GEOMETRIC STRUCTURES AND SUPERSYMMETRY

This 4-day conference on August 23–26, 2022 is organized under the framework of the TMS/TFS project "Pure Mathematics in Norway" (https://www.puremath.no/meetings/)

Conference homepage: https://super-tromso.puremath.no/

Organizers: Boris Kruglikov (UiT), Andrea Santi (Roma Tor Vergata), Dennis The (UiT)

Conference location: UiT The Arctic University of Norway

	Aug 23, 24, 26 (main campus)	Aug 25 (Forskningsparken)
Conference room	MH U6.A1AUD1	FPARK B281
Lunch location	MH Lysgården	Mersmak kantina

SCHEDULE

(All talks are 45 min + 5 min questions.)

Tuesday, August 23, Room: MH U6.A1AUD1, session chairs:
Kruglikov (morning), Huerta (afternoon)

09:00	Serganova	Geometric methods in representation theory of supergroups
09:50		**Coffee break**
10:20	Grassi	New Cohomologies for Lie Superalgebras
11:10	Mazorchuk	Whittaker categories and Fock space categorification for Lie superalgebras
12:00		**Lunch break**
13:30	Papadopoulos	Twisted covariant form hierarchies
14:20	Palmkvist	Tensor hierarchy algebras and weak associativity
15:10		**Coffee break**
15:40	Lavau	Embedding tensors and differential crossed modules
16:30	Winther	Jet functors in noncommutative geometry

Wednesday, August 24, Room: MH U6.A1AUD1, session chairs: Lewandowski (morning), Palmkvist (afternoon)

09:00	Fioresi	Supergeometry and Quantization
09:50		**Coffee break**
10:20	Lazaroiu	The geometry and DSZ quantization of four-dimensional supergravity
11:10	Toppan	Hopf algebras, First Quantization and braided parastatistics
12:00		**Lunch break**
13:30	Cederwall	Extended geometry, superalgebras, teleparallelism and L_∞
14:20	Huerta	An Invitation to Higher Supergeometry
15:10		**Coffee break**
15:40	Suszek	A (physicist's) categorical approach to supersymmetry
16:30	Grigoriev	(Conformal) geometry as a gauge PDE

Thursday, August 25, Room: FPARK B281, session chair: Figueroa-O'Farrill

09:00	Alekseevsky	Geometry of special rank three Vinberg cones. Application to calculation of the entropy of BPS black holes in N=2 d=4 Supergravity
09:50		**Coffee break**
10:20	The	Exceptionally simple super-PDE
11:10	Marrani	Jordan meets Freudenthal : black hole exceptional symmetries
12:00		**Lunch**
13:00		**Bus departs (from Forskningsparken) for Sommarøy**
18:00		**Dinner at Sommarøy Arctic Hotel**
20:00		**Bus leaves Sommarøy**
21:00		**Bus arrives back in Tromsø (drop off in city center)**

Friday, August 26, Room: MH U6.A1AUD1, session chairs: Santi (morning), Schneider (afternoon)

09:00	Figueroa-O'Farrill	(Almost) all you ever wanted to know about four-dimensional supersymmetry, but were afraid to ask
09:50		**Coffee break**
10:20	Franchetti	Kaluza-Klein reductions of non-standard $D = 5$ maximally supersymmetric backgrounds
11:10	Jiang	Monoidally Graded Manifolds
12:00		**Lunch break**
13:30	Poletaeva	On finite W-algebras for superalgebras and super-Yangians
14:20	Kožić	Associating quantum vertex algebras with super Yangians for $\mathfrak{gl}(m\|n)$
15:10		**Coffee break**
15:40	Wang	Curvature restrictions on a manifold with a flat Higgs bundle
16:30	Svanes	Partition Functions of Heterotic Potentials

TITLES & ABSTRACTS

Dmitri Alekseevsky (IITP Moscow)

Title: **Geometry of special rank three Vinberg cones. Application to calculation of the entropy of BPS black holes in N=2 d=4 Supergravity**

Abstract: The talk is based on joint works with Vicente Cortés (TG, 2021) and Alessio Marrani and Andrea Spiro (JHEP-2021).

E. B. Vinberg developed a theory of homogeneous convex cones based on his theory of matrix T-algebras. A T-algebra M_n of rank n consists of rank n matrices $X = \|X_{ij}\|$ where $X_{ii} \in \mathbb{R}$ and off-diagonal entries $X_{ij} \in V_{ij}$ belong to different Euclidean vector spaces V_{ij} with $V_{ji} = V_{ij}^*$. The matrix multiplication is defined by a system of isometric bilinear maps $\mu_{ijk} : V_{ij} \times V_{jk} \to V_{ik}$ and must satisfy some axioms. A bilinear map between Euclidean vector spaces $\mu : V \times S_0 \to S_1$, $(v, s_0) \to \mu_v s_0 = v \cdot s_0$ is called isometric if $|v \cdot s_0| = |v| \cdot |s_0|$. The main axiom is that the subalgebra $\mathfrak{g}$ of upper triangular matrices is associative and hence defines the group $G \subset \mathfrak{g}$, and this group acts in the subspace $Herm_n \subset M_n$ of hermitian matrices. Associated to M_n homogeneous convex cone $C = Herm_n^+$ is defined as the G-orbit of the identity matrix.

If $\dim S_0 = \dim S_1$, then the isometric map μ is extended to a representation of the Clifford algebra $Cl(V)$ in $S = S_0 + S_1$ and μ become the Clifford multiplication for a $\mathbb{Z}_2$-graded Clifford module $S = S_0 + S_1$. Any $\mathbb{Z}_2$-graded Clifford $Cl(V)$-module $S = S_0 + S_1$ with a $Spin(V)$-invariant Euclidean metric defines a rank 3 T-algebra $M_3(S)$ with $V_{12} = V$, $V_{23} = S_0$, $V_{13} = S_1$ and the associated cone $C = Herm^+(S)$ is called a special rank 3 Vinberg cone.

We describe the differential geometry of such a cone C and its dual cone C^*, calculate the basic polynomial invariants $p_1.p_2, p_3$, defined by Vinberg, and the cubic determinant $d(C)$ which plays an importantg role in Supergravity. It turns out that the cubic determinants for the cones C and C^* coincide: $d(C) = d(C^*)$. The determinant hypersurface $C_1 = \{d(x) = 1\} \cap C$ is homogeneous very special real manifold in the sense of B. de Witt and A. Van Proeyen and it describes the target space of scalar multiplets in $N = 2d = 5$ Supergravity. The supergravity r-map associates to C_1 a homogeneous special Kaehler manifold $K = r(C_1)$, which is the target space of the $d = 4$ Supergravity, obtained by dimensional reduction from the $d = 5$ Supergravity. From mathematical point of view, $r(C_1)$ is the Siegel domain of the first kind defined by the cone C.

We shortly recall the description of stationary BPS black holes in $d = 4$ supergravity and calculate explicitly the entropy of such black hole, using Shmakova formula. The problem reduces to the inversion of the quadratic map $h_d : W \to W^*$, $X \to d(X, X, \cdot)$, $W = Herm_3(S)$, associated to the (polarized) cubic determinant $d(X) = d(X, X, X)$.

Martin Cederwall (Chalmers University of Technology)

Title: **Extended geometry, superalgebras, teleparallelism and** L_∞

Abstract: I will review of the connection between extended geometry (double and exceptional geometry etc.) and tensor hierarchy algebras, a class of non-contragredient superalgebras. Starting with ordinary gravity, the complex containing diffeomorphism parameters, vielbeins, field strengths (torsion), Bianchi identities and so on can be identified with a superalgebra, $W(d)$ in the classification by Kac. Field taking values in the superalgebra can be equipped with a nilpotent derivative, the starting point for an L_∞ algebra. I will explain how this picture is generalised when $GL(d)$ of gravity is changed to other structure groups, leading to extended geometry, and maybe point out some interesting properties and challenges when the structure group becomes infinite-dimensional.

José Figueroa-O'Farrill (University of Edinburgh)

Title: **(Almost) all you ever wanted to know about four-dimensional supersymmetry, but were afraid to ask.**

Abstract: I will give an overview of four-dimensional (N=1, rigid) supersymmetry, concentrating on two approaches I have followed recently. One approach is based on lorentzian spin geometry and, particularly, in the geometric construction of supersymmetry algebras and their associated geometries. This is based on works with Paul de Medeiros, Andrea Santi and, more recently, also Andrew Beckett. The second approach is based on the superisation of homogeneous kinematical spacetimes and is based on work with Ross Grassie.

Rita Fioresi (University of Bologna)

Title: **Supergeometry and Quantization**

Abstract: An important class of Kahler supermanifolds consists of the homogeneous superspaces of simple supergroups. We construct hermitian super line bundles on such supermanifolds and view their holomorphic sections as representations of real forms G of a given simple supergroup G_C. Using the method of geometric quantization, we construct a model X, that is, all irreducible representations of a compact real form G occur in the space of holomorphic sections of super line bundles on X. When G is non compact, extra care must be exerted and we have partial results towards the construction of models.

Guido Franchetti (University of Bath)

Title: **Kaluza-Klein reductions of non-standard $D = 5$ maximally supersymmetric backgrounds**

Abstract: Taking the definition of a Killing spinor to be that of a section of the spinor bundle parallel with respect to some suitably chosen "superconnection", Figueroa–Santi have shown that the Lie superalgebra generated by Killing spinors has a particular algebraic structure, and classified objects with such a structure. To each of them a supergravity-like theory can be associated, with the notions of bosonic background and maximally supersymmetric background formulated entirely in terms of the superconnection curvature.

For certain dimensions this approach not only recovers standard supergravity, but other possibilities arise. One such case is D = 5, and the aim of this talk is to consider Kaluza-Klein reductions by a timelike or spacelike vector field of the maximally supersymmetric backgrounds which do not arise in standard 5D supergravity. These are the locally symmetric spaces $\mathbb{R} \times S^4$, $\mathbb{R} \times AdS^4$ and certain Cahen–Wallach spaces. For each of them we will classify the possible reductions and determine the amount of preserved supersymmetry.

Pietro Grassi (Università del Piemonte Orientale)

Title: **New Cohomologies for Lie Superalgebras**

Abstract: In the talk, we present new results in the computations of cohomologies for Lie Superalgebras. In particular, we will give the first explicit example of an invariant pseudoform representing a cohomology class for the Chevalley-Eilenberg cohomology for $\mathfrak{osp}(2|2)$ superalgebra emerging in $N = 2$ string theory.

Maxim Grigoriev (Lomonosov Moscow State University)

Title: **(Conformal) geometry as a gauge PDE**

Abstract: Gauge PDE is a geometrical object underlying what physicists call a local gauge field theory defined at the level of equations of motion (i.e. without specifying Lagranian) in terms of BV-BRST formalism. This notion generalises the BV-BRST formulation in terms of jet-bundles on the one hand and the geometrical approach to PDEs on the other hand. In the case of standard "geometrical" gauge theories such as Einstein gravity and conformal gravity the respective gauge PDE directly relates the usual (metric-like) and Cartan-like (frame-like) descriptions of the gauge systems and the underlying geomeries. In the language of gauge PDEs the Lagrangian formulation is encoded in the compatible graded presymplectic structure defined on the equation manifold, giving an invariant approach to Lagrangians of gauge systems. We illustrate the construction with the examples of Einstein gravity and Yang-Mills theory and give a more detailed account of the conformal gravity. In particular, we present a presymplectic AKSZ formulation of the conformal gravity in terms of the minimal model of its BRST complex. We also discuss applications of these structures in studying conformal invariants.

John Huerta (Technical University of Lisbon)

Title: **An Invitation to Higher Supergeometry**

Abstract: The philosophy of higher structures is to import ideas from homotopy theory and higher category theory into other parts of mathematics. This import enriches the mathematical objects and makes them more flexible, enabling new perspectives and new theorems, at the cost of new machinery. Though that machinery can be burdensome, supersymmetry demands higher structures. This is most vividly illustrated by the homotopy actions of the supersymmetry algebra constructed by Elliott, Safronov and Williams, but it also appears in the superstring 2-group of the author. Using nothing more sophisticated than a chain complex, we describe these examples, and sketch future directions.

Shuhan Jiang (MPI Leipzig)

Title: **Monoidally Graded Manifolds**

Abstract: We give a generalization of the theory of $\mathbb{Z}_2$-graded manifolds to a theory of $\mathbb{J}$-graded manifolds, where $\mathbb{J}$ is a commutative semi-ring with some additional properties. We prove Bachelor-type theorems in this generalized setting. To our knowledge, such proofs are still missing except for some special cases.

Slaven Kožić (University of Zagreb)

Title: Associating quantum vertex algebras with super Yangians for $\mathfrak{gl}_{m|n}$

Abstract: One important problem in the theory of vertex algebras is to associate certain vertex algebra-like structures, the *quantum vertex algebras*, to various classes of quantum groups, such as quantum affine algebras or double Yangians. Roughly speaking, the goal is to establish a correspondence between these structures that goes in parallel with the connection between affine Kac–Moody Lie

algebras and vertex algebras. In this talk, I will illustrate this problem in the context of Etingof–Kazhdan's construction of quantum vertex algebras associated with Yangians for $\mathfrak{gl}_n$. Moreover, I will discuss possible generalizations of these results to super Yangians for $\mathfrak{gl}_{m|n}$ and their applications.

Sylvain Lavau (Aristotle University of Thessaloniki)

Title: **Embedding tensors and differential crossed modules**

Abstract: Gauging procedures in supergravity theories depart from classical gauge theories because the gauge algebra is a subalgebra $\mathfrak{h}$ of the algebra $\mathfrak{g}$ of global symmetries of the system, and the gauge fields usually take values in the fundamental representation V of $\mathfrak{g}$. For the consistency of the theory, one then needs to introduce: 1. a set of differential p-forms taking values in a chain complex of $\mathfrak{g}$-modules that physicists call the *tensor hierarchy*, as well as 2. a linear map $V \longrightarrow \mathfrak{g}$ surjective on $\mathfrak{h}$ called the *embedding tensor* and which turns V into a *Leibniz algebra*. Two such algebras V and $\mathfrak{g}$, together with their embedding tensor, form a triple called a *Lie-Leibniz triple*, of which differential crossed modules are particular cases. Then we can prove that any Lie-Leibniz triple canonically induces a differential graded Lie algebra – its associated tensor hierarchy – whose restriction to the subcategory of differential crossed modules is the canonical assignment associating to any differential crossed module its corresponding unique 2-term differential graded Lie algebra. This shows that Lie-Leibniz triples form natural generalizations of differential crossed modules and that their associated tensor hierarchies can be considered as some kind of 'lie-ization' of the former. As Courant algebroids and G-algebroids form natural classes of Leibniz algebroids, we will conclude by discussing the "oidization" of Lie-Leibniz triples and how the former are possibly related to the latter.

Calin Lazaroiu (IFIN-HH Bucharest)

Title: **The geometry and DSZ quantization of four-dimensional supergravity**

Abstract: We develop the Dirac-Schwinger-Zwanziger (DSZ) quantization of four-dimensional bosonic ungauged supergravity on an oriented four-manifold of arbitrary topology and use it to obtain its manifestly duality-covariant gauge-theoretic geometric formulation in terms of connections on a certain kind of principal bundles.

Alessio Marrani (University of Murcia)

Title: **Jordan meets Freudenthal : black hole exceptional symmetries**

Abstract: I will review some aspects of the electric-magnetic duality in extremal black hole solutions of Maxwell-Einstein-scalar theories (which can be regarded as the purely bosonic sector of ungauged extended supergravity) in four space-time dimensions. Such aspects include the attractor mechanism, the geometry of the scalar manifolds, the duality orbits and the moduli space associated to the various classes of attractors. In particular, I will highlight the role of Jordan algebras of rank three, of the corresponding reduced Freudenthal triple systems and of their exceptional symmetries, and I will elucidate the relation between the Hessian of the black hole entropy and the pseudo-Euclidean, rigid special (pseudo)Kähler metric of the pre-homogeneous spaces associated to the duality orbits. I will then introduce the Freudenthal duality map acting on the electric-magnetic fluxes, and present the non-linear invariance of the Bekenstein-Hawking black hole entropy. I will then consider the axiomatization of groups "of type E_7" as introduced by Brown, highlighting their role as electric-magnetic duality groups, as well as their relation to pre-homogeneous vector spaces. Finally, relying

on some results of Dynkin and Solomon, I will present various novel (numerably) infinite classes of groups "of type E_7". I will conclude with an outlook on further developments.

Volodymyr Mazorchuk (Uppsala University)

Title: **Whittaker categories and Fock space categorification for Lie superalgebras**

Abstract: In this talk we will realize several catgories of Whittaker modules over a type I Lie superalgebra as cokernel categories that fit into the framework of properly stratified categories. For the general linear Lie superalgebra this leads to a categorification of the q-symmetrized Fock space over a quantum group of type A with an explicit realization of the canonical and the dual canonical bases. This is a report on a joint work with Chih-Whi Chen and Shun-Jen Cheng.

Jakob Palmkvist (Örebro University)

Title: **Tensor hierarchy algebras and weak associativity**

Abstract: Tensor hierarchy algebras are infinite-dimensional generalisations of Lie superalgebras of Cartan type, which have proven useful in the description of certain gauge structures in physics. In particular, they have turned out to play a crucial role in the framework of extended geometry, where gauge transformations are unified with diffeomorphisms. In my talk, I will present a new construction of some tensor hierarchy algebras, where the brackets originate from the commutator in an algebra which is not associative, but satisfies a weaker form of associativity.

George Papadopolous (King's College London)

Title: **Twisted covariant form hierarchies**

Abstract: I shall demonstrate that the conditions on the form bilinears imposed by the Killing spinor equations of any (super) gravity theory can be organised as a twisted covariant form hierarchy. Amongst other things, this implies that the bilinears satisfy the conformal Killing-Yano equation with respect to the hierarchy connection. I shall also explore the consequences that this has on the integrability of dynamics of probes, which include geodesic flows, on supersymmetric gravitational backgrounds.

Elena Poletaeva (University of Texas Rio Grande Valley)

Title: **On finite W-algebras for superalgebras and super-Yangians**

Abstract: Let $Q(n)$ be the queer Lie superalgebra, W^n be the finite W-algebra for $Q(n)$ associated with the principal even nilpotent coadjoint orbit, and $YQ(n)$ be the super-Yangian of $Q(n)$. There exists a surjective homomorphism from $YQ(1)$ onto W^n. We classify simple W^n-modules (they are all finite-dimensional) and simple finite-dimensional $YQ(1)$-modules. We determine conditions under which two 1-dimensional modules over $YQ(1)$ and over W^n can be extended nontrivially. It is a joint work with V. Serganova.

Vera Serganova (University of California, Berkeley)

Title: **Geometric methods in representation theory of supergroups**

Abstract: We will discuss how geometric properties of homogeneous superspaces can be used in representation theory of an algebraic supergroup G with a reductive underlying group. The main goal is to associate with any representation M of G a geometric object which we call the support variety of M. This support variety is a subset in the odd part of the Lie superalgebra of G. The construction involves certain cohomological tensor functors from the category of representations of G to vector superspaces. Computation of these functors in the space of functions on different homogeneous superspaces G/H allows us to prove projectivity detection for our support varieties. It also provides potentially a new approach to a Borel-Weil-Bott theorem in super case.

Rafał R. Suszek (University of Warsaw)

Title: **A (physicist's) categorical approach to supersymmetry**

Abstract: A careful study of the lagrangean dynamics, formulated in terms of Cheeger-Simons–type differential characters, of extended distributions of charge – such as the bosonic string – in ambient metric manifolds (*aka* targets) permeated by external $(p+2)$-form fields (*i.e.*, integral de Rham $(p+2)$-cocycles) leads to the emergence of geometric higher categories – such as the bicategory of 1-gerbes over a given manifold – as structures which most neatly model and classify the geometric phases of the $(p+1)$-dimensional worldvolume field theory and self-intersecting defects between them. The pay-off for the rigour is immense: The higher categories provide us with a (pre)quantisation scheme for the dynamics, distinguish its quantum-mechanically consistent boundary conditions, determine the gauging of its global symmetries and critically affect the so-called emergent target geometry and effective field theory associated with the fluctuating distribution of charge. Remarkably enough, they also capture (and categorify) prequantisable symmetries of the field theory, and dualities between field theories. This has particularly far-reaching consequences in field theories with highly symmetric targets given by homogeneous spaces of Lie groups.

In the talk, a research programme shall be delineated in which the above lessons from the ungraded geometry are taken seriously in the setting of supergeometry with supersymmetry, with focus on Green-Schwarz–type dynamics of super-p-branes in homogeneous spaces G/H of a supersymmetry Lie supergroup G as super-targets. In its course, we shall discuss super-geometrisations of physically relevant classes in G-invariant de Rham cohomology of G/H and take the first (simplicial) step towards the full super-geometric defect bicategory for the superstring. We shall also provide an account of a higher-geometric emanation of the so-called κ-symmetry and, time permitting, touch upon a proposal for a categorification of an İnönü-Wigner–type relation between curved and flat super-targets.

Eirik Svanes (University of Stavanger)

Title: **Partition Functions of Heterotic Potentials**

Abstract: We compute the one loop partition function of the superpotential and Kähler potentials of the geometric sector of six-dimensional heterotic compactifications. The results are interesting both from an ordinary mirror symmetry and heterotic (0,2) mirror symmetry point of view.

Dennis The (UiT The Arctic University of Norway)

Title: **Exceptionally simple super-PDE**

Abstract: In 1893, Cartan and Engel gave the first geometric realisations of the (complex) exceptional simple Lie algebra G_2, namely as the symmetries of various differential geometric structures. I'll describe generalizations of this story to the other exceptional simple Lie algebras, and more recently the exceptional simple Lie superalgebras $G(3)$ (joint work with Kruglikov & Santi) and $F(4)$ (joint work with Santi).

Francesco Toppan (Centro Brasileiro de Pesquisas Fisicas)

Title: **Hopf algebras, First Quantization and braided parastatistics**

Abstract: The formalism of graded Hopf algebras endowed with a braided tensor product is applied to define multiparticle sectors and parastatistics of quantum mechanical models. I discuss three recent applications of this framework: the proofs of the detectability of the Rittenberg-Wyler $\mathbb{Z}_2 \times \mathbb{Z}_2$-graded parafermions and parabosons, and the braiding of the Majorana fermions regarded as $\mathbb{Z}_2$-graded qubits. The talk is based on:
F.T., $\mathbb{Z}_2 \times \mathbb{Z}_2$-graded parastatistics in multiparticle quantum Hamiltonians, J. Phys. A: Math. Theor. 54, 115203 (2021).
F. T., Inequivalent quantizations from gradings and $\mathbb{Z}_2 \times \mathbb{Z}_2$ parabosons, J. Phys. A: Math. Theor. 54, 355202 (2021).
F.T., First quantization of braided Majorana fermions, Nucl. Phys. B 980 (2022) 115834.

Xu Wang (Norwegian University of Science and Technology)

Title: **Curvature restrictions on a manifold with a flat Higgs bundle**

Abstract: We shall prove a semi-negative curvature property for a manifold with a flat admissible Higgs bundle.

Henrik Winther (Masaryk University)

Title: **Jet functors in noncommutative geometry**

Abstract: We construct an infinite family of endofunctors J^n_d on the category of left A-modules, where A is a unital associative algebra over a commutative ring k, equipped with an exterior algebra $\Omega^\bullet_d$. We prove that these functors generalize the corresponding classical notion of jet functors. The functor J^n_d comes equipped with a natural transformation from the identity functor to itself, which plays the rôle of the classical prolongation map. This allows us to define the notion of linear differential operator with respect to $\Omega^\bullet_d$. These retain most classical properties of differential operators, and operators such as partial derivatives and connections belong to this class. Moreover, we construct a functor of quantum symmetric forms S^n_d associated to $\Omega^\bullet_d$, and proceed to introduce the corresponding noncommutative analogue of the Spencer δ-complex. We give necessary and sufficient conditions under which the jet functor J^n_d satisfies the jet exact sequence, $0 \to S^n_d \to J^n_d \to J^{n-1}_d \to 0$. This involves imposing mild homological conditions on the exterior algebra, in particular on the Spencer cohomology $H^{\bullet,2}$. This is a joint work with K.Flood and M.Mantegazza.
