Verbetering toets Complexe getallen deel 3:

1.Los volgende oefeningen op in C:

a) $(\boldsymbol{x}+\boldsymbol{y}\boldsymbol{i})^2 = \mathbf{3}+\mathbf{4}\boldsymbol{i}$

$$\begin{cases} x^2 - y^2 = 3 \\ 2xyi = 4i \end{cases}$$

$$\begin{cases} x^2 - y^2 = 3 \\ xy = 2i \end{cases}$$

$$\begin{cases} x^2 - y^2 = 3 \\ x = \dfrac{2}{y} \end{cases}$$

$$\begin{cases} \dfrac{4}{y^2} - y^2 = 3 \\ x = \dfrac{2}{y} \end{cases}$$

$$\begin{cases} -y^4 - 3y^2 + 4 = 0 \quad (beide\ leden\ maal\ y^2) \\ x = \dfrac{2}{y} \end{cases}$$

Delers van 4 zijn 1,-1,2,-2,4,-4.

Voor y = 1 of y = -1 klopt de vergelijking.

$$\begin{cases} y = 1\ v\ y = -1 \\ x = \dfrac{2}{1}\ v\ x = -\dfrac{2}{1} \end{cases}$$

De oplossingen zijn dus: $2 + i\ v\ -2 - i$

b) $(\boldsymbol{x}+\boldsymbol{y}\boldsymbol{i})^2 = -\mathbf{5}+\mathbf{12}\boldsymbol{i}$

$$\begin{cases} x^2 - y^2 = -5 \\ 2xyi = 12i \end{cases}$$

$$\begin{cases} x^2 - y^2 = -5 \\ xy = 6i \end{cases}$$

$$\begin{cases} x^2 - y^2 = -5 \\ x = \dfrac{6}{y} \end{cases}$$

$$\begin{cases} \dfrac{36}{y^2} - y^2 = -5 \\ x = \dfrac{6}{y} \end{cases}$$

$$\begin{cases} -y^4 + 5y^2 + 36 = 0 \quad (beide\ leden\ maal\ y^2) \\ x = \dfrac{6}{y} \end{cases}$$

Het is moeilijk om alle delers van 36 af te gaan.

Daarom gaan we het oplossen via de discriminantmethode (**x en y zijn element van de reële getallen**).

$-y^4 + 5y^2 + 36 = 0$

$-t^2 + 5t + 36 = 0 \ \ We\ stellen\ y^2 = t$

$D = b^2 - 4ac = 25 - 4.(-1).36 = 169$

$$t_1 = \frac{-b + \sqrt{D}}{2a}, t_2 = \frac{-b - \sqrt{D}}{2a}$$

$$t_1 = \frac{-5 + 13}{-2}, t_2 = \frac{-5 - 13}{-2}$$

$t_1 = -4, t_2 = 9$

$\sout{y^2 = -4}, y^2 = 9\ (y \in \mathbb{R})$

$y = 3 \vee y = -3$

$$\begin{cases} y = 3 \vee y = -3 \\ x = \dfrac{6}{y} \end{cases}$$

$$\begin{cases} y = 3 \vee y = -3 \\ x = 2 \vee x = -2 \end{cases}$$

De oplossingen zijn dus: $2 + 3i \vee -2 - 3i$

c) $\boldsymbol{(x + yi)^2 = -11 + 60i}$

$$\begin{cases} x^2 - y^2 = -11 \\ 2xyi = 60i \end{cases}$$

$$\begin{cases} x^2 - y^2 = -11 \\ xy = 30i \end{cases}$$

$$\begin{cases} x^2 - y^2 = -11 \\ x = \dfrac{30}{y} \end{cases}$$

$$\begin{cases} \dfrac{900}{y^2} - y^2 = -11 \\ x = \dfrac{30}{y} \end{cases}$$

$$\begin{cases} -y^4 + 11y^2 + 900 = 0 \quad (beide\ leden\ maal\ y^2) \\ x = \dfrac{30}{y} \end{cases}$$

Het is moeilijk om alle delers van 900 af te gaan.

Daarom gaan we het oplossen via de discriminantmethode (**x en y zijn element van de reële getallen**).

$-y^4 + 11y^2 + 900 = 0$

$-t^2 + 11t + 900 = 0 \;\; We\ stellen\ y^2 = t$

$D = b^2 - 4ac = 121 - 4.(-1).900 = 3721$

$t_1 = \frac{-b + \sqrt{D}}{2a}, t_2 = \frac{-b - \sqrt{D}}{2a}$

$t_1 = \frac{-11 + 61}{-2}, t_2 = \frac{-11 - 61}{-2}$

$t_1 = -25, t_2 = 36$

~~$y^2 = -25$~~, $y^2 = 36 \; (y \in \mathbb{R})$

$y = 6 \; v \; y = -6$

$\begin{cases} y = 6 \; v \; y = -6 \\ x = \frac{30}{y} \end{cases}$

$\begin{cases} y = 6 \; v \; y = -6 \\ x = 5 \; v \; x = -5 \end{cases}$

De oplossingen zijn dus: $5 + 6i \; v \; -5 - 6i$

2.Bereken r en θ:

a) $\mathbf{2 - 2i}$

$r = \sqrt{a^2 + b^2}, \sin(\theta) = \frac{b}{r}, \cos(\theta) = \frac{a}{r}$

$r = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$

$\sin(\theta) = \frac{-2}{2\sqrt{2}} = \frac{-1}{\sqrt{2}} = \frac{-\sqrt{2}}{2}$ (*noemer wortelvrij maken, beide leden maal wortel* 2)

$\cos(\theta) = \frac{a}{r} = \frac{2}{2\sqrt{2}} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$ (*noemer wortelvrij maken, beide leden maal wortel* 2)

Als $\cos(\theta) = \frac{\sqrt{2}}{2}$ *dan is* $\theta = \frac{\pi}{4}\ v\ \theta = -\frac{\pi}{4}$

Nu we weten dat de sinus negatief is $\left(\frac{-\sqrt{2}}{2}\right)$ *dus* $\theta = -\frac{\pi}{4}$

b) $\boldsymbol{\sqrt{3} + i}$

$r = \sqrt{a^2 + b^2}, \sin(\theta) = \frac{b}{r}, \cos(\theta) = \frac{a}{r}$

$r = \sqrt{3+1} = \sqrt{4} = 2$

$\sin(\theta) = \frac{1}{2}$

$\cos(\theta) = \frac{\sqrt{3}}{2}$

Als $\cos(\theta) = \frac{\sqrt{3}}{2}$ *dan is* $\theta = \frac{\pi}{6}\ v\ \theta = -\frac{\pi}{6}$

Nu we weten dat de sinus positief is $\left(\frac{1}{2}\right)$ *dus* $\theta = \frac{\pi}{6}$

c) $\mathbf{1 - i}$

$r = \sqrt{a^2 + b^2}, \sin(\theta) = \frac{b}{r}, \cos(\theta) = \frac{a}{r}$

$r = \sqrt{1+1} = \sqrt{2}$

$\sin(\theta) = \frac{-1}{\sqrt{2}} = \frac{-\sqrt{2}}{2} (noemer\ wortelvrij\ maken, beide\ leden\ maal\ wortel\ 2)$

$\cos(\theta) = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2} (noemer\ wortelvrij\ maken, beide\ leden\ maal\ wortel\ 2)$

Als $\cos(\theta) = \frac{\sqrt{2}}{2}\ dan\ is\ \theta = \frac{\pi}{4}\ v\ \theta = -\frac{\pi}{4}$

$Nu\ we\ weten\ dat\ de\ sinus\ negatief\ is \left(\frac{-\sqrt{2}}{2}\right) dus\ \theta = -\frac{\pi}{4}$

d) $\mathbf{-1 - i}$

$r = \sqrt{a^2 + b^2}, \sin(\theta) = \frac{b}{r}, \cos(\theta) = \frac{a}{r}$

$r = \sqrt{1+1} = \sqrt{2}$

$\sin(\theta) = \frac{-1}{\sqrt{2}} = \frac{-\sqrt{2}}{2} (noemer\ wortelvrij\ maken, beide\ leden\ maal\ wortel\ 2)$

$\cos(\theta) = -\frac{1}{\sqrt{2}} = -\frac{\sqrt{2}}{2} (noemer\ wortelvrij\ maken, beide\ leden\ maal\ wortel\ 2)$

Als $\cos(\theta) = -\frac{\sqrt{2}}{2}$

$dan\ is\ \theta = \frac{5\pi}{4} (antisupplementair) v\ \theta = \frac{3\pi}{4} (supplementair)$

$Nu\ we\ weten\ dat\ de\ sinus\ negatief\ is\ \left(\frac{-\sqrt{2}}{2}\right)\ dus\ \theta = \frac{5\pi}{4}$

e) $\boldsymbol{2i}$

$r = \sqrt{a^2 + b^2}, \sin(\theta) = \frac{b}{r}, \cos(\theta) = \frac{a}{r}$

$r = \sqrt{0 + 2} = \sqrt{4} = 2$

$\sin(\theta) = \frac{2}{2} = 1$

$\cos(\theta) = \frac{0}{2} = 0$

Als $\sin(\theta) = 1\ dan\ is\ \theta = \frac{\pi}{2}$

3.Bereken a en b:

a) r = 2 en $\boldsymbol{\theta = \frac{\pi}{4}}$

$r = \sqrt{a^2 + b^2}, \sin(\theta) = \frac{b}{r}, \cos(\theta) = \frac{a}{r}$

$\sin(\theta) = \frac{\sqrt{2}}{2} = \frac{b}{2} => b = \sqrt{2}$

$\cos(\theta) = \frac{\sqrt{2}}{2} = \frac{a}{2} => a = \sqrt{2}$

b) r = 2 en $\boldsymbol{\theta = \frac{\pi}{6}}$

$r = \sqrt{a^2 + b^2}, \sin(\theta) = \frac{b}{r}, \cos(\theta) = \frac{a}{r}$

$\sin(\theta) = \frac{1}{2} = \frac{b}{2} => b = 1$

$\cos(\theta) = \frac{\sqrt{3}}{2} = \frac{a}{2} => a = \sqrt{3}$

c) r = 1 en $\boldsymbol{\theta = \frac{\pi}{2}}$

$r = \sqrt{a^2 + b^2}, \sin(\theta) = \frac{b}{r}, \cos(\theta) = \frac{a}{r}$

$\sin(\theta) = 1 = \frac{b}{1} => b = 1$

$\cos(\theta) = 0 = \frac{a}{1} => a = 0$