Finding the right grain-size for measurement in the classroom

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Outline

- What do I mean by grain-size?
- Measurement at the meso level:
  - example of BASS
- Measurement at the micro level
  - example of TOTS
- Linking measurement at the meso level to micro level
  - BASS-TOTS
- Conclusion

What do I mean by “grain-size”?

micro level ⇔ Fine-grained assessments, within-instruction observations

meso level ⇔ Testing for instructional planning in the classroom, narrower standards

macro level ⇔ Summative testing, Standardized testing, broader standards

Finding an appropriate level of detail/grain-size

Micro level ⇔ Fine-grained assessments, within-instruction observations

about: Relatively fine grain-size knowledge, “in-pieces”
(definition of density, initial buoyancy experiences,
what it means to measure a length)
for: Teacher, computerized teaching software-level, student,
how often: Brief education time-periods
(i.e., sequence of concepts within a unit, etc.),
purposes: Smaller-scale instructional decisions
(what tactic to use next in discussing a certain idea with students)

Finding an appropriate level of detail/grain-size

Meso level ⇔ Testing for teaching, Broader standards:

about: Teachable Constructs

(e.g., buoyancy, variability, measurement)

for: Teacher-level, student-level (when old enough)

how often: Instructional-scale time-periods and up

(days, weeks, teaching units, etc.)

purposes: Instructional decisions

(planning for day/week's topics, what topics to revisit, which students need extensive help)

Finding an appropriate level of detail/grain-size

Macro level ⇔ Summative testing, Standardized testing:

about: Coarse, administrative, composite constructs
(Science, Language Arts, Geometry, etc.)
for: Teacher/Parent level and *up*
(i.e., administrators/policy-makers, etc.)
how often: Longer education time-periods
(Years, Semesters, program length, etc.)
purposes: Large-scale decisions
(Pass-Course, Advancement, course-placement, etc.)

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Measurement at the meso/micro levels: An Example Project

- NSF Collaborative Research Project: Modeling Assessment to Enhance Teaching and Learning (MAETL?)
- Collaboration Among:
  - Rich Lehrer, Leona Schauble and Corey Brady (Vanderbilt University);
  - Mark Wilson and Perman Gochyyev (UC Berkeley)
  - A bunch of graduate students at both universities

The Project

- Products:
  - A shared conceptual framework for describing instructional goals and valued forms of teaching and learning—i.e., the learning progression**;
  - a set of electronic tools to help educators detect, share, analyze, and interpret data—BASS and TOTs; and
  - classroom and school-level community professional development structures to support and sustain the practice of assessing to guide instruction*.
- Topics:
  - Measurement of: Length, Area, Volume, Angle, and
  - Measured Quantities as entrée to Rational Numbers (Fractions as quantities, fractions as operators)

*Lehrer, R., Kim, M-J., Ayers, E., & Wilson, M. (2014). Toward establishing a learning progression to support the development of statistical reasoning. In A. Maloney, J. Confrey & K. Nguyen (Eds.) *Learning over Time: Learning Trajectories in Mathematics Education* (pp. 31-60). Charlotte, NC: Information Age Publishers.

** Wilson, M., & Lehrer, R. (2021). Improving learning: Using a learning progression to coordinate instruction and assessment. *Frontiers in Education, 6:* 654212. doi.org/10.3389/feduc.2021.654212

Having something to measure

- The Construct: **ToML**—*Theory of Measurement--Length*
- Describes how children come to constitute a theory of measure to compare magnitudes (extents) of lengths.
  - A theory of measure refers to the web of "big ideas" and procedures involved in developing these comparisons.
  - Expressed using a "construct map"

The Levels of the Construct Map: **ToML**—*Theory of Measurement-Length*

6. Generalizing relationships (e.g., Measure of A in B is the reciprocal of measure of B in A)
5. Partitioning and symbolizing involving 3-splits and composition of 2- and 3-splits
4. Partitioning, iterating, symbolizing partitioned units—2-splits
3. Iterating units and symbolizing distance traveled
2. Explaining properties of units and their role in accumulation
1. Directly comparing

A note on Construct Maps

- A ***construct map*** such as **ToML** is not intended to portray every nuance of learning about the mathematics of length measurement, but instead, to highlight critical conceptual and procedural attainments.
- Each *waypoint* of the construct is composed of *sub-levels* that collectively constitute a network of the ideas and procedures that describe the particular way of knowing and doing expressed in a level.
- Hence, the waypoints should be regarded not as a ladder, but as ideas and procedures that are coordinated to develop the form of thinking described by the overarching level.
  - Concepts and procedures described at lower waypoints are resources for forms of learning described at a higher level

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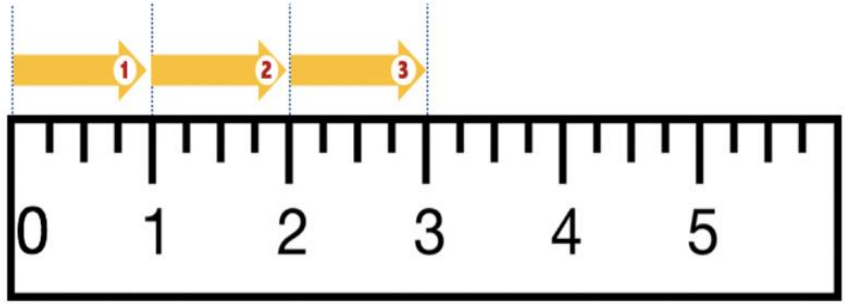


- For example, 3 on a standard ruler is marked at the endpoint of the third unit not at the center.
- This spacing distinguishes interpretation of a length as distance traveled rather than as merely counting a collection of units.
- The understandings constitute the beginnings of understanding a measurement scale—a way of specifying relations among units to mark quantities.
- Students understand that a measure of 10*u* means that the measured length is 10 times as long as the length of 1 unit (*u*). Also, a measure of 10*u* implies that the length can be subdivided into 10 congruent parts.
- However, they may not yet routinely understand the *reciprocal* relation, that 1 unit is 1/10 as long as 10 units.

Meso Level Online Assessment

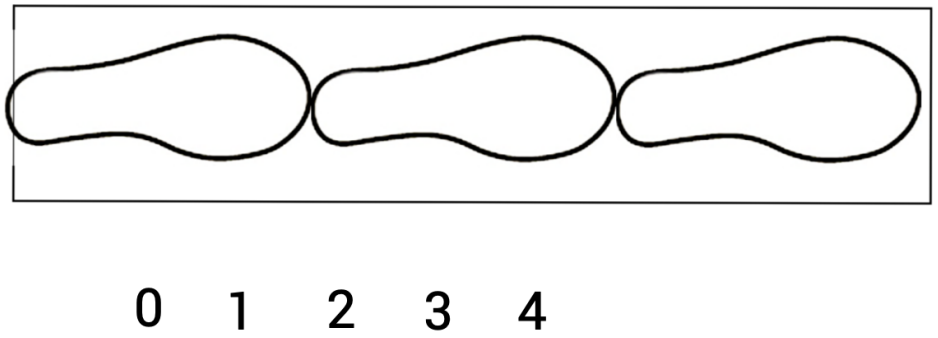
- Delivered online using the *Berkeley Assessment System Software* (BASS)
- Data analysis conducted as a multidimensional Rasch model using both Conquest and BASS software
- One dimension for each construct map: Length, Area, Volume, Angle.

An item at level 3: Labeling Foot Ruler

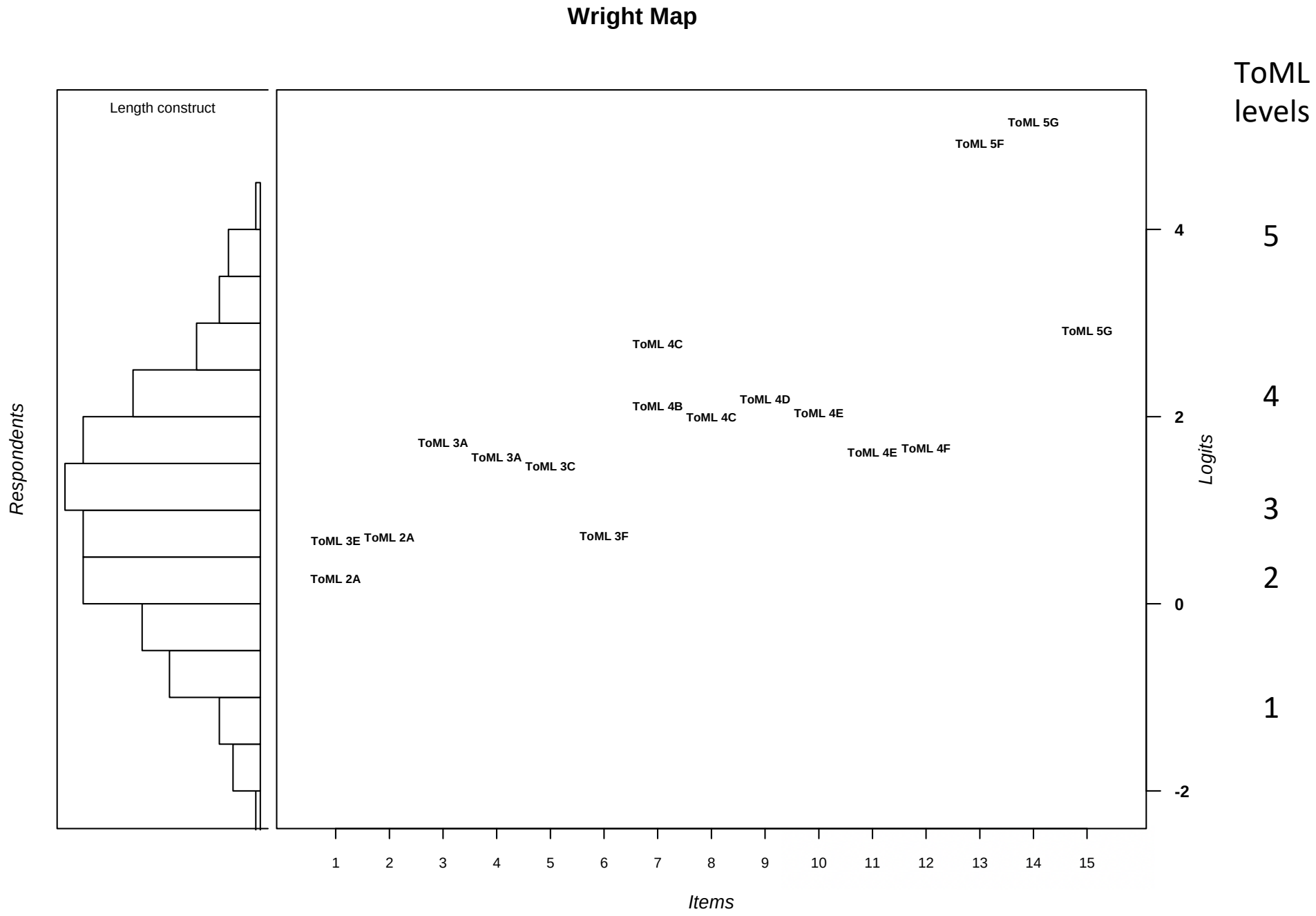
Here is one of Maria's footprints. 

Maria used copies of her footprint to make this foot-ruler. But Maria forgot to label her foot-ruler.

Drag the numbers onto the foot-ruler to label the units. Use only the numbers that you need

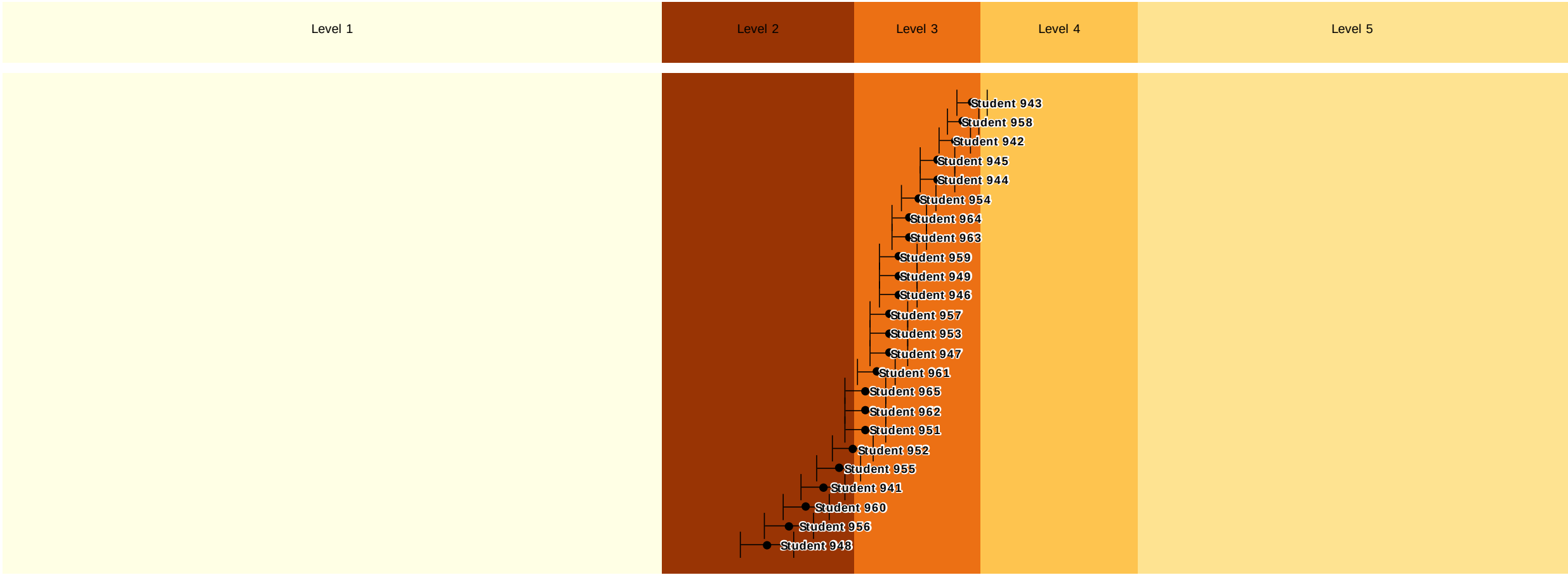


BASS Wright Map: Results from 2019 posttest



Example BASS Report for a class (2018/19 ToML data)

- Level 5
- Level 4
- Level 3
- Level 2
- Level 1



Micro Level Online Assessment

- A teaching/learning context-Day 1:
- In previous classroom activity, the teacher had students measure a distance in the hallway with clipboards, or with dowel rods, or with their feet.
- These activities were aimed primarily at ToML Level 2 with support for “edging” into Level 3.
- There were more than 1 of each type of unit (feet, naturally, were in inexhaustible supply).

A teaching/learning context-Day 1

- The class observed children as they conducted their measurements.
- With whole class conversation that followed the class established that shorter units resulted in greater measure of the same distance (and why),
- And they explained that the units had to be translated so that there were no *gaps* or *laps*.

“When is 4 feet 4 feet?”-Day 2

- New challenge (from teacher)
  - using just one standard unit, the length of *her* foot (*u*), create a path with measure 4*u*.
- Teacher took photos of student strategies
  - class discussed problems and prospects of a few of these strategies*.
- Each pair cut a strip of paper just as long as the path—what happened when they were compared?
- The class used the teacher’s photos to try to investigate why.

*She was looking for evidence of ToML 3A.

Micro Levels of the Construct Map
Earlier Levels as Resources for Current Level

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3A *	Re-use (iterate) a unit to measure.
2F	Qualitatively predict the inverse relation between size of unit and measure.
2E	Consider suitability of unit and explain why.
2D	Count with reservoir of identical units to tile a length and represent measure by the total. If units are not identical, distinguish among them.
2C	Use identical units and explain why.
2B *	Tile and explain why (the explanation is required).
2A	Associate measure with count.
1F	Develop and use local (classroom) conventions to distinguish or order two or more objects by a single attribute.

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Measurement at the micro level : *TOTs*

- TOTs = “ Teacher Observation Tools ”
- A mobile data-gathering *ipad* application
- Designed for teacher use while teaching in their classrooms
- A sample screenshot …

Reset | Clear Drawing | Change Color

Select Brush Size

No constructs

ToML 3A | ToML 3B | ToML 3C

Learning Construct Details

Once you select a learning construct using the toggle buttons above, the description will be displayed here.

Observation date

2020-04-03

Notes

Add Video/Audio note

Nice iteration of a unit; however, the pencils made gaps. This group's tape measure was used for discussion and we could see the pencils accounted for the "extra" length on their paper tape.

Add URL to video

Learning Construct Examples ToML 3A

"I just had one unit so I marked its end and then used it again, marked its end again, and kept doing that. It's 8 paper clips long" Note:

Students

Yareli O

[redacted]roman W

Asher S

Phil B

Nelli M

Raina P

Jaquelin C

Robert A

Alivia F

Perla M

Kevin M

Thiago A

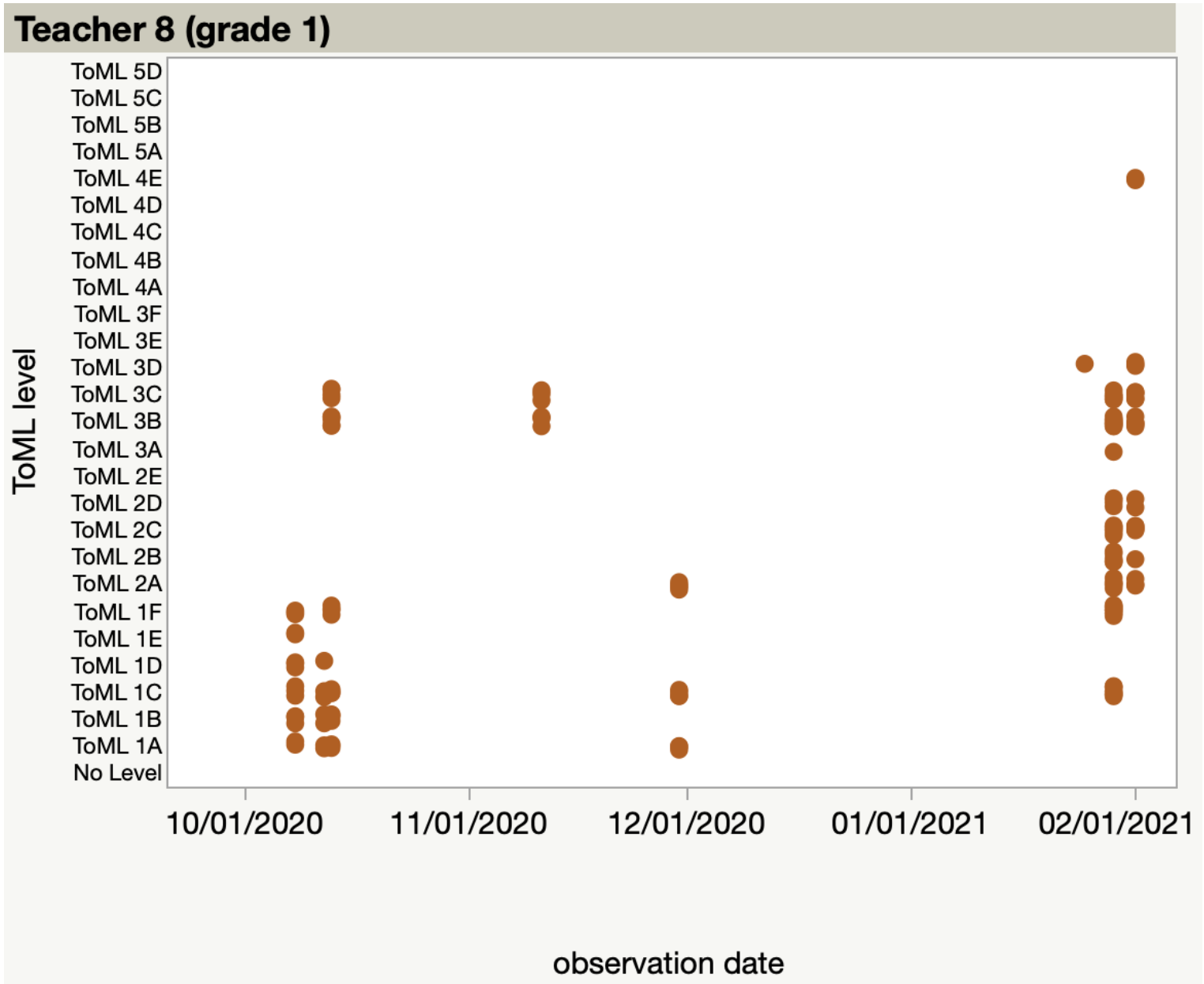
Evie F

Aaron E

Allen C

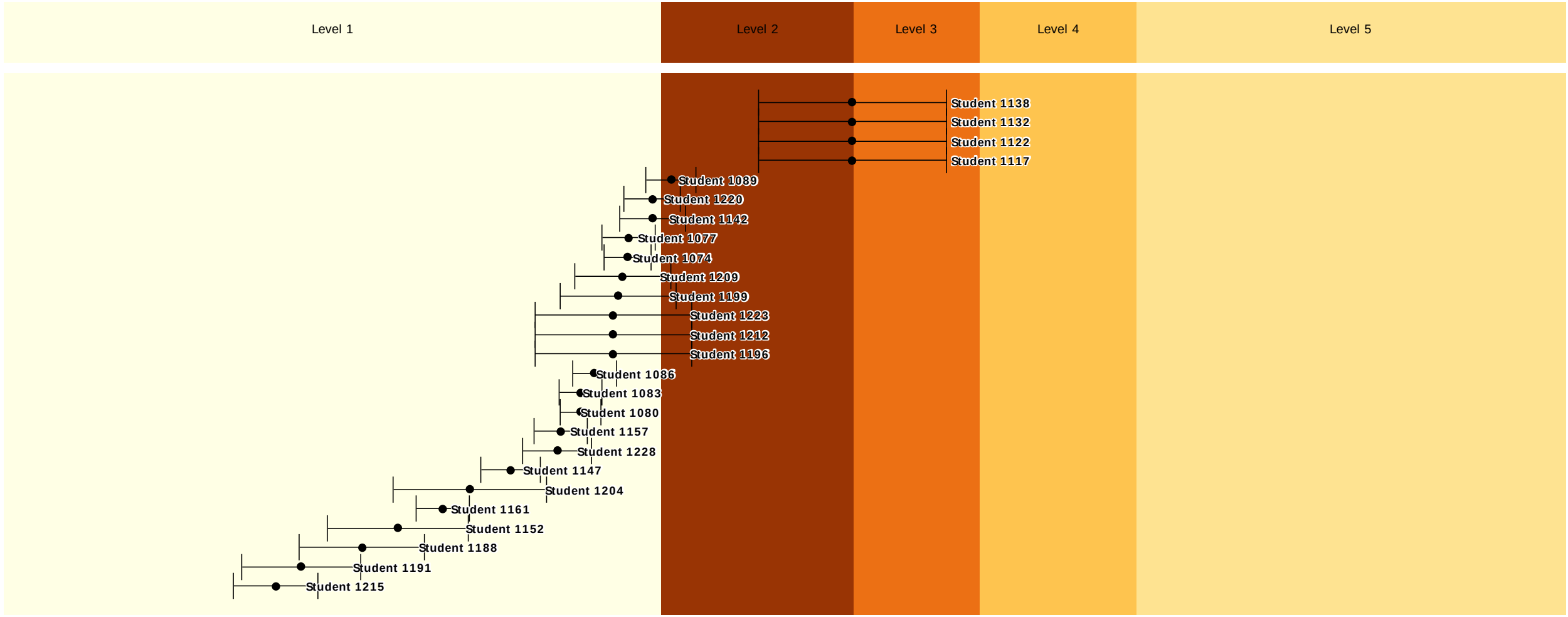
Jabe J

2020/21 Data: Evaluations across time for one class



20/21 Data: BASS Report for TOTs data

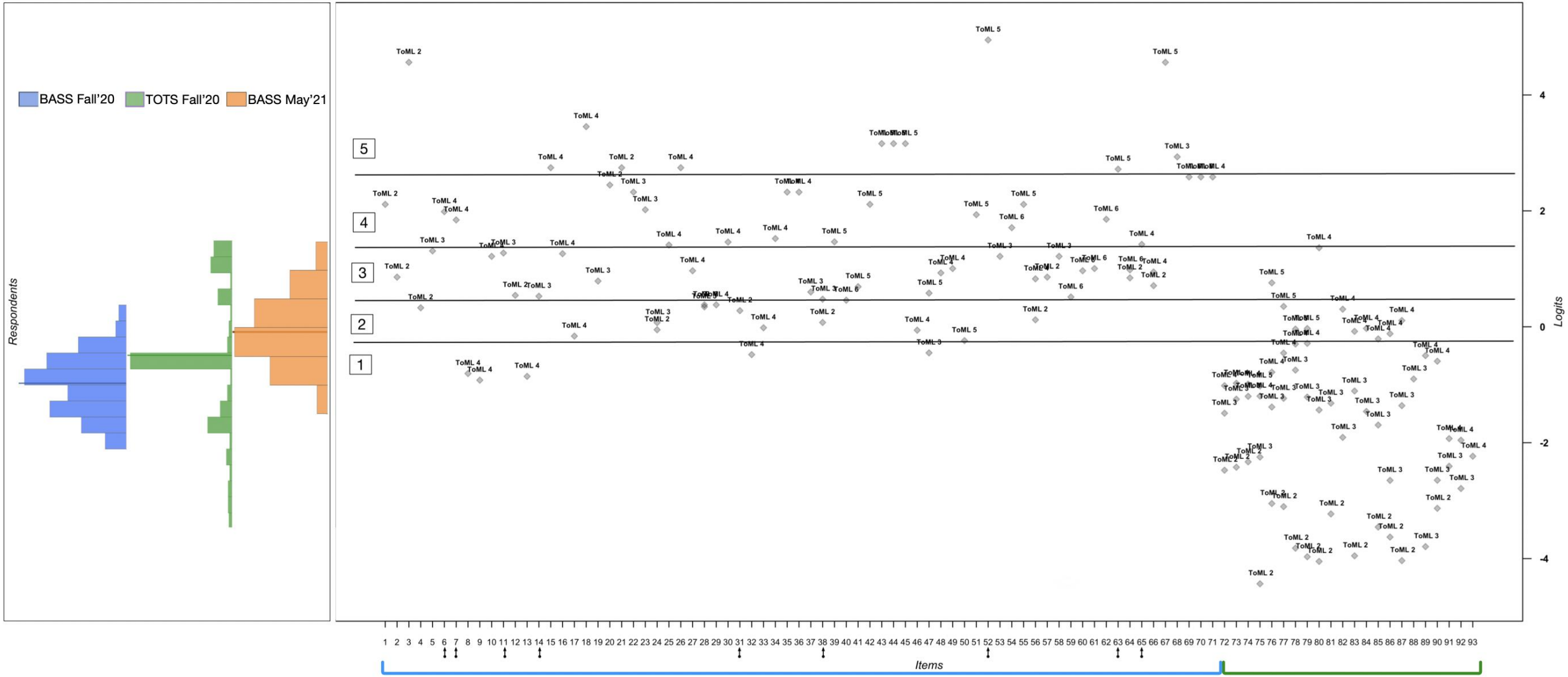
- Level 5
- Level 4
- Level 3
- Level 2
- Level 1



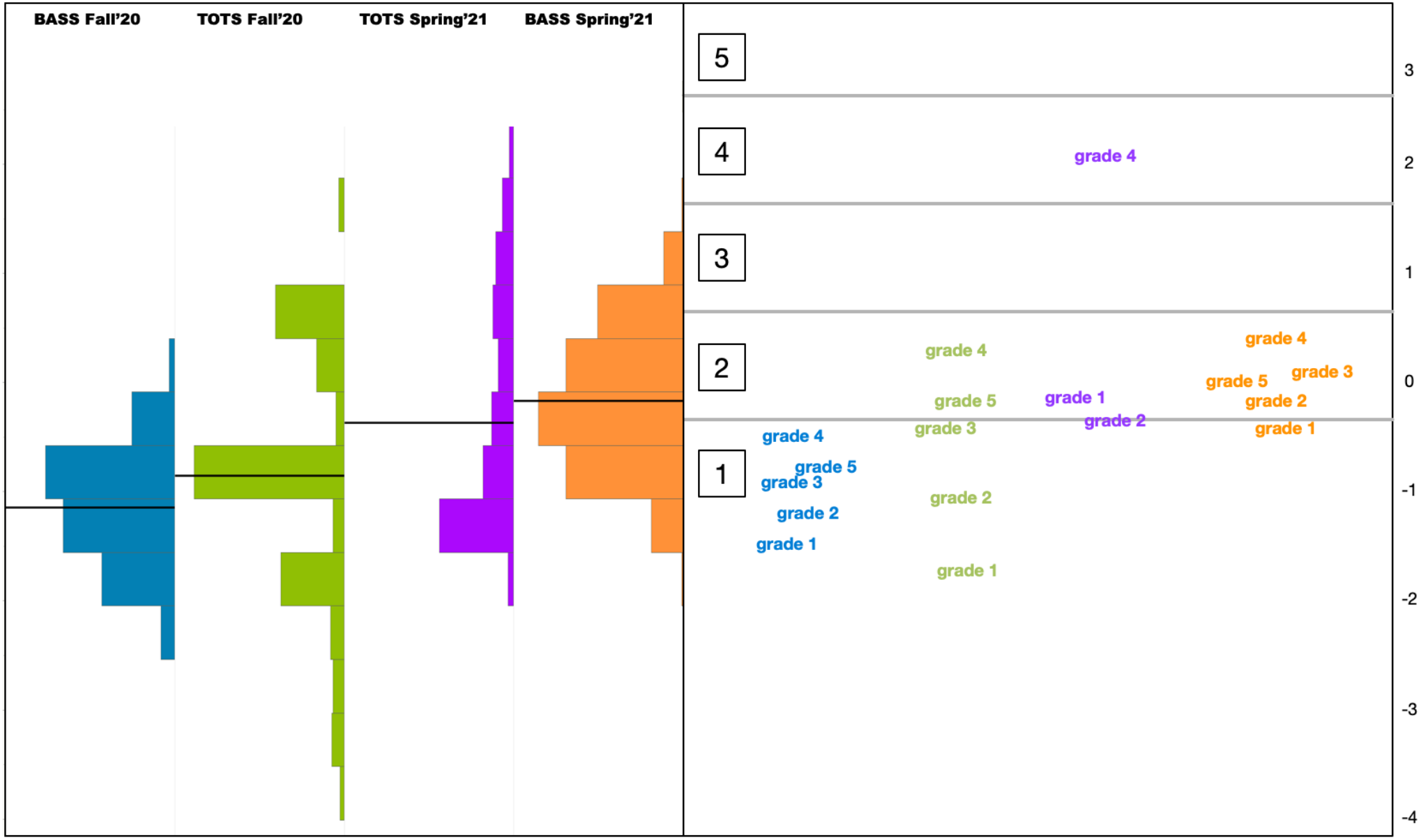
2020/21 Data: BASS Pretest

TOTs during year

BASS Posttest



Grade means


BASS Fall'20
TOTS Fall'20
TOTS Spring'21
BASS Spring'21
5
4
3
2
1
grade 4
grade 4
grade 5
grade 1
grade 4
grade 5
grade 3
grade 3
grade 2
grade 2
grade 1
grade 5
grade 3
grade 2
grade 2
grade 1
grade 1
3
2
1
0
-1
-2
-3
-4

Conclusion

This work helps us …

(a) create the possibility of a more robust classroom assessment system by integrating across grain-sizes to represent student performance, and

(b) perhaps more importantly, growth in student knowledge

(c) while being accountable[1] to students and teachers in the sense of creating systematic opportunities for conceptual development.

[1] I.e., accountable *to the classroom* (R. Lehrer)

Conclusion

Micro: The work at the micro level helps teachers
record their observations about instruction in-place
and plan for the next (micro) instructional event

Meso: The work at the meso level helps teachers
think about the outcomes from instruction, and
plan for the next phase

And so, my answer to the question of the *right* grain-size is
both micro and meso.

Thank-you

- MarkW@Berkeley.edu

ToML-2 Explaining properties of units and their role in accumulation

- At the second level, **ToML-2**, students focus on the nature of a unit.
- Units enable indirect measurements via accumulation and count (instead of directly comparing)
- . Units allow for both additive (how much longer?) and multiplicative comparisons (how many times longer?).
- Students must develop understandings of the properties that enable these uses. Hence, students at this level explain the roles of identical units and tiling.

- Students anticipate an inverse relation between unit length and the measure of a given length: shorter units, compared to longer units, result in a greater measure (i.e., larger numbers) representing the same magnitude (i.e., extent of the length).
- Students begin to develop a sense of the suitability of a particular choice of unit for the goal at hand.
- Students tile units and explain why no “gaps” or “laps.”

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Sub-Levels 2: More detail

3A	Re-use (iterate) a unit to measure.
2F	Qualitatively predict the inverse relation between size of unit and measure.
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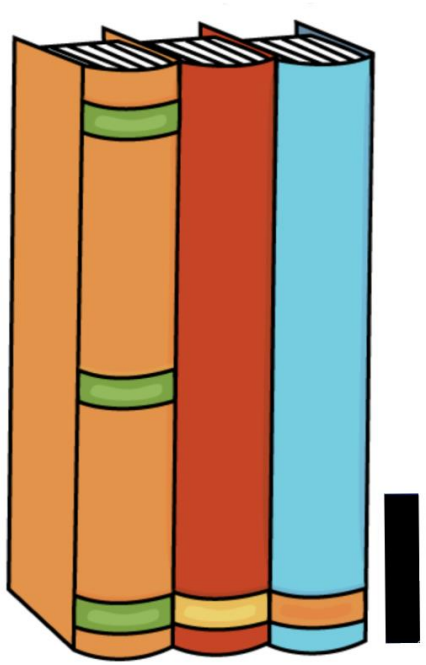
2A	Associate measure with count.
1F	Develop and use local (classroom) conventions to distinguish or order two or more objects by a single attribute.

An item at level 3: Height of Book

Carlos started measuring the height of the blue book, but he does not have enough units. Carlos says he cannot ! nish measuring the height.



Do you agree with Carlos?

- Yes
- No

2020/21 Data: Evaluations across time for one student