  $2\pi^2(2\pi + 1)$               D  $\pi^2(\pi + 4)$   
 E  $2\pi^2(\pi + 2)$



17. In the rectangle  $PQRS$ , the side  $PQ$  is of length 2 and the side  $QR$  is of length 4. Points  $T$  and  $U$  lie inside the rectangle so that  $PQT$  and  $RSU$  are equilateral triangles.

What is the area of the quadrilateral  $QRUT$ ?

- A  $\frac{6 - \sqrt{3}}{2}$                       B  $\frac{8}{3}$                               C  $4 - 2\sqrt{3}$                       D  $4 - \sqrt{3}$                       E 3

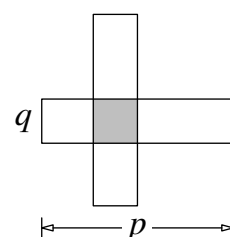
18. Which of these is closest in size to 1?

- A  $0.\dot{9}\dot{5}$                       B  $1.\dot{0}\dot{5}$                       C  $0.\dot{9}6\dot{0}$                       D  $1.\dot{0}4\dot{0}$                       E  $0.9\dot{5}$

19. The diagram shows two overlapping rectangles, each measuring  $p$  by  $q$ . The area of overlap is exactly one-quarter of the total area of the figure.

What is the ratio  $p : q$ ?

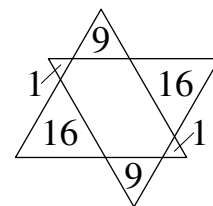
- A 5 : 2                      B 4 : 1                      C 3 : 1                      D 2 : 1                      E 3 : 2



20. Two straight lines have equations  $y = px + 4$  and  $py = qx - 7$ , where  $p$  and  $q$  are constants. The two lines meet at the point  $(3, 1)$ . What is the value of  $q$ ?

A 1                      B 2                      C 3                      D 4                      E 5

21. The diagram shows two congruent equilateral triangles whose overlap is a hexagon. The areas of the smaller triangles, which are also equilateral, are 1, 1, 9, 9, 16 and 16, as shown.



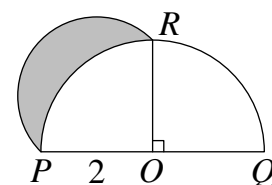
What is the area of the inner hexagon?

A 68                      B 58                      C 48                      D 38                      E 28

22. What is the result when we simplify the expression  $\left(1 + \frac{1}{x}\right)\left(1 - \frac{2}{x+1}\right)\left(1 + \frac{2}{x-1}\right)$ ?

A 1                      B  $\frac{1}{x(x+1)}$                       C  $\frac{1}{(x+1)(x-1)}$                       D  $\frac{1}{x(x+1)(x-1)}$                       E  $\frac{x+1}{x}$

23. The diagram shows a semicircle with centre  $O$  and radius 2 and a semicircular arc with diameter  $PR$ . Angle  $POR$  is a right angle.



What is the area of the shaded region?

A  $\pi - 2$                       B 2                      C  $\pi$                       D 3                      E  $2\pi - 2$

24. Sam writes on a white board the positive integers from 1 to 6 inclusive, once each. She then writes  $p$  additional fives and  $q$  sevens on the board. The mean of all the numbers on the board is then 5.3. What is the smallest possible value of  $q$ ?

A 7                      B 9                      C 11                      D 13                      E 15

25. Thomas has constant speeds for both running and walking. When a down-escalator is moving, Thomas can run down it in 15 seconds or walk down it in 30 seconds. One day, when the escalator was broken (and stationary), it took Thomas 20 seconds to run down it.

How long, in seconds, would it take Thomas to walk down the broken escalator?

A 30                      B 40                      C 45                      D 50                      E 60