GEOMETRIC STRUCTURES AND SUPERSYMMETRY

This 2-day virtual conference on August 2 & 3, 2021 is organized under the framework of the BFS/TFS project "Pure Mathematics in Norway" (https://site.uit.no/puremath/meetings)

For the conference Zoom link, please register by sending an email to: dennis.the@uit.no

Organizers: Boris Kruglikov (UiT), Andrea Santi (UiT), Dennis The (UiT)

SCHEDULE

(All talks are 45 min + 5 min questions.)

Monday, August 2

15:00	Papadopoulos	*Classification of AdS backgrounds*
16:00	Mazorchuk	*Simple supermodules over Lie superalgebras*
17:00	Cederwall	SL(5) *supersymmetry*
18:00	Poletaeva	*On representations of finite W-algebras and super Yangians*

Tuesday, August 3

15:00	Santi	G(3) *supergeometry and a supersymmetric extension of the Hilbert–Cartan equation*
16:00	Svanes	*Heterotic moduli without field redefinitions*
17:00	Toppan	*Detectability of parastatistics induced by Rittenberg-Wyler color Lie (super)algebras*
18:00	Serganova	*Volumes of supergrassmannians and their application to representation theory of algebraic supergroups.*

TITLES & ABSTRACTS

Martin Cederwall (Chalmers University of Technology)

Title: SL(5) supersymmetry

Abstract: In 5 dimensions, there is the peculiar possibility of introducing supersymmetry generators which are 2-forms (instead of spinors), which breaks gl(5) only to sl(5) (instead of so(5)). I discuss how supermultplets may be found from cohomology of superfields that depend on a constrained bosonic object λ, analogous to a pure spinor, in addition to x and θ. The supermultiplet obtained from a scalar superfield turns out to the coadjoint module of the exceptional, infinite-dimensional, superalgebra called E(5,10). On the other hand, the multiplet is encoded in the partition function of λ, and in turn related by a Koszul duality to the positive levels of a Borcherd superalgebra (or tensor hierarchy superalgebra) extension of sl(5) encountered in exceptional geometry. There is a curious relation between the different superalgebras, which will be discussed. I may also display some other supermultiplet.

Volodymyr Mazorchuk (Uppsala University)

Title: Simple supermodules over Lie superalgebras

Abstract: In this talk I will discuss how one can reduce the problem of classification of simple supermodules over Lie superalgebras to the problem of classification of simple modules over Lie algebras. This is based on joint projects with Chih-Whi Chen and Kevin Coulembier.

George Papadopolous (King's College London)

Title: Classification of AdS backgrounds

Abstract: I shall summarise some of the progress made so far towards the classification of supersymmetric AdS backgrounds. This includes some of the local and global techniques used as well as some of the results obtained. I shall present a classification of all AdS backgrounds in 10 and 11 dimensions that preserve more 16 supersymmetries.

Elena Poletaeva (University of Texas Rio Grande Valley)

Title: On representations of finite W-algebras and super Yangians

Abstract: A finite W-algebra is a certain associative algebra attached to a pair $(\mathfrak{g}, e)$, where $\mathfrak{g}$ is a complex semisimple Lie algebra and $e \in \mathfrak{g}$ is a nilpotent element. It is a generalization of the universal enveloping algebra $U(\mathfrak{g})$. There is a connection between finite W-algebras and Yangians. We classify irreducible representations of finite W-algebra for the queer Lie superalgebra $Q(n)$ associated with the regular even nilpotent coadjoint orbits. We use this result to obtain a classification of irreducible finite-dimensional representations of the super Yangian $YQ(1)$.

Andrea Santi (UiT The Arctic University of Norway)

Title: $G(3)$ supergeometry and a supersymmetric extension of the Hilbert–Cartan equation

Abstract: I will report on the realization of the simple Lie superalgebra $G(3)$ as supersymmetry of various geometric structures – most importantly super-versions of the Hilbert–Cartan equation and Cartan's involutive PDE system that exhibit $G(2)$ symmetry – and compute, via Spencer cohomology groups, the Tanaka–Weisfeiler prolongation of the negatively graded Lie superalgebras associated with two particular choices of parabolics. I will then discuss non-holonomic superdistributions with growth vector $(2|4, 1|2, 2|0)$ obtained as super-deformations of rank 2 distributions in a 5-dimensional space, and show that the second Spencer cohomology group gives a binary quadric, thereby providing a "square-root" of Cartan's classical binary quartic invariant for $(2,3,5)$-distributions. If time allows, I will outline an extension of Tanaka's geometric prolongation scheme to the case of supermanifolds. This is a joint work with B. Kruglikov and D. The.

Vera Serganova (University of California, Berkeley)

Title: Volumes of supergrassmannians and their application to representation theory of algebraic supergroups.

Abstract: The supergrassmannians and Q-supergrassmannians have G-invariant volume forms for a suitable compact supergroups. We determine in which cases the volumes of supergrassmannians are not zero. We use the Schwartz-Zaboronsky localization formula which generalizes the well-known Duistermaat-Heckman formula for certain compact supermanifolds. Then we explain the application of this computation to representation theory of algebraic supergroups, in particular, to the support variety theory.

Eirik Svanes (University of Stavanger)

Title: Heterotic moduli without field redefinitions

Abstract: I will review recent breakthroughs in understanding on-shell heterotic moduli from the past decade, many of which required thinking in terms of generalised geometry, doubled field theory, L-infinity algebras, etc. In most of these formulations an extra set of End(TX)-valued fields are needed for the description to make sense. These unphysical fields are often given the interpretation of field redefinitions. I will take some modest steps towards an understanding of the moduli problem without these spurious degrees of freedom. In particular, I will describe the cohomology which counts the infinitesimal massless modes.

Francesco Toppan (Centro Brasileiro de Pesquisas Fisicas)

Title: Detectability of parastatistics induced by Rittenberg-Wyler color Lie (super)algebras

Abstract: The $\mathbb{Z}_2 \times \mathbb{Z}_2$-graded "color" Lie (super)algebras introduced by Rittenberg and Wyler in 1978 produce testable consequences. Color superalgebras extend ordinary dynamical systems by allowing particles to be accommodated into n bits of information. In the ordinary case bosons and fermions are described by 1 bit (0 for bosons, 1 for fermions), while particles in $\mathbb{Z}_2 \times \mathbb{Z}_2$-graded physics, e.g., are described by 2 bits: $00, 10, 01, 11$. In layman's terms, these particles are "colored". It was proven in J. Phys. A: Math. Theor. **54**, 115203 (2021) that carefully selected quantum observables acting on suitable multiparticle states can discriminate whether a system under consideration is composed by ordinary bosons/fermions or by $\mathbb{Z}_2 \times \mathbb{Z}_2$-graded particles. The implication is that the colored parastatistics cannot be mimicked by black/white pictures of ordinary bosons/fermions. The multiparticle states are obtained via the braided tensor product introduced by Majid in 1995, within the Hopf algebra structure of Universal Enveloping graded-Lie Algebras.

Sometimes a general argument about the conventionality of parastatistics (and based on the existence of the Klein transformations) is invoked. In the talk I illustrate the mechanism which allows to circumvent the applicability of this argument in the color algebra setting.

Once established that $\mathbb{Z}_2 \times \mathbb{Z}_2$-graded Lie superalgebras produce testable predictions, the further task is to investigate their possible applications. One can mention fundamental physics (quantum gravity at the Planck's length scale, dark matter, etc.), laboratory physics (emergent phenomena such as (para)fermionic collective modes in condensed matter) and even direct construction (for instance, by building topological metamaterials with legos).

Comment: in the last couple of years colored Lie superalgebras received a renewed attention with the first systematic investigation of their invariant classical and quantum dynamical systems. Several researchers and groups are contributing to this line (Bruce, Duplij, Aizawa, Kuznetsova, ...). This talk is mostly based on the recent papers:

- F. T., J. Phys. A: Math. Theor. **54**, 115203 (2021); arXiv:2008.11554,
- Z. Kuznetsova and F. T., arXiv:2103.04385 (to appear in J. Math. Phys.),
- F. T., *Inequivalent quantizations from gradings and* $\mathbb{Z}_2 \times \mathbb{Z}_2$ *parabosons*, arXiv:2104.09692.
- N. Aizawa, Z. Kuznetsova and F. T., Eur. J. Phys. **C 80**, 668 (2020); arXiv:2003.06470,
- N. Aizawa, Z. Kuznetsova and F. T., Nucl. Phys. **B 967**, 115426 (2021); arXiv:2005.10759.