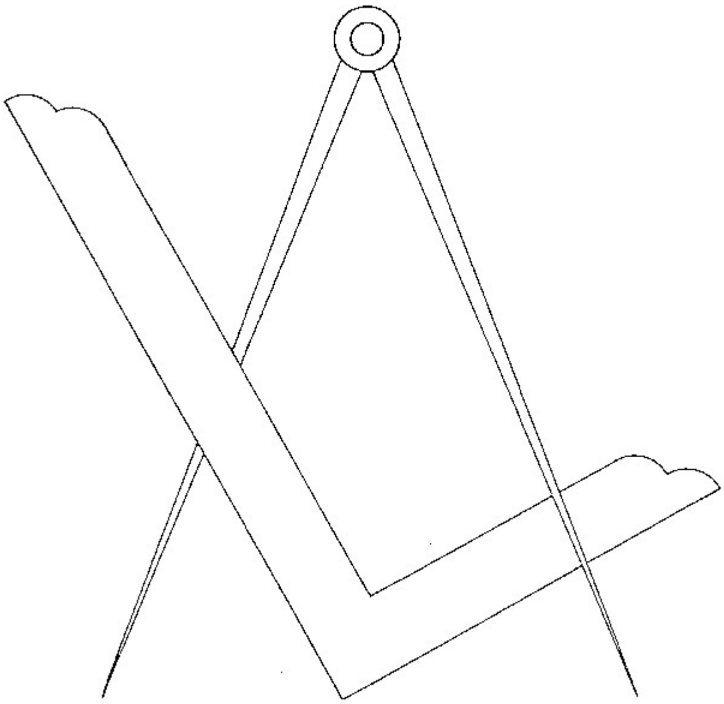
Geometrical Proportions in Classical Violin Design

A Report on the Rules of the Old Masters



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Contents

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Siamo alle solite

Introduction

This booklet is devoted to one of mankind's most beautiful as well as most mysterious creations: the instruments of the Violin Family. For centuries they played, and they still play, one of the leading parts in Western Music. Innumerable inspired words have been dedicated to them, their beauty and soulfulness of sound, their perfection of design – indeed, anything imaginable about them being beautiful and magnificent. Many questions have been asked as to the "secrets" of the instruments – in what way such a seemingly simply contrived object might be able to generate that immense richness of musical expression.

Consequently, reports and narratives about the production of the instruments, the work conditions and social situation of the makers, their education, traditions and "schools" etcetera have come into existence. By and by other kinds of problems were taken up in studies concerning, e.g., acoustics, woods, handicraft, as well as questions with reference to the music performed: playing technique, repertoire etc. Thus, the literature on violin instruments has grown very extensive and covers many branches.

Now, this text leaves aside most of the general story about the instruments, their manufacture and use, and concentrates exclusively on the following fundamental problem:

In which ways and according to which principles might the "old masters" have designed their instruments during the "classical" period, when common measuring units and tools (in our modern sense) differed locally and, not the least, according to the changing political situations, as well?

The question is crucial, and philosophical as much as practical since it deals with two different, important principles – "proportions" and "measurings" – in the designing of the instruments. We know that the latter principle was the winner in the industrial revolution of the 19th century, but we don't know, more in detail, either the form and application of the older principles, or why they lost the competition. Many scholars and instrument experts have pondered over the problem – and, as well, any musician holding a sincere interest in her/his instrument: the imagination of the "secrets" that might be hidden in the beloved instrument would seem to be a constantly present question ...

I

Backgrounds

The guilds

In the middle of the 16th century, when the typical traits of the violin family originated, there already existed a rich literature about "proportions" and "proportioning", all of it founded on *De architectura* by the Roman architect Vitruvius.[1]

His work was well known, although only in many different, more or less fragmentary copies, during the Middle Ages, but was actualized to a great extent at the beginning of the 15th century (as a part of the Renaissance culture) and turned to be the dominant authority concerning architecture. Unfortunately, Vitruvius's own illustrations have not been preserved, e.g., the figure announced in book III, section V, which in detail was intended to demonstrate the construction of the Ionic volute – a problem with bearing on, e.g., the design of the violin scroll. However, to many of the early editions of Vitruvius's work the editors added illustrations elaborated on the basis of his written descriptions. In addition to the mentioned architectural works other writings emerged dealing with methods more suitable for proportioning in the liberal arts and handicrafts: arithmetical methods inherited from the classical antiquity (including mysticism of numbers) and Euclidean geometry. Thus, the repertoire of methods was extensive although limited to Euclidean constructions using only compasses and square or ruler, methods useful in order to materialize arithmetical proportions as well. Hermann Graf (1958) summarizes:

> Als Hilfe zur Erziehlung guter Proportionen an Werken aller Art der bildenden Künste […] wurden mit mehr oder minder klaren Beweisen festgestellt oder nur als verwendet vermutet: Modulsysteme, Rechtecknetze, Rechtecke, die zwischen dem Quadrat und dem Doppelquadrat liegen, und ihre Diagonalverspannungen, Quadratur, Triangulatur mit verschiedenen Drei-ecken besonderer Art, Systeme der regelmässigen Vielecke […]

He adds, e.g., the "golden section", similar figures, rays, perspective partitions, aleatory, musical proportions and number series.

Nevertheless, instructions directed exclusively to the makers of musical instruments are sought after nearly completely in vain. However, this is not very remarkable since the art of the architects at that time was more theoretical than practical and could be dealt with in scholarly writings while the handicraftsmen – among them the instrument makers – inherited and developed their skills inside the more or less closed guilds. Jouven (1979)

1 *De architectura*, ten "books" (papyrus rolls) preferably dealing with architecture, written in Rome some decades B.C.

maintains that proportioning methods were in common use, however, that the geometrical methods were reserved for the guilds and were kept a profound secret by them.
Remarkably enough, Athanasius Kircher didn't with one single word touch the subject in his encyclopedic *Musurgia universalis*, Rome 1650, a central work on, e.g., music instruments at that time.

The fact that instrument makers made extensive use of mathematical methods when designing their instruments has been demonstrated by a number of modern scholars, especially Herbert Heyde, who provides (1986) a broad and excellent introduction to the whole field of problems. He also gives a comprehensive account of linear measure units in the 15th to 19th centuries and lists relevant literature. As to the violin instruments, Heyde primarily deals with the main proportions of the body, but he also discusses a number of the "interpretations" of the violin design launched by Adolf Beck (1923), Simone F. Sacconi (1972), Wolfgang Stalling (1983) and others. Ambitious attempts at an objective description and analysis of proportions, based on measuring were made by , e.g., Lars Frydén (1977), Heyde in an early study (1984) and Kevin Coates (1985). Seemingly there is a growing interest in the subject.[2] However, the proportioning of the soundholes and the scroll was not paid attention to until more recently, in Lomnäs 1997 and 1998, respectively, both summarized in the present book, Sections III and IV.

Thus far concerning the proportioning methods and the research work up to now. However, there still exist a number of important aspects waiting for further investigation. For example: To which extent were the proportioning methods mentioned above known and applied by the instrument makers in general? In what regions? During which periods? In different ways, with different execution, limitations or extensions?

The fact that, hitherto, no documents were found describing or even mentioning the existence of "rules" or methods does not mean or indicate that they did not exist.[3] As will be demonstrated in this book the "rules" could be very simple, easy to learn and remember, and but little time-consuming when applied. As a consequence, they did not need to be written down at all, just memorized – although with the severe fragility that they could be lost forever if they were neglected during one sole generation of makers.

2 Coates 1985 was paid attention to and had some followers, e.g., Gug 1990. Maybe other musicologists as well have studied the problems, yet without publishing the results; they are however unknown to me.
However, it is a deplorable fact that Heyde's incomparably more well-informed works (1984, and especially 1986) have (so far) not been met with appropriate response – not even Stalling 1992 does mention Heyde.

3 In Museo Civico, Cremona, are kept patterns and models, attributed to Antonio Stradivari, related to a number of violin instruments, including details such as soundholes, scrolls and pegboxes (Sacconi 1972, cf. in addition Pollens 1993, Woodraw 1991). In the same museum are also preserved, although not thoroughly examinated, utensils believed to come down from the workshop of the violin maker family Ceruti in Cremona (Santoro 1991: 135f. with reference to Bacchetta 1937).

However, the makers had to face the rapidly changing political and economical situation in Europe during the later decades of the 18th century. The market for individually designed instruments made by the traditionally working makers rapidly weakened in favour of cheaper products. More rational and useful for the maker now was the "copying" of instrument models that already had turned out to be successful on the market.

That was what happened, however, mainly outside of Italy: in France, Bohemia and Germany an important half industrial activity came about. The old guilds lost their foothold and disappeared by degrees. Instead of continuing the traditions several young makers left their profession completely.[4] In addition, the rationalism at that time looked down upon the obsolete "superstition" and "mysticism" that cleaved to *sectio divina*, number series and other within the guilds preserved and carefully hidden knowledges.

But the traditional craftsmanship survived, mainly as previously, in small-scaled family undertakings, in Italy seemingly in a more conservative way than elsewhere, and adhering to the old conventions. There is nothing known to prove that the guild rules would have been abandoned at that time, not even the fact that Antonio Bagatella in his ambitious effort was unsuccessful in his search for "the truth".[5] Instead he introduces an elementary measuring system – the body length of the instrument was divided in 72 equal parts – without any relation to the traditional proportioning methods which will be described here, in the present book. Peluzzi 1978: 104ff. quotes an anonymous (later possibly lost) manuscript that mentions the 72-parts system but nothing about traditional proportioning (yet gives a description of the violin scroll spiral). The reason for the meagre result of Bagatella's investigation, made in the years just before 1782, might have been quite simple: no tradition carrier could be found or was inclined to reveal his knowledge to a young stranger – especially not as the intention of that person was to publish the "secrets". The following case may add further aspects to the situation.

It seems probable that Giambattista Guadagnini (ca. 1711-1786), highly praised by posterity, was a thoroughly educated and capable master of his profession well acquainted with the traditions of his guild. However, when he from 1772 onwards, in constant need of money, started working for count Alessandro Cozio di Salabue (1755-1840),[6] he had to follow all instructions ("sotto i miei occhi"), even if they were contrary to his own opinions,

4 In addition to Heyde 1986 and Jouven 1979, cf. Osse 1994.

5 Bagatella 1786. Essay delivered 1782 to the Academy of Science in Padua. The subject was a description of a method for the production of high class violins founded on traditional knowledge. Seemingly, Bagatella (1755-1829) had some knowledge in violin making, and experience of his own. In addition he established contact with several of his contemporary violin makers in order to find out their methods.

6 For all particulars as to the circumstances related to Cozio di Salabue, cf. Cozio 1950; the citation is found on p. 426. The editor of the book, Giovanni Iviglia, comments on this prerevolutionary social class drama (footnote 1): "Il lettore può ora immaginarsi la lotta fra due titani, ognuno dei quali difendeva ragioni sacrosante: fortuna volle che Guadagnini, plebeo, ignorante, cocciuto, la vincesse zu Cozio, nobile, studioso, tenace la sua parte, il suo feticismo per Stradivari, però, quasi quasi stava per privarci di una differenziazione artistica preziosissima."

and in spite of Cozio's lack of professional knowledge and experience in violin making.[7] Cozio estimated Antonio Stradivari's works much more than Guadagnini's (however, Jakob Stainer's still more than Stradivari's!) and Guadagnini had to work on Stradivari's patterns, in 1775 purchased by Cozio from Stradivari's son Paolo. There was a split between the old Master in his profession (although illiterate, according to Cozio 1950: 395), and the self-confident and educated young nobleman. Is seems plausible that, under such circumstances, Guadagnini hardly was inclined to inform Cozio of the old guild rules, completely out-of-date and senseless from Cozio's point of view. It may as well be supposed that the relation between Cozio and the family of violin makers, Mantegazza, turned out in a similar manner. And perhaps, likewise, that the violin maker Marchi (mentioned below) was disturbed by Cozio's claims to be considered a violin expert (Regazzi 1986: 12f.).[8]

More examples of good reasons to omit passing on the old traditions (and particularly to publish them!) offers Giovanni Antonio Marchi (1727-1807).[9] He was a fully developed professional craftsman who in a very comprehensive manuscript, *Libro che tratta della Professione dei Violini*, finished in 1786, intended to summarize his own knowledge in violin making. His work was not printed until 1986, and Marchi gives motives for not publishing it in his own lifetime. In a letter of May 13, 1805, to Cozio di Salabue, he wrote, i.a.:[10]

> As regards the book, I have never published it and never will: it is not worthwhile, the world is poor of violin makers and therefore the book would not be taken into consideration. Another point is that my book would teach things to people who do not need to be taught […]

However, in his manuscript Marchi does not mention anything at all dealing with proportioning of the instruments. Had he, actually, nothing to report, or didn't he *want* to reveal the old "secrets" in his book? Instead, he presents his opinion that the violin makers ought to imitate the "outlines" of the old masters (Regazzi 1986: 124f.):

> The outlines I have seen by such Masters were not only very beautiful, but they were also extraordinarily proportioned, and I think no one will be able to do better.

The best guiding principles according to Marchi may be found with Jakob Stainer, Nicola Amati, Francesco Ruggieri 'il Per' and Giuseppe Guarneri 'del Gesù' (idem: 130f., 248f.,

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7 Probably, Cozio had at least limited knowledge in violin making, cf. Cozio 1950: 425, footnote 1.

8 From 1776 on, Cozio maintained very close contact with the violin maker Pietro Giovanni Mantegazza (c. 1740? - c. 1800) in Milano and his family (Cozio 1950: passim). By order of Cozio, members of the family made several alterings and reconstructions ("improvements") of older instruments (e.g., modernization of the neck according to the "new" Parisian mode). The family members are, as well as Marchi, typical of the time by nearly completely abandoning making instruments of their own, instead working with more profitable repair and reconstruction.

9 Regazzi 1986: 24-35.

10 Letter from Marchi to Cozio, in Cozio 1950: 435ff. and Regazzi 1986: 339ff. In the latter, the complete manuscript, together with attached letters, are rendered in Italian (original) and English.

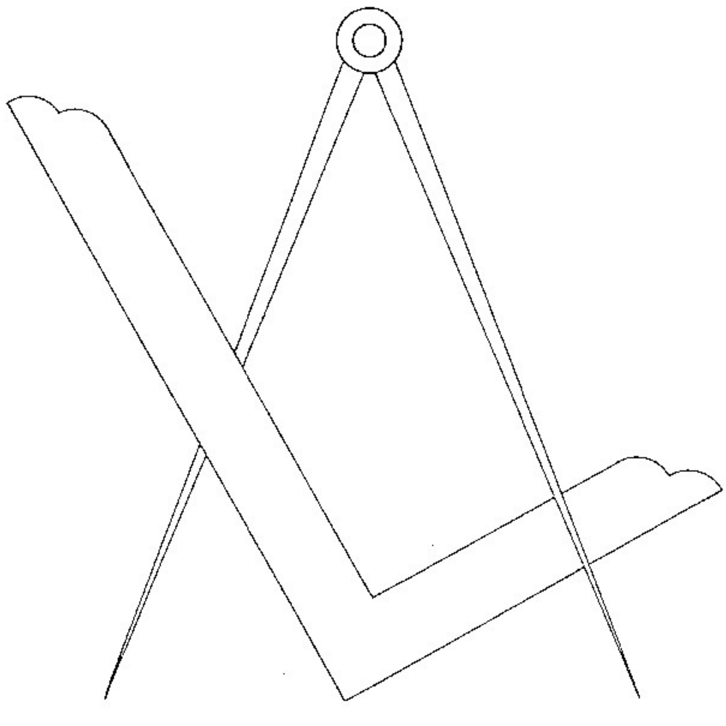
258f.). Only, if there is a real lack of knowledge or something else behind the citation below is scarcely possible to determine (idem: 124f.):

> The Old Masters used to change sometimes the dimension of their patterns, and we don't know the reason why […]

Giovanni Iviglia comments on Marchi's disinclination for publishing his book (Cozio 1950: 436, footnote 4): "Siamo alle solite: nessun liutaio vuole insegnare alle altri. Ma probabilmente perchè nessuno è assolutamente sicuro delle regole che applica." Or, perhaps Marchi merely expressed his and his fellow craftsmen's resignation to new ages with new valuations?

*

In absence of additional relevant sources on the old "secrets" about proportional design in violin making, the actual realization of them, and their surviving, we – like Iviglia above – "face the same old story", now however with a possible solution, namely, that the still existing old instruments themselves, when accessible for us, might turn out to be reliable and sufficient sources of information.



Principles of Geometrical Proportioning

In the preceding chapter are listed a number of constructions useful in order to bring about geometrical proportions, i.e. relations (ratios) of geometrical quantities.[11] All of them make use of the square and/or the circle. The proportions are expressed ("described") by means of the *geometrical constructions* themselves (and may often be more or less complicated when expressed by other means). The square and the circle are the "perfect" figures according to ancient thinking. (By Renaissance architects and artists, they are often reproduced inscribed in each other, a figure demonstrating "perfect proportions" and the human body as "the measure of everything".)

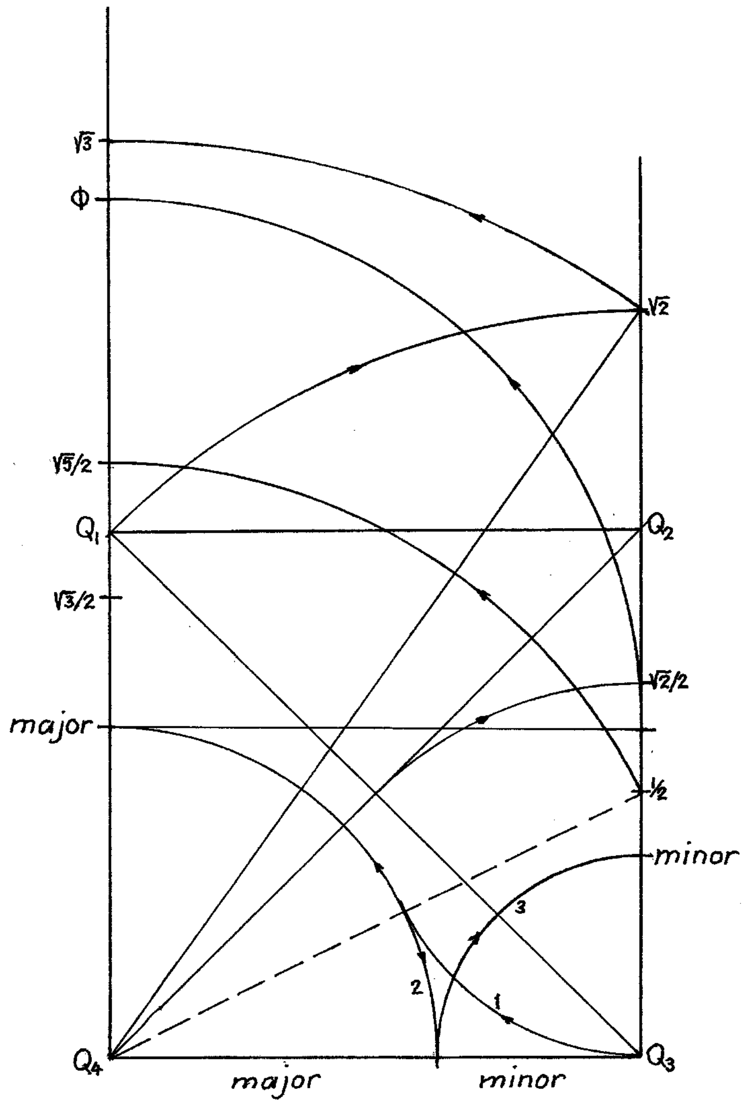
The geometrical construction (intended for a violin, e.g.) sets out from a square, the side of which is equal to the width of the lower bouts of the violin. (Later on, when it comes to the design of the soundholes and the scroll/pegbox, other dimensions will be chosen, cf. Sections III and IV of this book.)

In the following few pages the general principles of the geometrical proportioning is described, as illustrated by the figures, Fig. I: 1-6. They may be applied directly by proportioning the belly (front side) of a violin (observed from above, at square angle to the front) and with a few alterations when proportioning the soundholes and the scroll/pegbox.

11 In this book the term/concept 'geometrical proportioning' denotes that the position of a certain point/location in the outer design of an instrument is defined by means of a geometrical construction, in such a way that the distances of the point to two already existing points constitutes a certain mathematical relation, i.e., proportion, ratio.

Initially a square is drawn, Q_{1-4} in Fig. I: 1, and two parallel sides are prolonged.

Fig. I: 1



The sides of the square may be divided in, e.g., three, four or five equal parts, and in (equal parts of) square roots like √2, √3 och √5.[12] Methods of dividing in three parts etc. are demonstrated below, Fig. I: 2. As to dividing of the side of a square in *any* number of parts, e.g., five parts, cf. Fig. I: 3.

Ways of construction of the square roots mentioned can be seen above, in Fig. I: 1, as well as the dividing of a distance in the ratio *major* : *minor* – often named "the golden section". (The notation Φ in Fig. I: 1 designates 1 + *major*, ≈ 1.618.)

(Ratios of whole numbers, e.g., 3:4, may in the same way be materialized with the aid of geometry, cf. the introduction of Section III in this book.)

All methods of proportioning and dividing of distances mentioned above are based upon the properties of similar triangles; more examples, cf. Fig. I: 6 below.

12 Certainly more dividing principles might have been used. However, Jouven (1979) limits the number to those mentioned above, and Vitruvius and even Vignola only mention arithmetical ratios.

Fig. I: 2

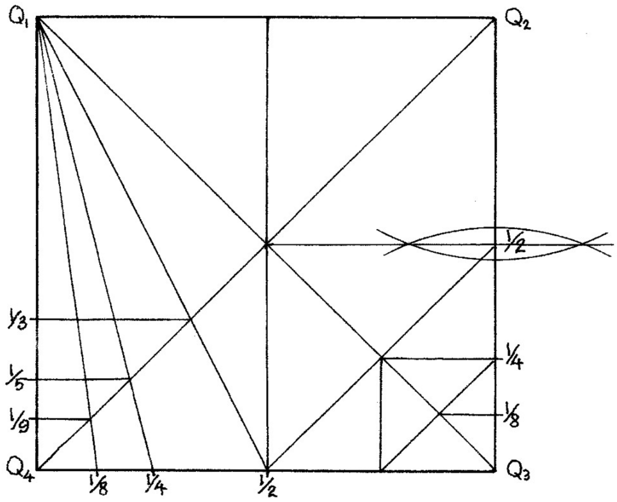


Fig. I: 3

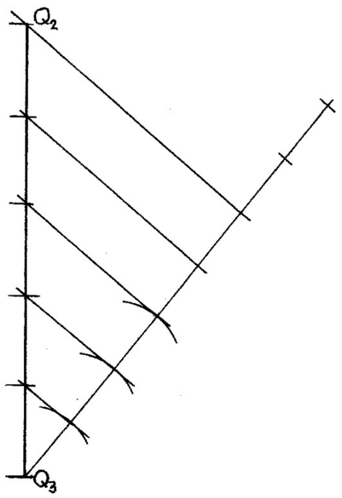
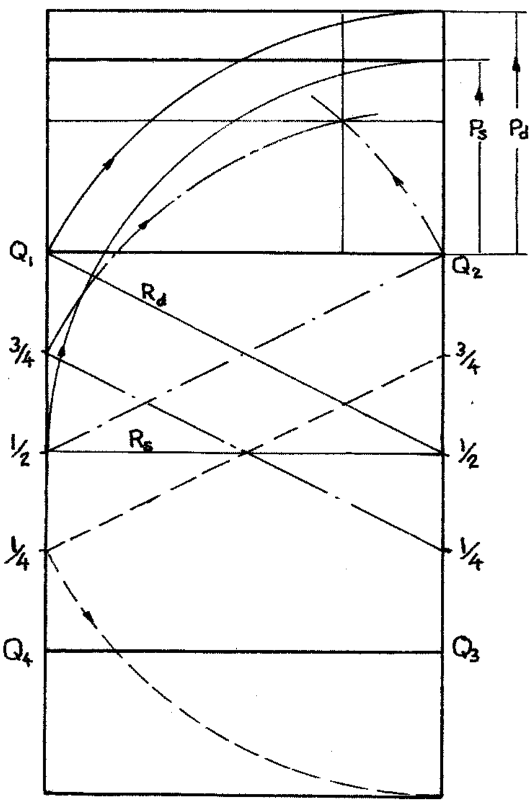


Fig. I: 4

In Fig. I: 4 some dividing points are marked.



The square may be prolonged into a rectangle both upwards and downwards. In order to settle a proportion of the prolongation a point (e.g., in Fig. I: 4 the dividing point ½) is chosen on one of the sides (in Fig. I: 4 the right side) of the square. Then a circle is drawn, the centre of which is the dividing point; the starting point of the circle line is another point, on the opposite side of the square (in Fig. I: 4 the dividing point ½).

Further on in this book will be demonstrated that in most cases only a very limited number of dividing points are necessary as circle centres (cf. footnote 12). Starting points of the circle lines are as a rule either a point *exactly opposite* the centre point, or the square corner Q_1 (*diagonally upwards* from the centre point), cf. Fig. I: 4, circle lines marked R_s and R_d, respectively. (As to the pegbox, the prolongation downwards of the square requires other dividing and starting points, cf. Section IV.)

In this manner it is possible to construct lots of vertical levels and distances, cf. the following table, Fig I: 5. (The listed dividing points will be sufficient in the following investigation except when it comes to the soundholes and the scroll, cf. Sections III and IV.)

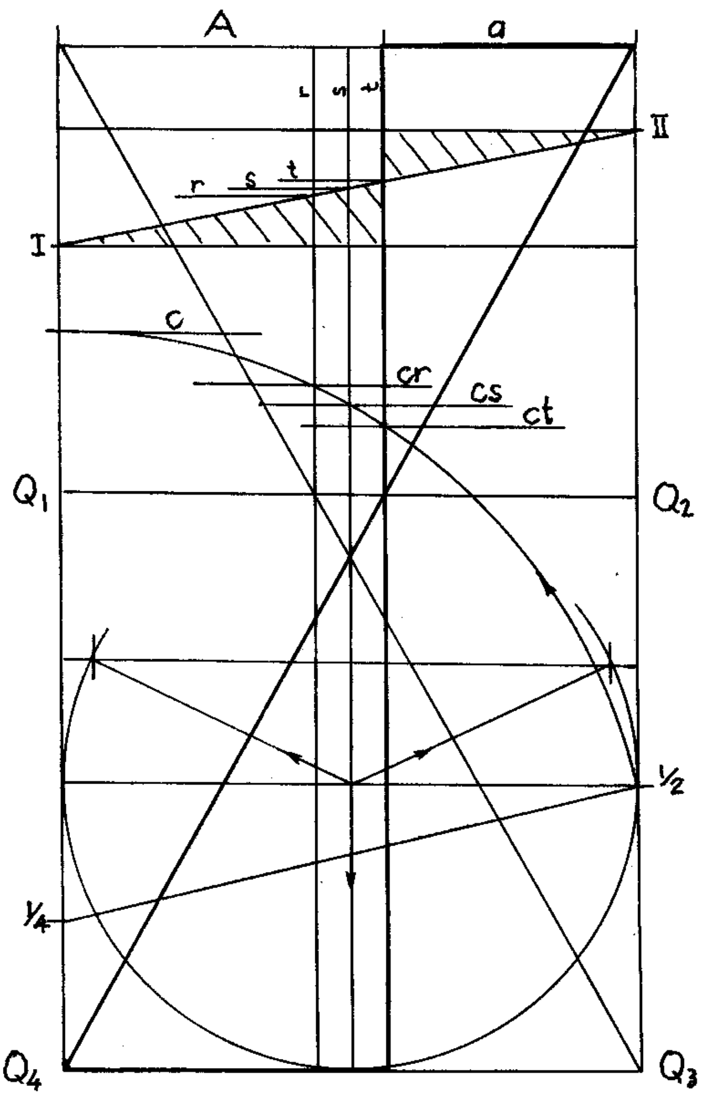
Fig. I: 5

Circle centre in dividing point	**R_s-value**	**R_d-value**	**Designation in Section IV**
1/8	0.125	0.454	*a / A*
1/7	0.143	0.460	-
1/6	0.167	0.468	*b / B*
1/5	0.200	0.481	*c / C*
1/4	0.250	0.500	*d / D*
$1 - \sqrt{2}/2$	0.293	0.518	*d´/ D´*
1/3	0.333	0.535	*e / E*
minor	0.382	0.558	*f / F*
2/5	0.400	0.566	*g / G*
1/2	0.500	0.618	*h / H*
3/5	0.600	0.677	*i / I*
major	0.618	0.688	*j / J*
2/3	0.667	0.721	*k / K*
$\sqrt{2}/2$	0.707	0.749	*l / L*
3/4	0.750	0.781	*m / M*
2x*minor*	0.764	0.791	*n / N*
4/5	0.800	0.820	*o / O*
$\Phi/2$	0.809	0.827	*p / P*
5/6	0.833	0.847	*q / Q*
6/7	0.857	0.867	–
7/8	0.875	0.883	*r / R*

In Fig. I: 6 (cf. next page) the square side Q_{1-2} divides the height of the rectangle in the proportion/ratio 3:4 (or, more generally, a:A). The diagonals of the rectangle intersect the side Q_{1-2} of the square, on both sides of the middle line. If vertical lines are drawn through these points, they will divide the width of the rectangle in identical proportions, 3:4 and 4:3, respectively (a:A and A:a, respectively). In this book they will be designated "vertical proportion lines" or, simply, "proportion lines". (Accordingly, "horizontal proportion line" designates the square side Q_{1-2}.)

In the same figure, the triangles drawn in thicker lines are "similar triangles", i.e., their proportions are identical. The proportion is 3:4 (a:A), implying that sides with identical positions in two (or more) triangles constitute the proportion 3:4 (a:A). Thus, by drawing a diagonal line in a rectangle, between the two height positions I and II, and crossing the proportion line marked **t**, two similar triangles (streaked in the figure) will be formed, in the proportion 3:4 (a:A), and the height of the rectangle will, as well, be divided in the proportion 3:4 (a:A). In the same way: if the proportion lines marked **r** or **s** are used the height is divided in the proportions 4:3 or 1:1, respectively.

Fig. I: 6



A defining property of the circle line, the constant radius, may be exploited in many ways. In Fig. I: 6 (the lower part) is demonstrated in which manner a vertical distance (e.g., ½ of the square side) may settle symmetrical intersections on a horizontal line. There is also demonstrated in which way a circle line (with the centre on one of the vertical sides of the square, and the starting point on the opposite side of the square) may be used for fixing height positions: the circle line intersects the side of the rectangle and the three central proportion lines on four different levels, marked **c**, **cr**, **cs** and **ct**, respectively.

Source material: general remarks

It is evident that source material to be used in the present investigation has to be representative of violin instruments produced and used during at least the 18th century in central parts of Europe. Likewise, it is evident that a collecting or study in the field of the instruments themselves on a sufficiently large scale is quite out of the question: preserved instruments of this period are rare, mostly in daily use or otherwise not very easily to be reached. In addition, most of them, if not all (especially the representative ones), have been repeatedly repaired, modified in different ways, or even remade.[13]

Obviously other kinds of source material have to be searched for, e.g., measurements and drawings found in violin literature would seem to be conceivable sources. However, this is a mistake: they have to be regarded with considerable precaution, and they may, for many reasons, very rarely be accepted. In addition, they occur only sporadically and do not allow of statistical consideration.

Instead, in the present study photographs and photographic reproductions of relevant instruments, primarily objects represented in the rich literature, will be tested as source material (as regards soundholes also thin wooden models, cf. Section III).[14]

It is necessary that the photographs in the selected literature have been chosen, and the reproduction quality accepted, by reliable editorial expertise, all of which may be considered to be proved by the mere publication of the material.

A search through of relevant literature reveals that pictures of sufficient quality referencing to earlier periods are more exceptional than good pictures of objects from later periods. This condition is not necessarily a disadvantage since it may give a better insight in the prevailing situation as regards to the application of old traditions in late 18th century when the craftsmen's guilds were on their decline.

The selection of photographic reproductions as source material however gives rise to several problems and limitations. Most important is that the double-bass, because of insufficient number of good pictures, has to be excluded from the study. Further, that otherwise acceptable photographs of the body of the instruments seldom include sufficiently good side-views to allow of qualified geometrical analysis of the rib heights, neither of the archings of the belly and back. The same problem applies to the neck length, owing to the fact that extremely few instruments reproduced in the literature have their original necks preserved.

Summing up, the above discussion of available and conceivable source material clearly indicates that the present investigation has to be restricted to violins, violas and violoncelli, and to the geometrical proportioning of (the face of) the belly, soundholes and scroll & pegbox.

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13 More detailed viewpoints, cf. Huber 1998, 25ff.

14 Using two-dimensional (reproductions of) photographs as sources involves a process of re-interpretation of the pictures, including paying regard to perspective factors, cf. Lomnäs 1998 and, briefly, Section IV.

II

Proportioning of the belly

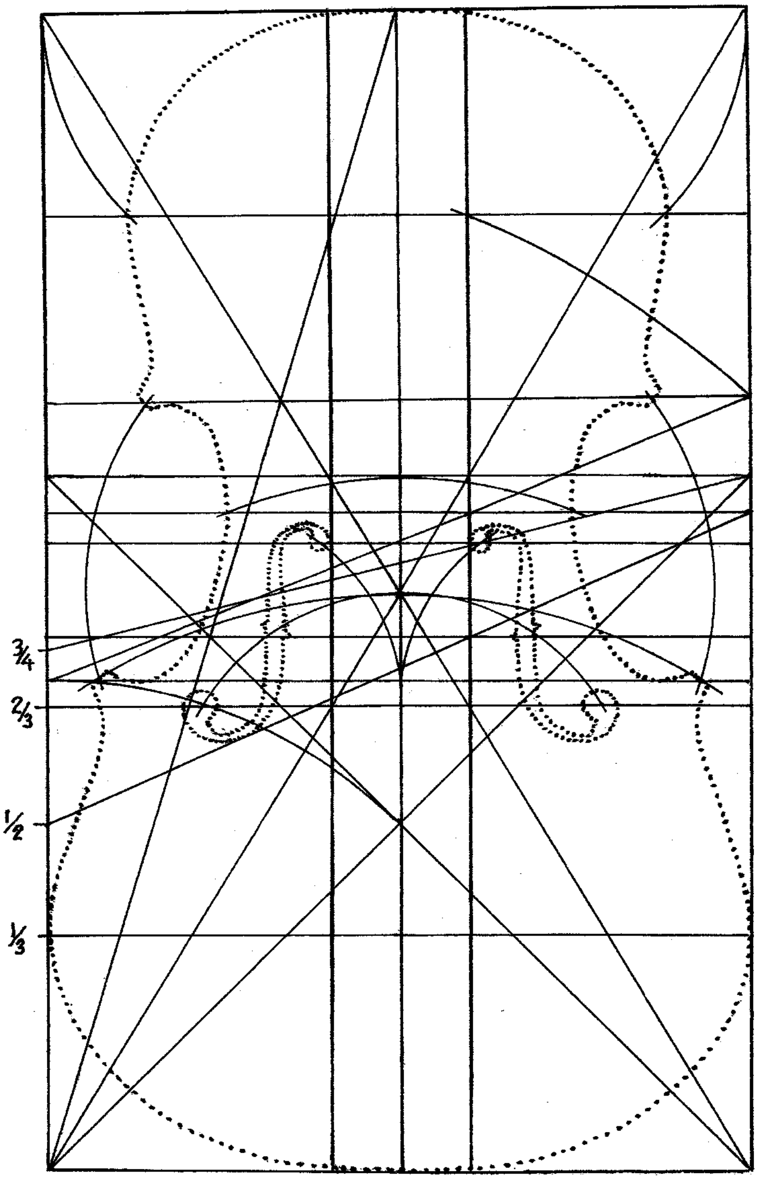

Fig. II: 1. **This example** of geometrical proportioning, cf. Key on p. 30, demonstrates the simplicity and beauty of the designing method and is included right here in order to encourage the intended reader of the present study to assimilate the following rather laborious chapter. – Starting with one single distance, the width of the lower bouts, all essential points of the design are determined by means of simple geometrical constructions. Neither measuring systems nor measuring tools are required!

Source material

Sources are photographs and photographic reproductions (printed), selected from the literature, and one poster, listed below. The selection of sources is mainly based on the reproduction quality of the pictures. The numbering refers to the list of all selected sources (# numbers) in Appendix A. Every separate picture may be identified in the literature by means of data given in the list.

1-10 Sacconi, Simone F.: *I 'segreti' di Stradivari.* Cremona 1972.
11-15 *Alte Meistergeigen*, ed. by Verband schweizerischer Geigenbaumeister. Frankfurt/M 1978-82. Vol. 1.
16-43 *Alte Meistergeigen*, ed. by Verband schweizerischer Geigenbaumeister. Frankfurt/M 1978-82. Vol. 2.
44-50 *Alte Meistergeigen*, ed. by Verband schweizerischer Geigenbaumeister. Frankfurt/M 1978-82. Vol. 3-4.
51-94 *Alte Meistergeigen*, ed. by Verband schweizerischer Geigenbaumeister. Frankfurt/M 1978-82. Vol. 5-8.
95-98 *The Strad*: posters, and *The Strad* 1984: Sept. p. 344f.
99-113 Hamma, Walter: *Meister italienischer Geigenbaukunst*. Wilhelmshaven 1993.
114 Hamma, Fridolin: *Meisterwerke italienischer Geigenbaukunst*. Stuttgart 1931.
115-204 Hamma, Walter: *Geigenbauer der deutschen Schule des 17. bis 19. Jahrhunderts*. Tutzing 1986. Vol. 1-2.
205-234 Pilař, Vladimír, & Šrámek, František: *Umění houslařů*. Prague 1989.

In the list, Appendix A, the maker's name and residence together with the dating of the instrument are given according to the reading of the source.

The datings are noted according to following period code:

-1620: a	1701-1720: d	1761-1780: g	1801-1810: j
1621-1660: b	1721-1740: e	1781-1790: h	1811-1820: k
1661-1700: c	1741-1760: f	1791-1800: i	1821-1850: l

Instrumental codes are: b = bassetto, va = viola, vc = violoncello, vl = violin

L/W designates the proportion (ratio) of the belly, i.e., length : largest width (= width of the lower bouts), according to the following code:

1.500: a	1.636: d	1.688: g	1.721: j	1.764: m	1.820: p
1.600: b	1.667: e	1.700: h	1.732: k	1.781: n	1.833: q
1.618: c	1.676: f	1.707: i	1.750: l	1.800: o	1.847: r

(Concerning geometrical equivalents to the proportions, cf. Section I, Fig. I: 5.)

Selection of essential, postulated proportioned points (PPPs) of a belly design

In Section I of this book the traditional square and rectangle used in proportioning violin instruments are described. On next page, Fig. II: 2, the outlines of a violin belly have been inscribed in a similar proportioning system.[15] It is clear from the figure that points on the outlines suitable for proportioning may be defined as crossings of horizontal and vertical lines or circles drawn according to Fig. I: 6 (lower part). Such points are (from the bottom) V, W/W, Cw/Cw, Mw/Mw, Dw/Dw, Pw/Pw and S.[16] Here they are, hypothetically, called "proportioned".

Between the proportioned points the outlines are constantly curved, often more or less asymmetrically, and can't be defined by simple geometrical methods. Therefore they are considered non-proportioned and instead supposed to have been open to *free design*.

In addition to the points noted above the positions of the centres of the two eyes of the soundholes have to be defined, Ew/Ew by using the method mentioned above, Fw/Fw by more advanced methods, see below. (The choice of the eyes and nothing else is discussed in Section III.)

By tradition the length position (the level) of the inner notches of the soundholes has to be considered essential; the length position is marked by the line GG in the figure.

Analytical method applied to each individual source # 1-234

A. Proportioning square/rectangle. Designations etc. according to Fig. II: 2

The source (i.e. the photographic reproduction of an instrument belly, if required enlarged to scale >1:2) has to be inscribed in a proportioning rectangle, as further elaborated in accordance with the description in Section I.

The positions of proportioned points (cf. above) will be settled and the points marked W, Cw, Mw etc. together with every other designation according to Fig. II: 2.

In Fig II: 2 several designations differ from those used in Section I. For example the square corners $Q_{1\text{-}4}$ are designated KKAA, the upper side of the rectangle LL, the dividing points of the square named H/h/h′ = 1/4, I/V/S = 1/2, J/j/j′ = 3/4; Y/y/y′ = 1/3, Z/z/z′ = 2/3, and the middle point of the distance A-L marked B. The remaining designations will probably be self-explanatory.

The distances AL and AA will be measured and the "main proportion", i.e. the ratio length : width of the belly (designated **L/W**, cf. Fig. II: 2), calculated and registered in Appendix A. In Fig. I: 6 the main proportion is demonstrated as the ratio (A+a):A.

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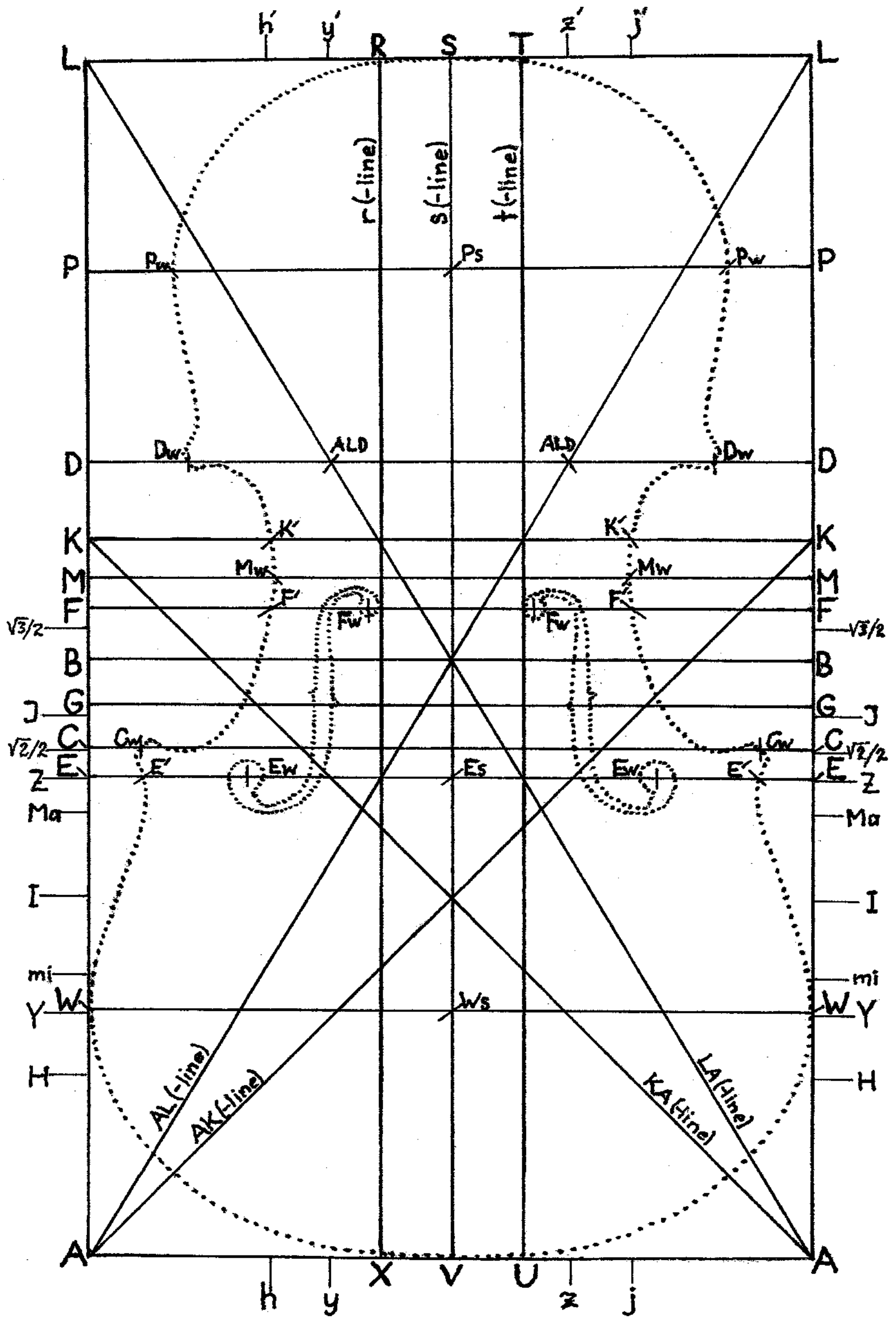
15 It has to be emphasized that several of the numerous lines and designations in the figure are included *only* because they are indispensable for the following analysis. The outcome of the investigation will demonstrate that very few of the lines etc. are required when the suggested proportioning method is put into practice.

16 Two more couples of similar points occur in the vicinity of the upper and lower corners (above Dw/Dw and below Cw/Cw). In this study they have been neglected, however, partly because their positions are complicated to define, partly since they seem less plausible and unnecessary as proportioned points.

Fig. II: 2



B. Levels of W, C, M, D, P, E, F and G

Level lines WW, EE, CC etc. through all postulated proportioned points (PPPs) and the inner notches of the soundholes plus level lines KK and BB have to be drawn (cf. Fig. II: 2) together with level lines HH, YY, mimi etc. (not shown in Fig. II: 2, neither the designations Hs, Ys, mis etc., cf. Ws, Es, Ps in Fig. II: 2).

The levels W, C, M, D, P, E, F, G will all be settled by one of the following two methods, a) and b).

a) Diagonal method, cf. Fig. I: 6, upper part:
A diagonal line from a point on the left rectangle side to a point on the opposite side,[17] crossing one of the vertical lines **r**, **s** or **t** on a level coinciding with that of a PPP will be registered (cf. **D** below), using a code, e.g., IMt (= level G in Fig. II: 2). There are three cases: a) both ends of the diagonal are given, b) only one of the ends is given, together with the crossing of one of the verticals, c) only the crossing of one of the verticals is given.

In addition, some other crossing lines have to be tested, one ascending from the lower left part of the rectangle, the second one descending from the upper side of the rectangle. If the level of the crossing point coincides with that of a PPP, this has to be registered, using a code, e.g., ALSA (= level D in Fig. II: 2).

b) Circle method, cf. Fig. I: 6, middle part:
A circle line with the centre on the left rectangle side and starting point on the opposite side, crossing the left side or one of the vertical lines **r**, **s** or **t** on a level coinciding with that of a PPP, has to be registered, using code, e.g., AIct (= level M in Fig. I: 2).

C. Widths of Cw, Mw, Dw, Pw, Ew, Fw

Circle method, cf. Fig. I: 6, lower part:
If a circle line having its centre in one point and the circle line is starting in another point, both on the **s**-line, and the circle line is passing through a couple of PPPs, this circle has to be registered, using code, e.g., BI (= widths Ew in Fig. II: 2). In the code the designations of the points are simplified, e.g., CP instead of CsPs (not in the table below, however) and in the situation BCwDw and BDwCw, cf. Fig. II: 3.

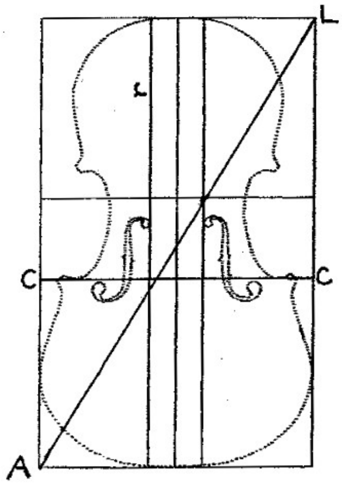
In addition, a number of alternatives with circle centres in Ew and starting points elsewhere have to be registered. The levels of the Fw points in particular are very diversified which calls for further simplification of the codes:

F'Fw=E'w	a	FsDs	e	BsKs	k	GsKs	o	CsKs	s	EwK'	1	CsIs	-
CsPs	b	FsKs	f	BsMs	l	GsMs	p	CsMs	t	EwMw	2	EwWl	-
KsBs	c	FsMs	g	BsGs	m	GsEs	q	EsKs	u	EwBs	3	JsMs	-
MsDs	d	FsBs	h	BsCs	n	GsMas	r	EsMs	v	EwGs	4	ZsEw	-
		FsGs	i					EsWs	w	EwCs	5		
		FsCs	j					EsEw	x	EwEs	6		
								EsEw=FwFw	y	EwWs	7		
								x + y =	z				

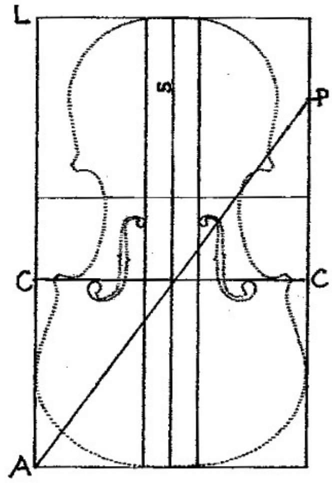
Examples of codes when applying the analytical methods above, cf. Fig. II: 3 (6 pages starting on next page). (long = level/length, lat = width)

17 Exceptionally, other end points will be tested, e.g., Pw, cf. example in Fig. II: 3.

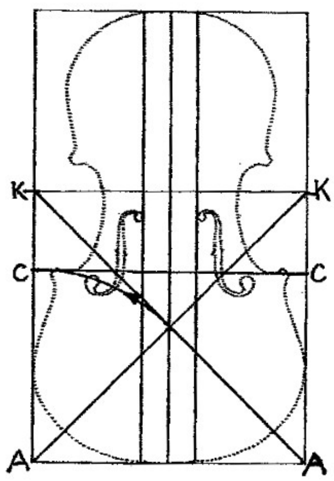
Fig II: 3 (1)



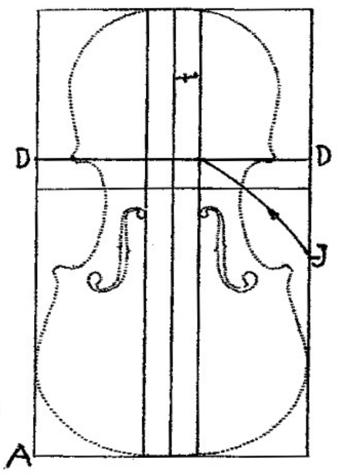
C long: ALr



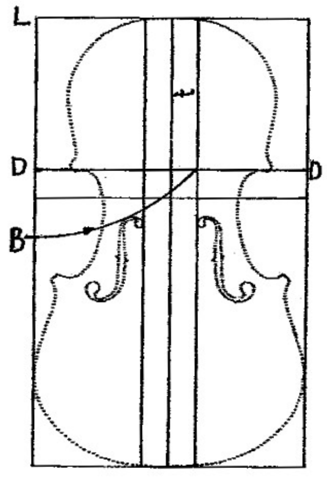
C long: APs



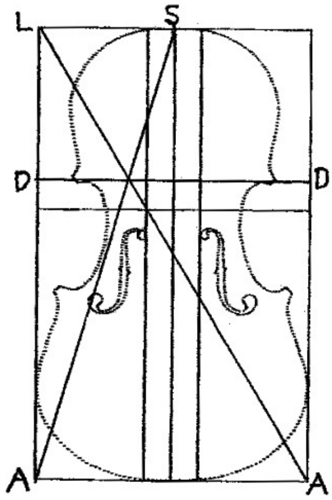
C long: AKKAc



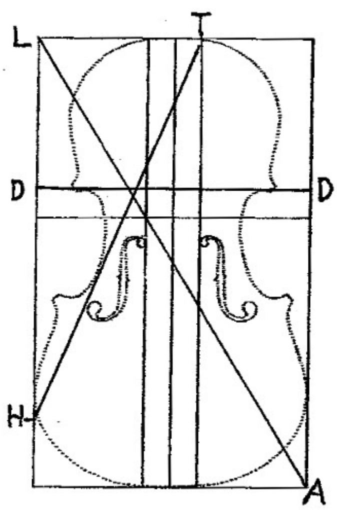
D long: AJct



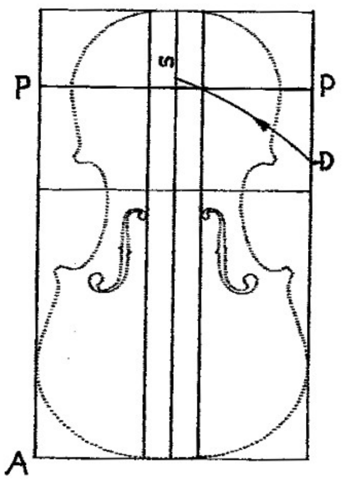
D long: LBct



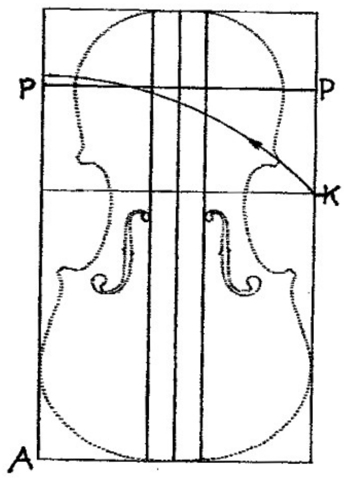
P long: ASLA



D long: HTLA

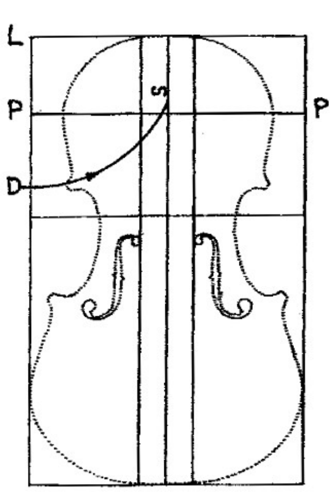


P long: ADcs

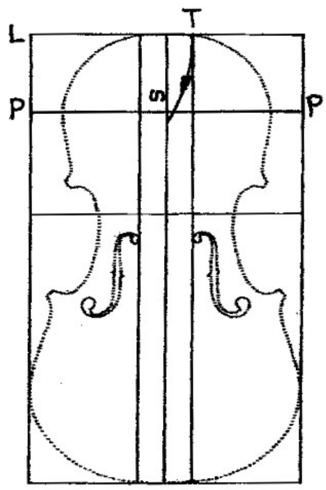


P long: AKc

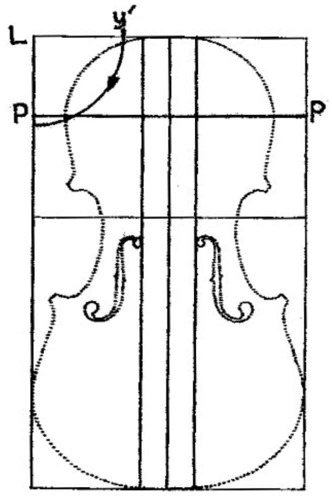
2)



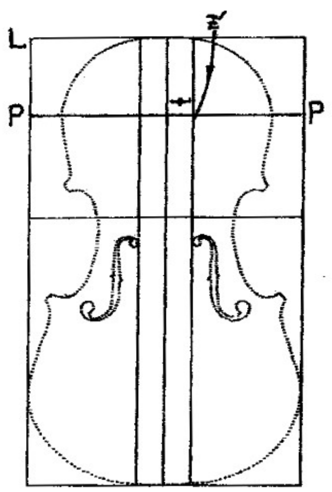
P long: LDcs



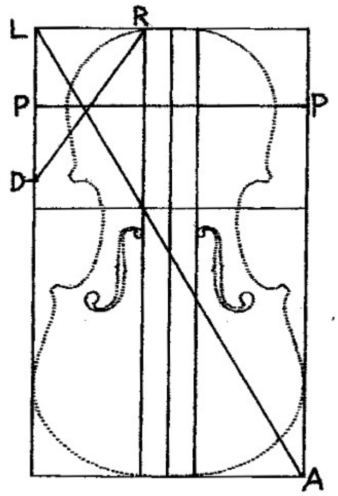
P long: LTcs



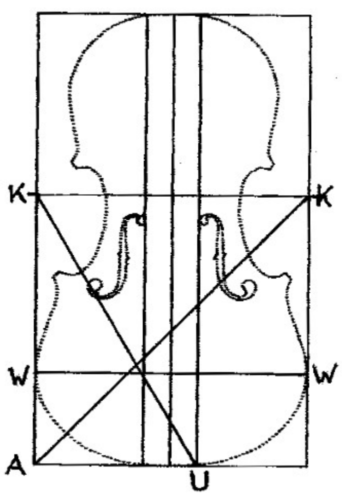
P long: Ly´c



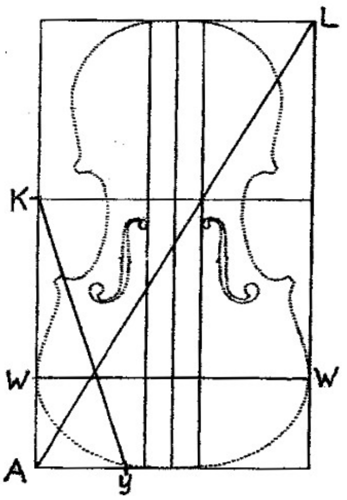
P long: Lz´ct



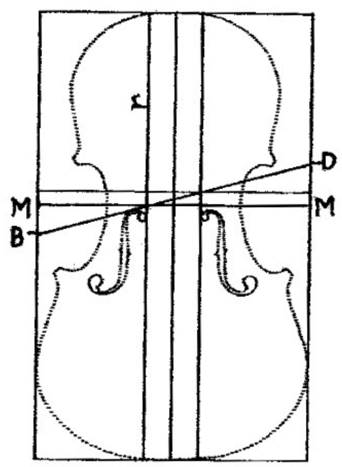
P long: DRLA



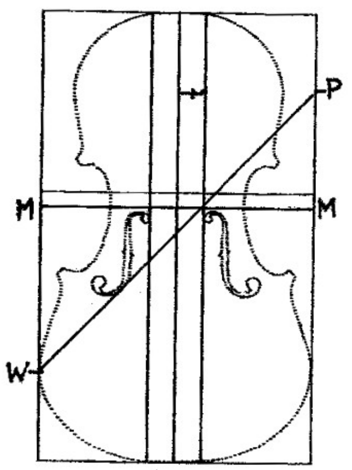
W long: AKKU



W long: ALKy

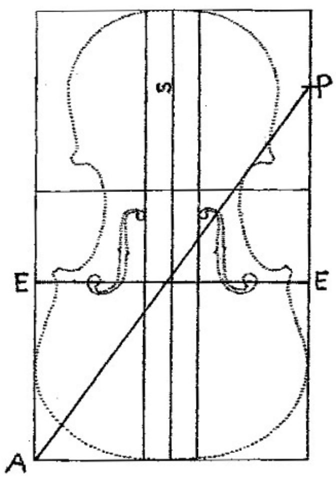


M long: BDr

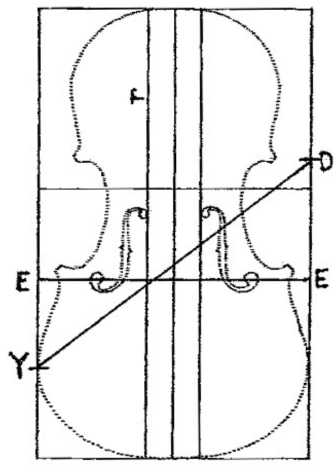


M long: WPt

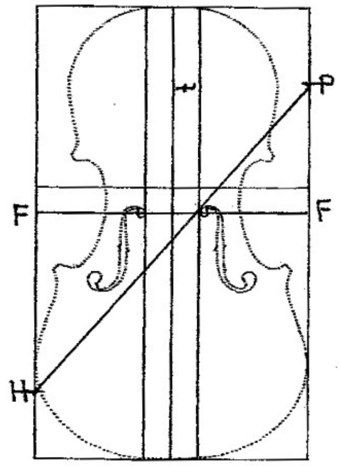
(3)



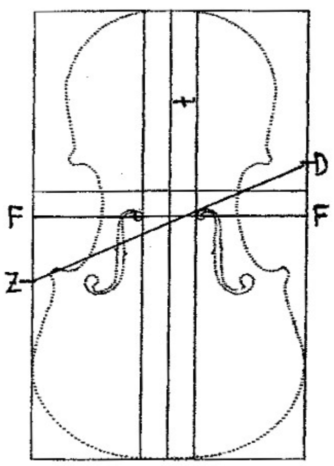
E long: APs



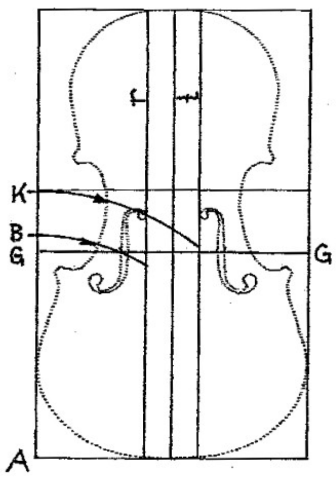
E long: YDr



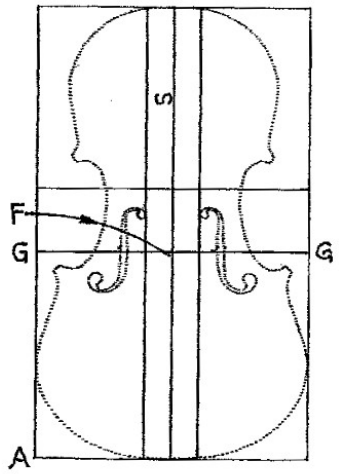
F long: HPt



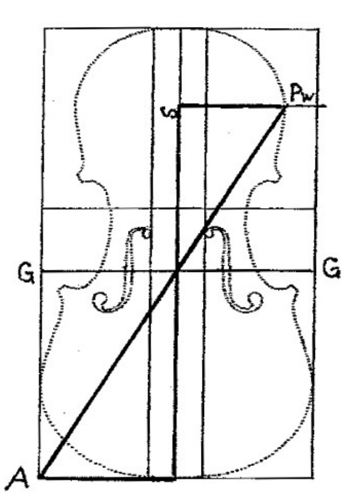
F long: ZDt



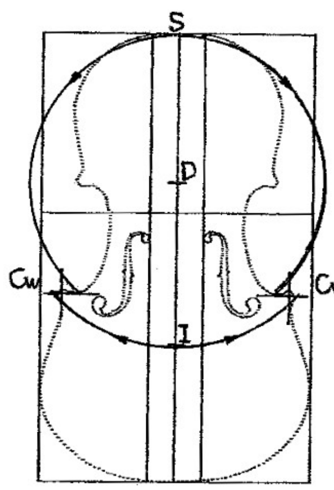
G long: ABcr and AKct



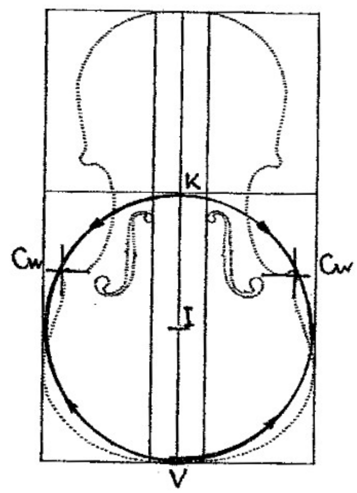
G long: AFcs



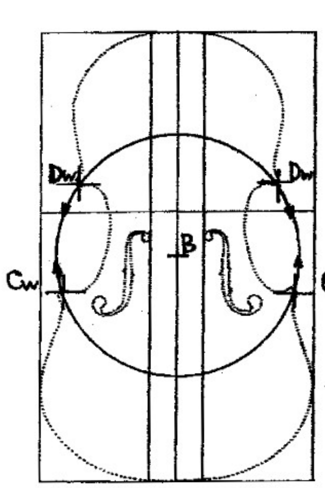
G long: APws



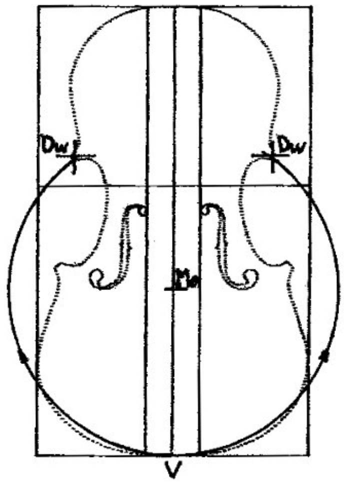
Cw lat: DI and DS



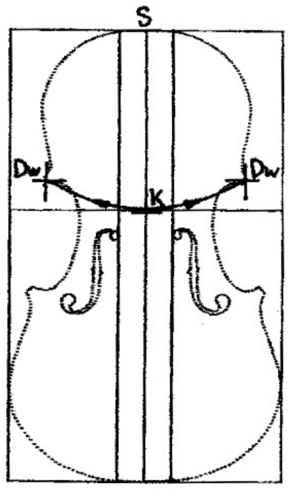
Cw lat: IK and IV



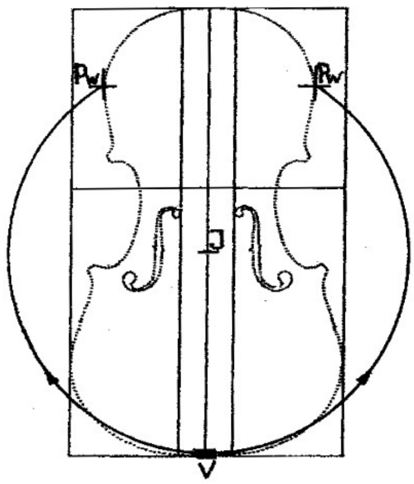
Cw lat: BDw and Dw lat: BCw



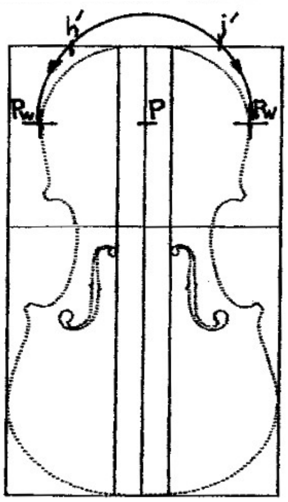
Dw lat: MaV



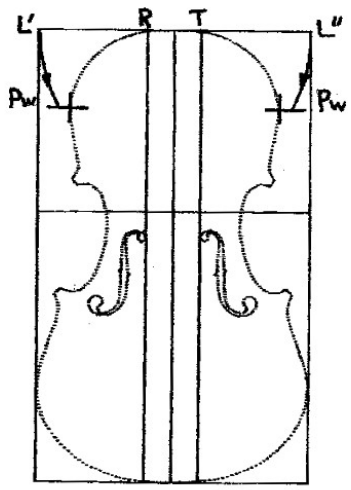
Dw lat: SK



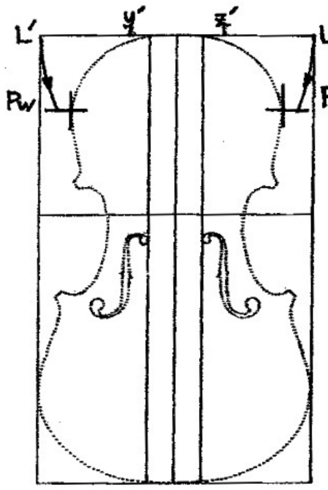
Pw lat: JV



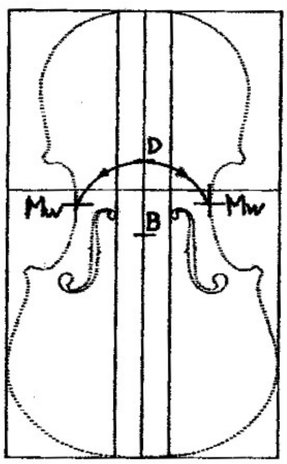
Pw lat: Ph´/j´



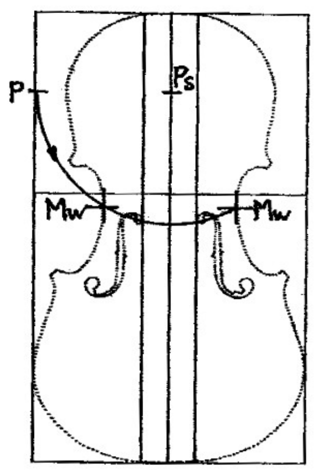
Pw lat: TL´ and RL″



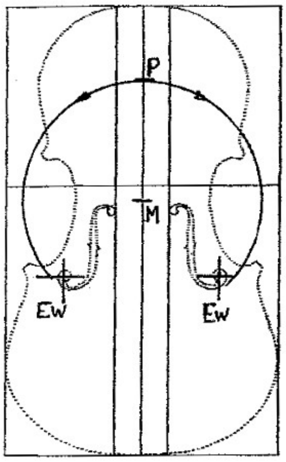
Pw lat: z´L´ and y´L″



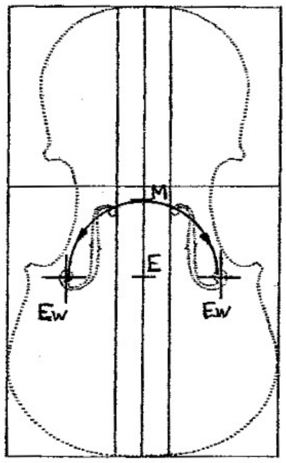
Mw lat: BD



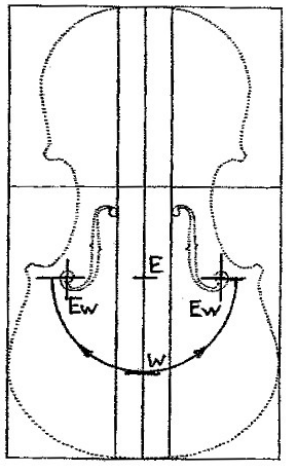
Mw lat:PsP



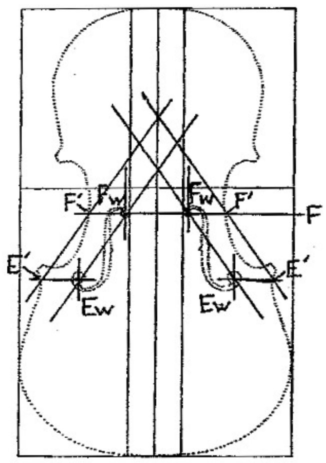
Ew lat: MP



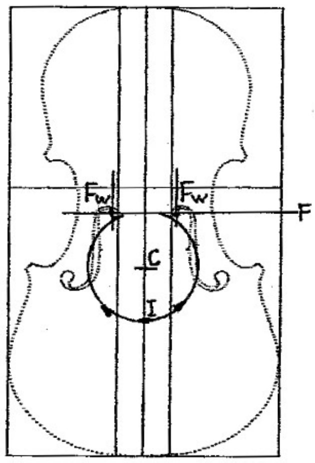
Ew lat: EM



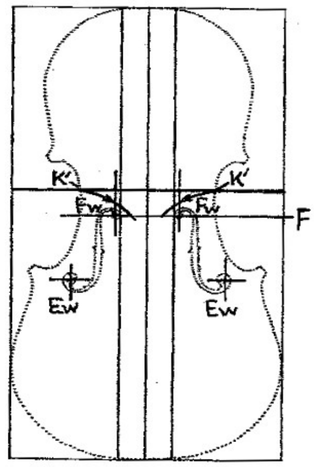
Ew lat: EW



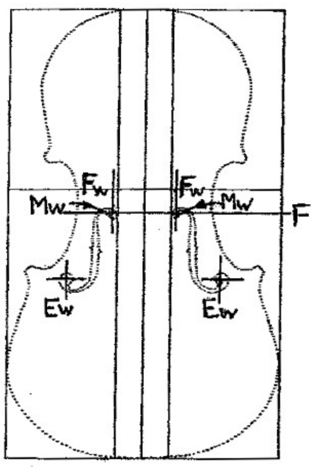
Fw lat: F′Fw=E′Ew



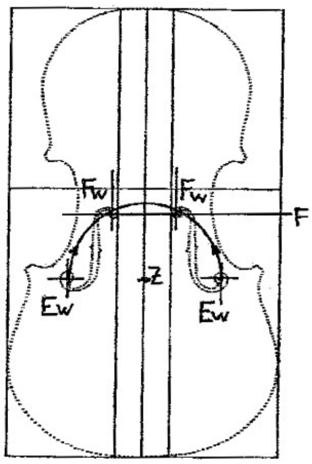
Fw lat: CI



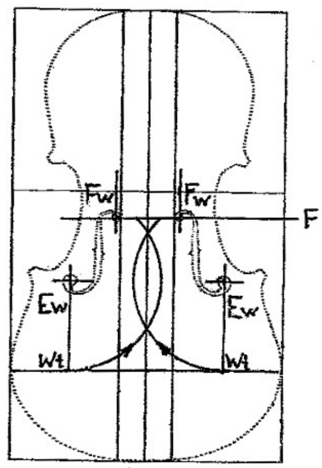
Fw lat: EwK′



Fw lat: EwMw

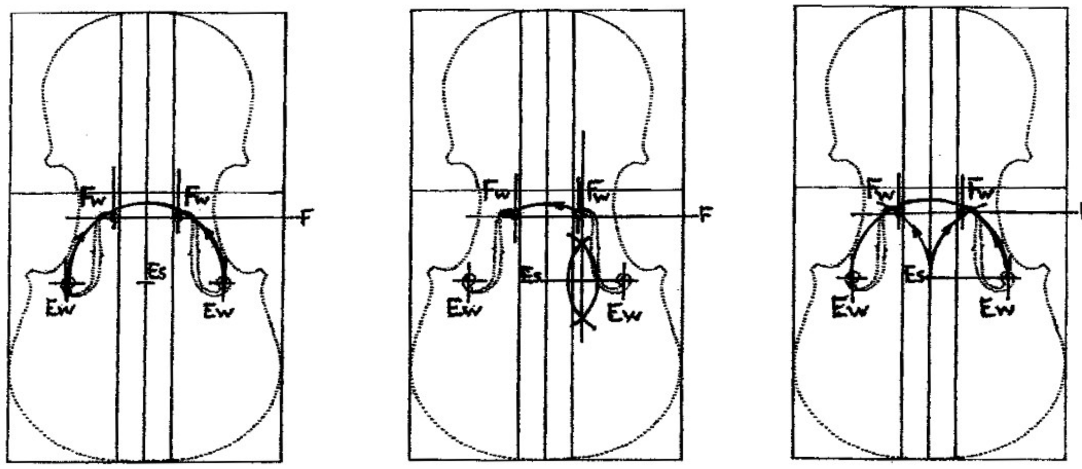


Fw lat: ZEw



Fw lat: EwWt

(6)



Fw lat (x): EsEw=EsFw   Fw lat (y): EsEw= FwFw   Fw lat (z): Fw(x)+Fw(y)

Concerning the Fw codes x, y and z above: if either code 6 or (as in the figure) code x is satisfied at the same time as code y, the situation coded z will appear (cf. the figure to the right). Then the distance EsFs = EsEw · √3/2 (i.e. that the triangle is equilateral). This is possible only if, by chance or consciously, EsFs and EsEw are "adapted" to each other. (From the present study it will be clear that this method was not unfamiliar at any time in the period.)

D. Listing of satisfying (coinciding) proportioning tests. Appendices A and B

In all test (trial) situations A-B above principally every designated point alongside the rectangle and, in C situations, along the middle **s**-line, cf. Fig. II: 2, has to be tested as start and end point, circle centre and compasses gap. The codes of all satisfying tests will be entered in the list, Appendix A, enlarged with columns for every proportioned point (PPP), level (length/longitude) and width (latitude), in all 23 columns. The completed list (holding more than 30 000 entries), however, is not reproduced in this report. Instead, the contents of the list will be rendered in a more systematic and surveyable way, see next chapter.

Although this investigation deals with "general" rules and aspects, not individual applications, a complete list of registered codes are given for eight individual sources, cf. Appendix B, arranged vertically, not in columns.

E. Considerations as to individual qualities of the sources. "Uncertainties"

In Section I general difficulties involved in using photographs as source material was discussed, e.g., perspective distortions. Several other kinds of insufficiencies that have to be regarded are paid attention to in Lomnäs 1997 and 1998 (e.g., the possibility that even the *realized* design of the instrument itself may differ from the maker's *intentions*). Indeed, it seems impossible to calculate in which manner all possible "uncertainties" may interact.

In this investigation, instead, will be tried a method successfully applied in the above studies (cf. further in Sections III and IV): all points made use of in the analytical process described above are considered "uncertain", i.e. as being "tolerance areas", approximately circular, measuring, in real source instrument size, diameter circa 6 mm (cello) or 3 mm (violin/viola). Whether this choice is suitable or not, i.e. if it has the quality of collecting similar items and separate dissimilar, will be discussed/indicated in the following synthesizing evaluations.

Systematic evaluation of analytical data (codes)

Merely a preliminary survey of the codes listed in Appendix A, columns 7-23, makes clear that
a) all individual PPPs may have been positioned, level and width, by means of one or more of the proportioning methods described in Section I and above,
b) some methods (i.e. with identical codes) are frequent in "short" instruments but infrequent in "long" ones, or vice versa, i.e. that the frequency seems to be depending on the L/W ratio (the "main proportion").

A couple of consequences of these facts are that
a) an evaluation based on the complete number of the codes of all sources may be considered generally valid,
b) the frequency of a certain method has to be compared with the frequencies of other methods used in sources with (practically) identical L/W value.

Method of accounting for proportioning alternatives (PAlts) and their frequencies

L/W value grouping of all sources

Several L/W values (cf. Section I, Fig. I: 5) are positioned quite close together. Is seems reasonable, then, in the context at hand (i.e. with many "uncertainties"), to arrange the sources in groups, each however big enough to allow for some statistical consideration. Here the following eight groups will be used:

I (8: b-d), II (23: e-f), III (43: g-h), IV (29: i), V (21: j), VI (37: k), VII (30: l), VIII (43: m-o)

(In parenthesis the number of sources in the group, and L/W value codes, cf. above.)

Account of PAlt frequencies, Appendix C

1. Each PPP, level and width, is accounted for separately in Appendix C (graphs) and below (comments).
2. All items have to be arranged in L/W value groups I-VIII (see above).
3. In each separate group identical PAlts are counted.
4. The number of each PAlt (in a group) is converted to % of the total number of sources in the group.
5. The PAlts with the highest frequencies are accounted for in graphs, Appendix C, some other important PAlts just mentioned below.

Evaluation of the PAlt frequency graphs, Appendix C

Owing to the fact that the number of sources within each group is relatively small, each separate registered code represents a relatively big part of the group. Therefore, the frequencies indicated in the diagrams have to be considered "uncertain" within limits equivalent to (at least) one (single, registered) PAlt, i.e. ca. +/- 10 % in group I, +/- 3 % in group III and VIII, and +/- 5 % in the other groups. The "uncertainty" should be considered also "between" the groups. Thus, the graph lines have to be interpreted as "bands", broad stripes, of slightly varying width.

Some or most of the "leaps" of the PAlt frequencies may be explained by the fairly coarse grouping of the L/W ratios. Consequently the wave movements of the "bands" should be imagined as being more gentle. However, the graph lines often show
a) a rather continuous progression when associated to one individual PAlt, and
b) definitely different progressions when associated to different PAlts.
Both circumstances may be interpreted as indicating that the "uncertainty" values applied in the analytical work have served their purposes. (In the following statistical report the PAlt frequencies according to the graphs will be used, however transformed to average values.)

Statistics

The evaluation is limited to and focused on the central task: to decide whether the positions of the proportioned levels and widths (PPPs) may or may not be defined by the geometrical constructions and methods described above, in Section I (PAlts).

L/W group I is too small, compared to groups II-VIII, to be handled similarly and is excluded from the following statistics (cf. the graphs, Appendix C).

In the schedule, Fig. II: 4, is accounted for each proportioned level and width individually, in the same order as previously in this text. (This order does not imply that the levels and points have, or may have been, defined in any definitive order at all, cf. comments further below.) Each proportioned level or width is accounted for on 2–6 lines in 5 columns.

Column 1:
a) L/W groups accounted for
b) L/W group having the lowest PAlt frequency sum (Σf), cf. column 3
c)* PAlts: Diagonal methods
d)* PAlts: Circle methods (occur only on levels D, F and G)
e) PAlts exploiting only square dividing points (DPs)
f) PAlts exploiting both DPs and (already) proportioned level PPPs
* These two lines required only on levels D, F and G
Column 2:
Number of PAlts
Column 3:
Total of all in the graphs registered PAlt frequencies (Σf), calculated as an L/W group average value (GAV)
Column 4:
Group average value (GAV), calculated as a PAlt average value
Column 5:
Number of different PAlts required to reach 100 % PAlt frequency; the numbers are compared with the actual numbers of PAlts in column 2

Fig. II: 4

PPP	1	2	3	4	5
C	a) Groups II–VIII	17	393	23	4.3 < 17
	b) Group VIII only (lowest Σf)	17	309	18	5.5 < 17
	e) PAlts: DPs only	8	180	23	4.4 < 8
	f) PAlts: DPs and level PPPs	9	213	24	4.2 < 9
D	a) Groups II–VIII	16	373	23	4.3 < 16
	b) Group VIII only (lowest Σf)	16	237	15	6.8 < 16
	c) PAlts: Diagonal methods	9	194	22	4.6 < 9
	d) PAlts: Circle methods	7	179	26	3.9 < 7
	e) PAlts: DPs only	12	295	25	4.1 < 12
	f) PAlts: DPs and levels (C, D etc.)	4	79	20	5.1 > 4
P	a) Groups II–VIII	13	294	23	4.4 < 13
	b) Group V only (lowest Σf)	13	272	21	4.8 < 13
	c) PAlts: Diagonal methods	6	108	18	5.6 < 6
	d) PAlts: Circle methods	7	186	27	3.8 < 7
	e) PAlts: DPs only	7	111	16	6.3 < 7
	f) PAlts: DPs and level PPPs	6	183	30	3.3 < 6
W	a) Groups II–VIII	12	283	24	4.2 < 12
	b) Group VIII only (lowest Σf)	12	263	22	4.6 < 12

	e) PAlts: DPs only	8	199	25	4.0 < 8
	f) PAlts: DPs and level PPPs	4	85	21	4.7 > 4
M	a) Groups II–VIII	9	221	25	4.1 < 9
	b) Group V only (lowest Σf)	9	186	21	4.8 < 9
	e) PAlts: DPs only	2	49	25	4.1 > 2
	f) PAlts: DPs and level PPPs	7	172	25	4.1 < 7
E	a) Groups II–VIII	13	293	23	4.4 < 13
	b) Group VIII only (lowest Σf)	13	215	17	6.0 < 13
	e) PAlts: DPs only	4	103	26	3.9 ≈ 4
	f) PAlts: DPs and level PPPs	9	190	21	4.7 < 9
F	a) Groups II–VIII	8	173	22	4.6 < 8
	b) Group V only (lowest Σf)	8	144	18	5.6 < 8
	e) PAlts: DPs only	1	12	12	8.3 > 1
	f) PAlts: DPs and level PPPs	7	161	23	4.3 < 7
G	a) Groups II–VIII	14	319	28	4.4 < 14
	b) Group II only (lowest Σf)	14	279	20	5.0 < 14
	c) PAlts: Diagonal methods	11	262	24	4.2 < 11
	d) PAlts: Circle methods	3	57	19	5.3 > 3
	e) PAlts: DPs only	6	111	19	5.4 < 6
	f) PAlts: DPs and level PPPs	8	208	26	5.8 < 8
Cw	a) Groups II–VIII	11	208	19	5.3 < 11
	b) Group II only (lowest Σf)	11	195	18	5.6 < 11
Dw	a) Groups II–VIII	11	237	22	4.6 < 11
	b) Group VIII only (lowest Σf)	11	163	15	6.8 < 11
Pw	a) Groups II–VIII	6	145	21	4.8 < 7
	b) Group VIII only (lowest Σf)	6	107	15	6.5 > 6
Mw	a) Groups II–VIII	14	196	14	7.1 < 14
	b) Group VII only (lowest Σf)	14	176	17	7.9 < 14
Ew	a) Groups II–VIII	13	266	20	4.9 < 13
	b) Group VIII only (lowest Σf)	13	174	13	7.5 < 13
Fw	a) Groups II–VIII	15	300	20	5.0 < 15
	b) Group VIII only (lowest Σf)	15	262	17	5.7 < 15

Comments

PAlts exploiting diagonal lines allow for interpretation in three different ways, a), b) and c), cf. p. 19. In the statistics, however, only the conventional method, a), is taken into consideration since the other cases are rather few and most of them of little or moderate importance. Yet, one spectacular example should be pointed out.

In Appendix C, level W, three possible b) cases occur, all related to the positioning of the soundholes, the PAlts AEt, AFr and AGs, indicating the possibility of defining the level of the eyes centres and the innermost notches in a very rational way. Another highly probable b) case among others is found in level P, the PAlt FIt, which might be very valuable when positioning level F.

The most central findings, however, are accounted for in the schedule, Fig. II: 4, on the lines marked a) and b) of each PPP, in column 5. There is demonstrated that all PPPs may be defined by means of the proportioning methods described in Section I, facts expressed even in the L/W groups with the lowest PAlt frequency totals.

As a rule less than half the number of the (in the graphs) registered PAlts are required to define the respective positions. This overflow of satisfying PAlts may be explained as a consequence of an unnecessary great number of tested DPs and PPPs in the investigation, and possibly of a conscious desire of the violin makers as well.

In addition to the evidence of proportional design obtained in this investigation, and reported above, several more or less sophisticated examples of proportional "play", or exhibition, may be extracted from the graphs in Appendix C, not the least the intricate calculations suggested in the proportioning of the soundholes. This, however, would imply an exploration far beyond the limits for the present investigation.

*

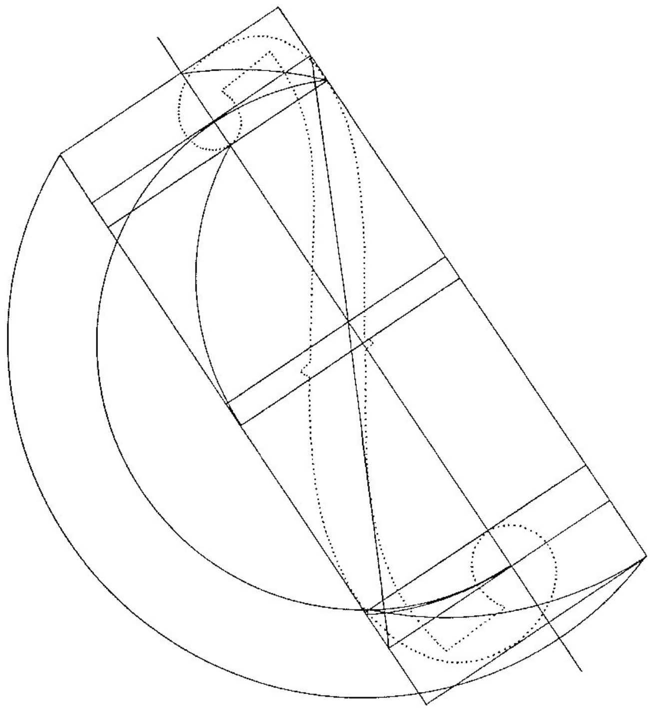
Key

to Fig. II: 1, making clear in which simple way the proportioning "rules" explained above may be applied in order to generate a proportioned, designed, violin belly.

L/W = 5:3 (group II)		P	ADct	E	Z	Cw	VB	Mw	YK
C	AKKAc	W	Y	F	JKt	Dw	BCw	Ew	BI
D	ASLA	M	CDt	G	IMt	Pw	RL, TL	Fw	EwE (6)

III

Proportioning of the soundholes[18]



18 This section of the present investigation is in the main a condensed version of Lomnäs 1997. Major divergences will be accounted for. (Extensive lists of intermediate analysing results are excluded.)

General comments

The special shape of the soundholes grew up as a parallel to the shapes of the scroll and the peg box, more closely described in Section IV. Both iconographies and preserved copies of lira da braccio (Jones: passim) and other string instruments (Geiser 1974: 36f. and 111f.) show that S-shaped holes existed early in the 16th century (at least ca. 1525). By and by they were furnished with one or two notches at the middle and had about 1600 developed "zur elegant ausgebuchteten Schallöffnung mit kreisförmig ausgeweiteten Enden und rautenartigen Mittelkerben, dem f-Loch" (Geiser 1974: 112). By Andrea Amati and Gasparo Bertolotti this "completed" shape was established in violins already (latest) in the 1560s (cf. e.g., Geiser 1974: 38ff. and illustrations IX and X).

Whether the soundholes were proportioned or not is a question not taken up in any literature known to me. In the present study, however, will be investigated if the main dimensions and certain for the shape crucial points may have been defined by implying proportioning principles similar to those demonstrated in Section II as valid for the violin belly, and if there possibly were still more or different "rules" at hand.

As pointed out in Section I, further aspects of the soundholes will be completely left aside: they may consider the function, e.g., but can't "explain" the extravagant shape of the soundholes. However, that the violin makers themselves always have been conscious of the importance of both size and design is beyond question.

Sources

The general aspects mentioned in Section I are, of course, equally valid here. However, in this connection printed illustrations (as in Section II) are insufficient: they don't pay attention to the fact that the soundholes have three dimensions.

Instead another kind of source material was chosen: the collection of soundhole models, carved in wood by the well known violin maker Hans Edler of Munich, as (posthumously) reproduced in Edler, Hans (1976): *Geigen-F-Modelle nach den Originalen alter Meister.* Siegburg/Rhld: Franz Schmitt OHG. 2. Auflage.

124 soundholes (scale 1:1) are reproduced and listed, including Edler's attributions (left out here) in Lomnäs 1997.[19] The collection was considered a representative selection of instruments made by influential violin makers in the period 1600–1800.

Edler certainly made the models as accurate as possible, with the originals at hand. In spite of that, some "uncertainties" – deviations from the original – probably occurred, discussed more in detail in Lomnäs 1997, where a general tolerance of ± 0.2 mm (violin soundholes) was settled and applied in the investigation.

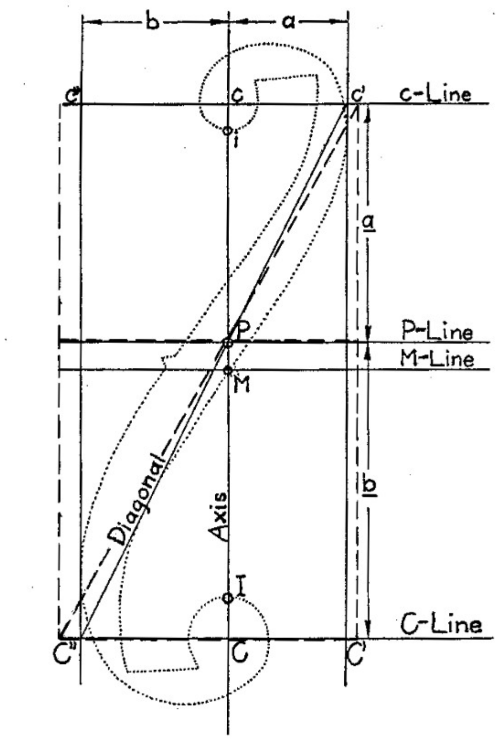
19 The families Amati, Gagliano, Guadagnini, Klotz, Stradivari and Widhalm are represented in more than 50 % of the collection. All instruments are violins except for two violas and one cello.

Proportioning pre-requisites

Above, in Section II, was established that the positions of the eyes of the soundholes may have been determined by positioning constructions (and hardly any more points related to the design of the holes). This means that the distance between the eyes is the only definition for the soundholes.

A right line, **Axis**, drawn through the centres of the eyes will divide the contour of the soundhole in two halves, both like reflected images of each other, although one of them "upside down". The contour may be inscribed in a rectangle, cf. Fig. III: 1, in which a diagonal line intersects the Axis in point **P**. A transverse line through P will complete a geometrical construction similar to those in Section I, Fig. I: 1-7. In Fig. III: 1 the proportioning square, the rectangle sides and a diagonal line are drawn in dashed lines. The Axis is in Section I similar to the vertical proportion line (**t line**). In the figure the **P-Line** is a horizontal proportion line (= the upper side of the proportioning square) and the **M-Line** is the (horizontal) middle line, i.e. that the distances cM and MC are equal. (Cf. the BB level line in Fig. II: 2.)

Fig. III: 1



In Fig. III: 1 the proportion of the contour curve to the right and to the left of the Axis is the ratio **a:b** (cf. Section I, explanations to Fig. I: 1-7), and the ratio <u>**a**</u>**:**<u>**b**</u> is the proportion vertically, above and below the P-Line. It is obvious that the ratios are equal (according to the rules of similar triangles). Below the ratio a:b is designated a **P value**.

(This geometrical construction may have been executed separately by the makers, on a paper sheet etc., with the aid of ruler and compasses only, and used for both soundholes.)

Investigation

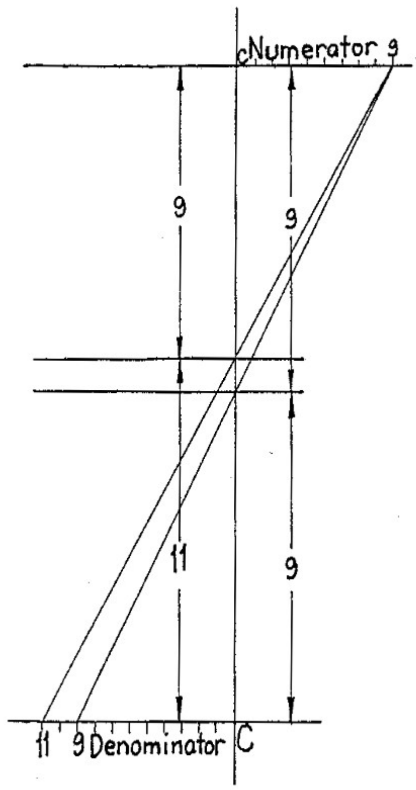
P values of the sources

The investigation has to begin with a registration of the P values of all 124 soundholes.
Measuring process of each soundhole copy separately:
At first a network has to be drawn according to Fig. III: 1, marking the contour and the centres of the eyes. The values a and b are measured and the ratio a:b calculated and registered, i.e. the P value.

Fig. III: 2

1.000	1/1	Rs		*x*
0.905	9/10	Rd		*w*
.900	9/10	Rs		*v*
.895	8/9	Rd		*u*
.889	8/9	Rs		*t*
.883	7/8	Rd		*s*
.875	7/8	Rs	√3/2 Rd	*r*
.867	6/7	Rd	√3/2 Rs	*q*
.857	6/7	Rs		*p*
.847	5/6	Rd		*o*
.833	5/6	Rs		*n*
.827	Φ/2	Rd		*m*
.820	4/5	Rd		*l*
.809	Φ/2	Rs		*k*
.800	4/5	Rs		*j*
.791	2xminor	Rd		*i*
.781	3/4	Rd		*h*
.764	2xminor	Rs		*g*
.750	3/4	Rs		*f*
.749	(Φ/2)√2/2	Rd		*f*
.721	2/3	Rd		*e*
.707	(Φ/2)√2/2	Rs		*d*
.688	major	Rd		*c*
.667	2/3	Rs	3/5 Rd	*b*
.618	major	Rs	1/2 Rd	*a*

Fig. III: 3



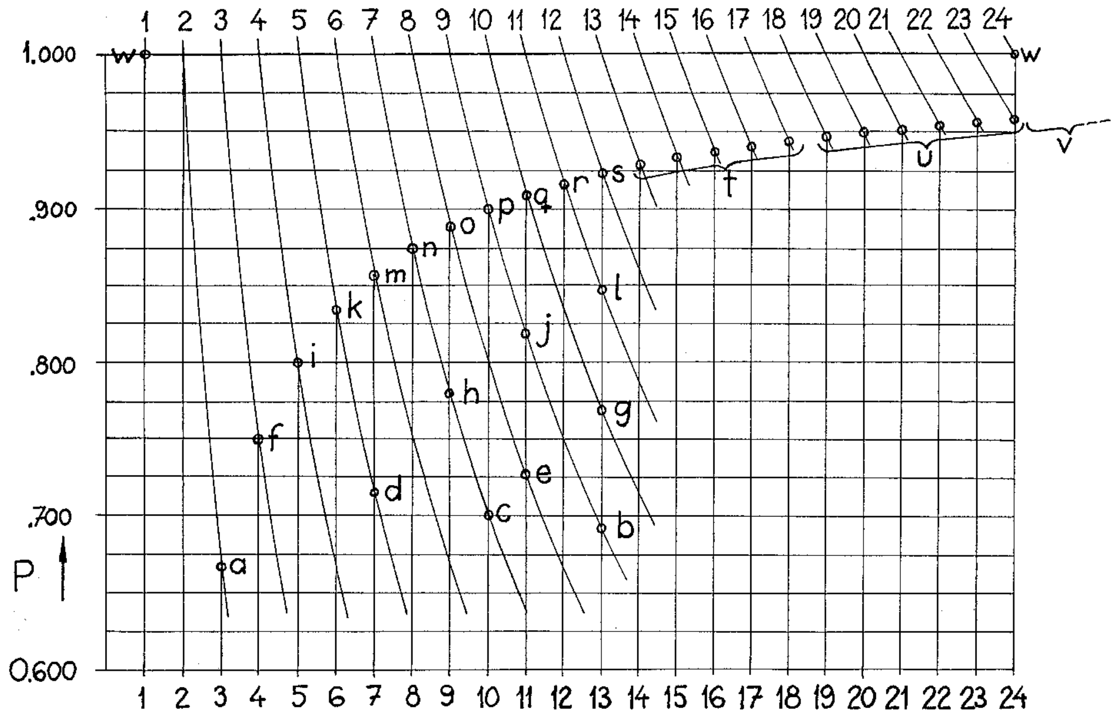
Evaluation of the listed P values

Earlier, in Section I, a geometric method for the construction of P values was demonstrated, cf. Fig. I: 4 and table, Fig. I: 5. Above, in Fig. III: 2, an identical table is shown, however otherwise written: P value, square division point and circle radius, alternative construction and (in italics) designations used below, cf. Fig. III: 4.

Another method is demonstrated in Fig. III: 3: Axis (cf. Fig. III: 1) is divided in the ratio 9:11 (= P value in the figure) and the ratio 11:11 (= M proportion = 1:1).

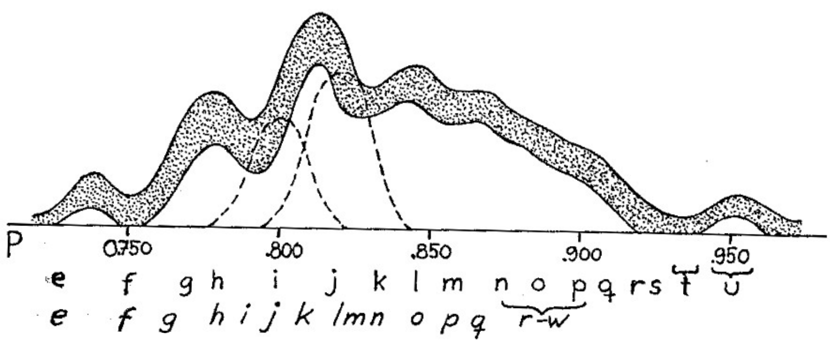
The following diagram, Fig. III: 4, display ratios (P values, with numerator above and denominator below the graph). The ratios are designated with (upright) letters used below.

Fig. III: 4



The listed P values may be grouped together, in three series with intervals alternatively 0.005, 0.010 and 0. 015. If the numbers of the P values in each group are counted, the result may be displayed in a graph, cf. Fig. III: 5. (The "width" of the curve depends on the "uncertainties" mentioned above, partly compensated by the triple grouping of the P values.)

Fig. III: 5



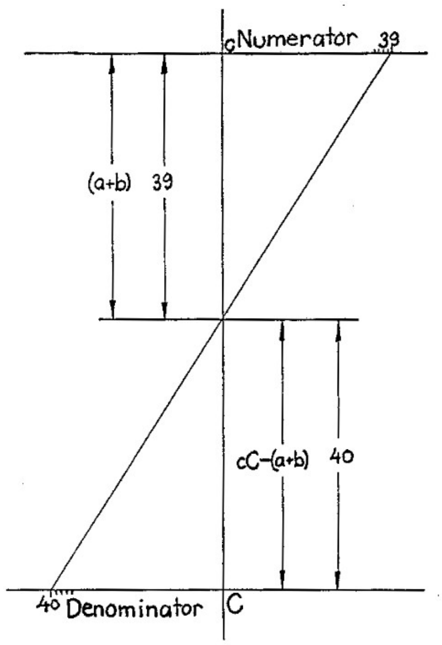
The P value scale is completed according to Fig. III: 2 and Fig. III: 4 above. Obviously P values form "accumulations" around certain values between **g** and **q** (*g* and *w*), especially around **i** and **j** (*j*, *k* and *l*). This distribution may be explained by "summing up" of (hypothetical) bell-shaped statistical curves at separate P values. For example, the "hump" at ca. 0.815 may be explained as the "summing up" of the bell-shaped curves at the P values **i** and **j**. It is evident from the graph that the geometric proportioning method (italic letters) is insufficient whereas the arithmetic method "covers" the scale completely.

With regard to "uncertainties" mentioned above, separation of individual P values is often impossible, e.g the values 0.778 (ratio 7:9) and 0.786 (ratio 11:14). However, presumedly low dividing numbers (numerators, denominators) were preferred to higher numbers when the values were close to similar. Consequently, all P values registered may be explained as ratios of whole numbers, most frequently 10:13, 7:9, 4:5, 9:11, 5:6, 11:13, 6:7, 7:8, 8:9 and 9:10.

B values of the sources

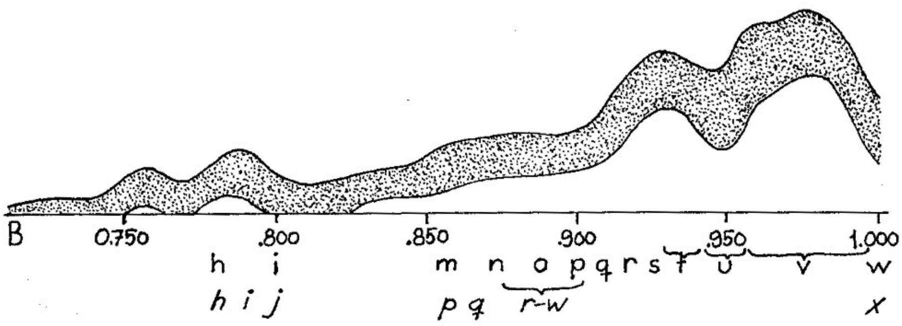
The width of a soundhole, a+b (cf. Fig. III: 1) might have been proportioned from the distance cC in a similar way as the P value. This proportion, the B value, may be written as the ratio (a+b) : [cC-(a+b)], according to Fig. III: 6.

Fig. III: 6



All 124 sources are measured, the B values calculated and registered. They may be treated similarly to the B values above, i.e. grouped together and the number of B values in each group counted. A graph analogous to Fig. III: 5 is drawn, cf. Fig. III: 7 below.

Fig. III: 7



This figure clearly shows that the dominating quantity of the sources display B values between 0.900 and 1.000. However, separate B values within that range can hardly be produced by the methods described above. Another kind of method is required.

B values derived from P values by geometrical construction

This method is demonstrated below, cf. Fig. III: 8 (next page) and more in detail discussed and described in Lomnäs 1997: principles, method of drawing the figures, mathematical treatment of the constructions. Here they are presented in a kind of generic order.

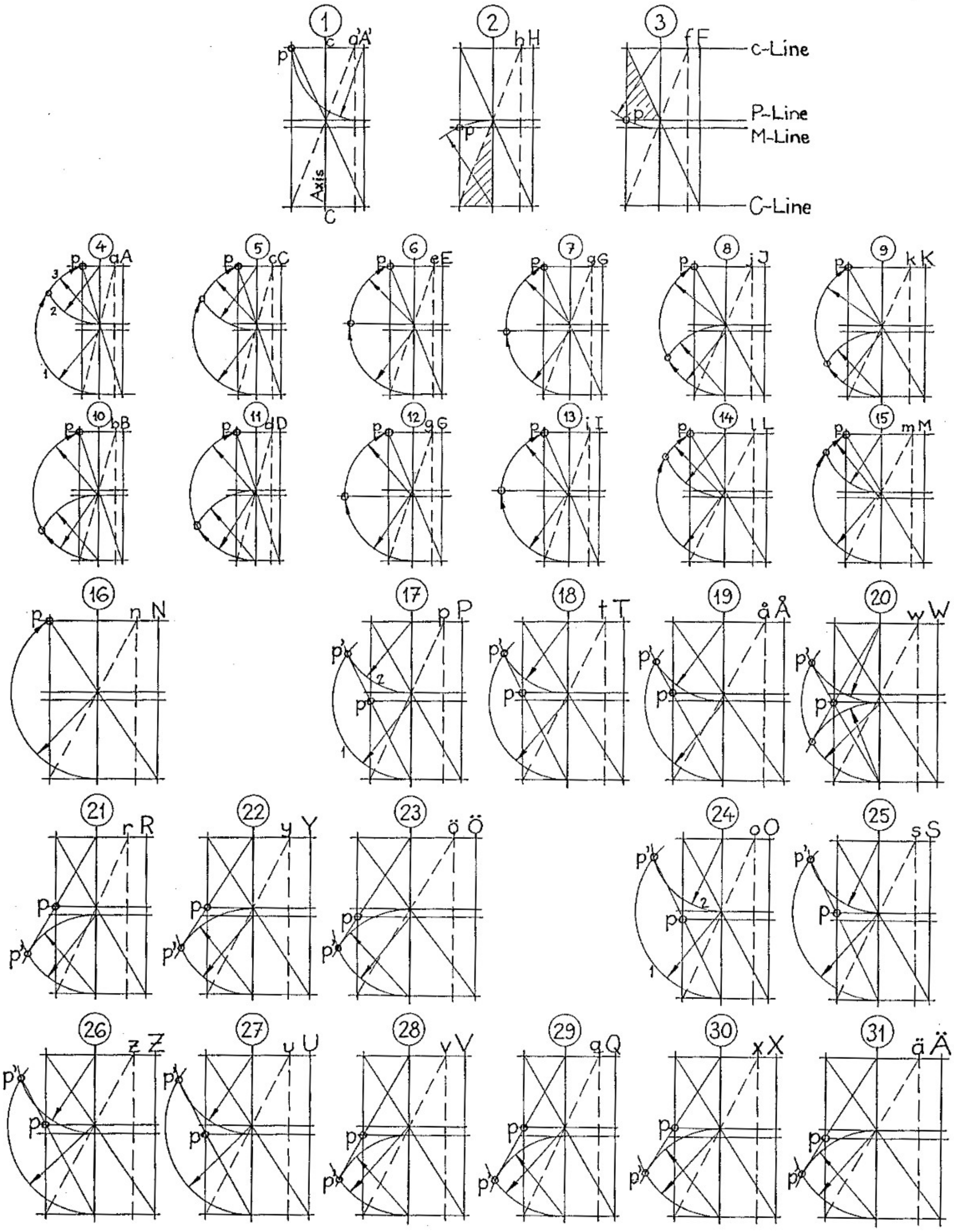
The constructions start with the vertical **Axis**, yet no. 1 with the **right side line**. c- and C-lines have to be drawn at right angles to the Axis, together with P and M lines according to Fig. III: 1 and 3. (The P value may be chosen at will.) In all cases the continuation is the drawing of one or more circle curves. Centre points and radii are set according to the figures. In a few "model" constructions the order of the circle curves is marked, as well. In constructions 1–16 the ends of the circles are marked **p**, i.e. intersections with the c, P or M lines. Yet, in constructions 17-31 there are circle curves crossing in **p'**, from where a right line is drawn to **c** or **C**; this line intersects the P or M lines in **p**.

In all constructions a vertical line, the **left side line**, is drawn through **p.** In construction 1, however, a diagonal line is drawn from **p** to the (original) **right side line**; the diagonal line intersects the P line in the point marked **P** (cf. Fig III: 1); through this point the vertical **Axis** will be drawn.

In all remaining cases a right line is drawn diagonally from the intersection of the **left side line** and the **c line**, through the **P** point, and ending with the intersection of the **C line**; through this intersection point the **right side line** is drawn.

Now a network similar to that in Fig. III: 1 is generated. In all constructions an additional diagonal line may be drawn from the intersection of the **left side line** and the **C line**, through the **P** point, and continued to the **c line**; through this intersection point the **right side line** may be drawn. This narrow alternative, in Lomnäs 1997 denominated **B'**, is in all constructions drawn in dashed lines. Below the alternatives are designated in capitals and small letters, respectively. (The constructions 7 and 12 are similar and given identical designations.)

Fig. III: 8



The execution of the present method might seem to be complicated but is in fact very simple, and necessary tools are as before only ruler and compasses. The next step is to demonstrate if the method might have been used when settling the B value.

Registered B values compared to the B values of the constructions

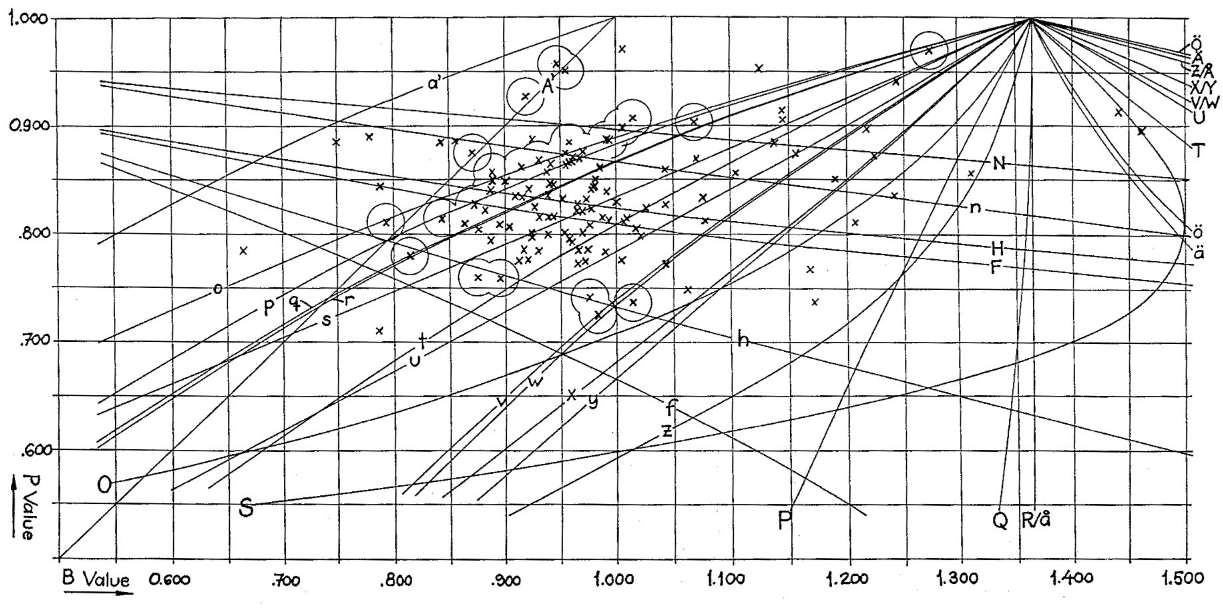
In connection to all constructions the functions B=f(P) and B'=f(P,B) can be mathematically drawn up (cf. Lomnäs 1997, Appendix I) and graphs produced, cf. Fig. III: 9 and 10 below. Fig. III: 9 displays B values of the constructions 1-3 and 16-31, Fig. III: 10 the B values of the constructions 4-15. In both figures the curves are designated with letters (cf. above).

Letter designations and numbers of the constructions

a'/A'	a/A	b/B	c/C	d/D	e/E	f/F	g/G	g/G	h/H	i/I	j/J	k/K	l/L	m/M	n/N
1	4	10	5	11	6	3	7	12	2	13	8	9	14	15	16
o/O	p/P	q/Q	r/R	s/S	t/T	u/U	v/V	w/W	x/X	y/Y	z/Z	å/Å	ä/Ä	ö/Ö	
24	17	29	21	25	18	27	28	20	30	22	26	19	31	23	

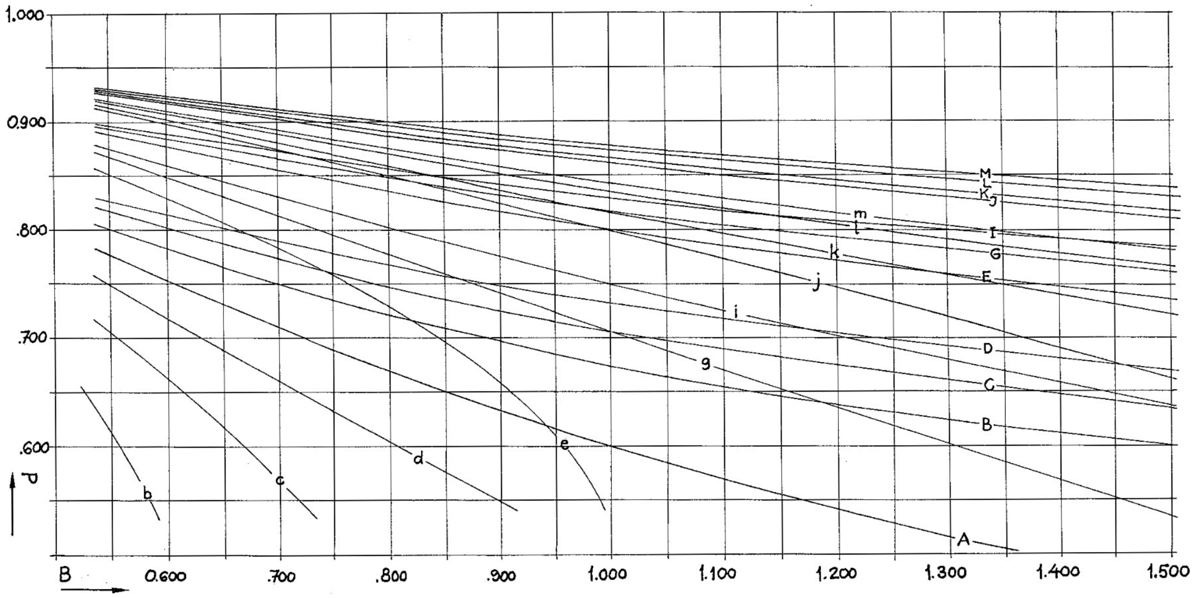
In Fig. III: 9 registered P and B values of all 124 soundholes are indicated with small cross marks, a number of them together with adherent "uncertainty" areas, marked with circle curves. (With the intention to make the image more clear, the radii are settled to ±0.015 instead of ±0.025.)

Fig. III: 9



In the graph, Fig. III: 10 (next page), representing the constructions 4-15, a couple of limitations in these constructions are revealed: a) a small alteration of the P value produces a (relatively) big change of the B value (a weakness found in the constructions 2, 3 and 16, as well), b) the constructions "cover" two regions of (registered) P values insufficiently. (However, this is no reason to exclude them as possible alternatives.)

Fig. III: 10



In the graph, Fig. III: 9, some circumstances should be observed:

a) the P/B values of a number of soundholes adhere to the **A'** curve (constr. 1);

b) several values are positioned close to the **h** and **H** curves (constr. 2);

c) many of them seem to be attached to the **F** curve (constr. 3);

d) a number may be assigned to the **n** and **N** curves (constr. 16);

e) several are grouped close to the **o** curve (constr. 24) in such a way as to be regarded a limit, outside of which are only satellites of curves mentioned above;

f) most of the P/B values (particularly within the central "cloud") may be judged attached to more than one curve, i.e. "explained" by more than one construction;

g) a small number of the P/B values are only weakly attached to any curve.

The distribution of the P/B values on the curves displays several obvious groupings indicating intentional construction. In the table below are registered frequencies of P/B values assigned to the separate curves (in % of all 124 soundholes).

Constr. no.	Curve	Frequency	Constr. no.	Curve	Frequency
1	a'/A'	17	23	ö/Ö	2
2	h/H	61	24	o/O	50
3	f/F	57	25	s/S	60
16	n/N	44	26	z/Z	2
17	p/P	49	27	u/U	32
18	t/T	51	28	v/V	17
20	w/W	17	29	q/Q	62
21	r/R	62	30	x/X	6
22	y/Y	4	31	ä/Ä	2

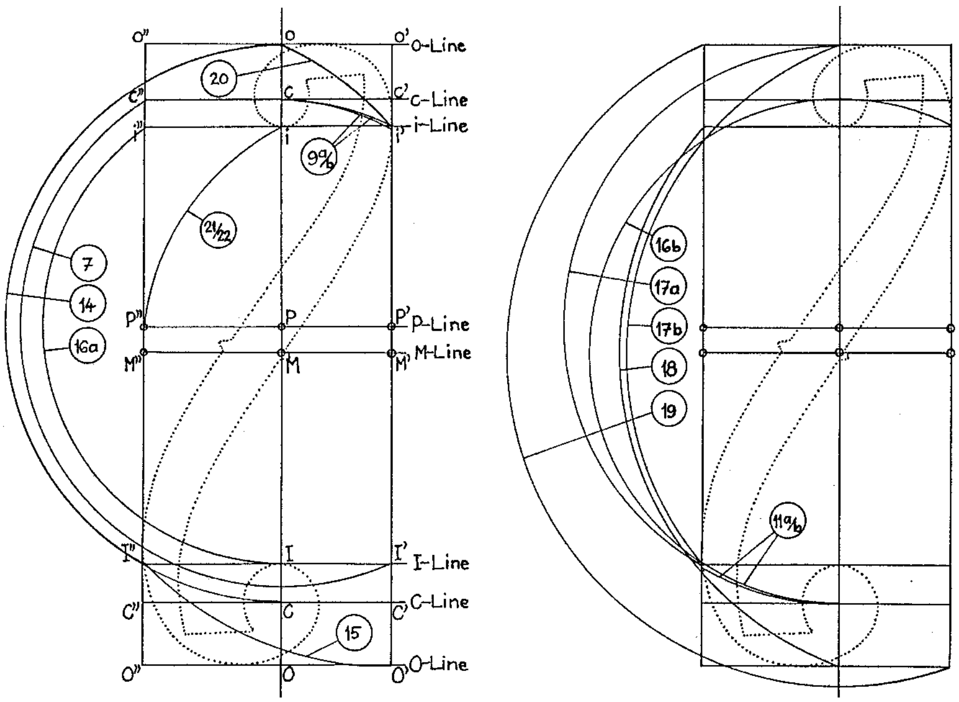
Of the account given above is obvious that the demonstrated geometrical derivation of the B value from the P value can "explain" the design of next to all soundholes in this study (more in detail in Lomnäs 1997, Addendum IV); the "odd" examples are by Edler attributed to Fuchs, Knitl, Maggini, Metelka, Petz and Soquet.

In most cases only ca. 3/4 of the demonstrated constructions may have been used, which one of them in the individual case is impossible to decide, however. Possibly the maker preferred the most simple and time-saving method, or an arithmetical one if the result would be the same.

Length proportioning of the soundhole contour

At first the network of Fig. III: 1 has to be completed with (at least) the horizontal tangents to the contour, in Fig. III: 11 designated **o-Line**, **i-Line**, **I-Line** and **O-Line**.

Fig. III: 11



Now proportioning circle lines will be tested in analogy to the circle methods used with the belly, cf. Section II. As circle centres are chosen the six well defined intersection points on the P-and M-Lines: **P"**, **P**, **P'**, **M"**, **M** and **M'**. Initially the circle lines have to start in points on the c- or C-Lines, later on, when the i- and o-Lines or I- and O-Lines are positioned, the additional intersection points, too, may be used as starting points.

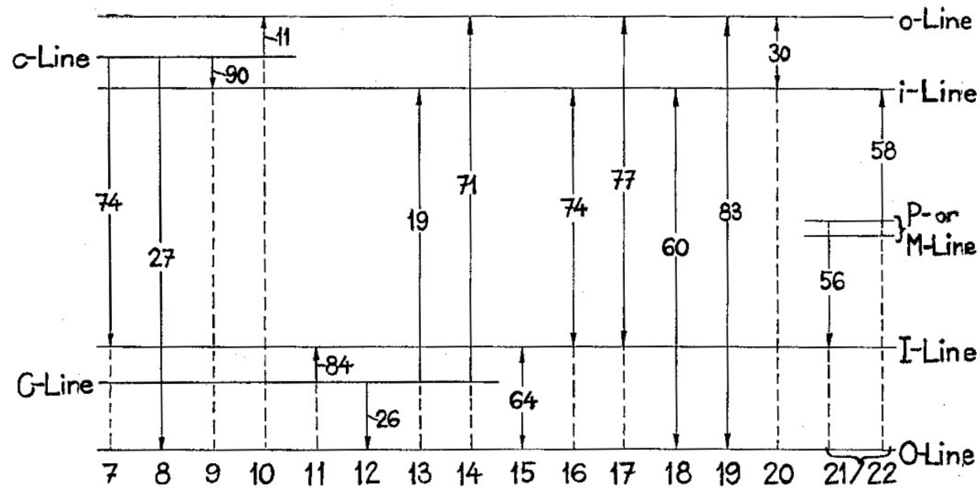
If all 124 soundholes are tested using circle lines in accordance with Fig. III: 11 (the alternatives distributed in two images), it will be obvious that each circle (i.e. circles designated with identical figure) will fit many soundholes, see table below, the column **Number**. (This table is limited to the circles in Fig. III: 11.)

Circle no.	Starts in	Centre	Ends in	Number
7	c″	P	I'	37
9	c	P or M	I'	104
11	C	P or M	I″	100
14	C	P	o	29
15	I″	P'	O'	37
16a	I	P	I″	30
16b	I″	M	I'	60
17a	I″	P	o	24
17b	I″	M'	o	32
18	O	M'	I″	53
19	O'	M	o″	46
20	I'	M″	o	29
21/22	P″ or M″	M'	I	36

A complete table (4 pages) of all 124 soundholes, covering all circles (numbered 7-22) is given in Lomnäs 1997, Appendix III, but not reproduced here. Instead the frequencies (in % of 124) of all appropriate circles are reported in the schedule below, Fig. III: 12.

Each vertical line represents the total of circle lines that may be drawn between points on two separate horizontal lines in the network according to Fig. III: 11. (Thus, e.g., line 19 represents all circle lines between any of the points O, O′, O″ and any of the points o, o′, o″. The frequency sum is 83 %.) The high percentages, above all, indicate that at least most of the circle options, if not all, are *intentionally* used. It is also evident that the results of two or more circles may coincide.[20]

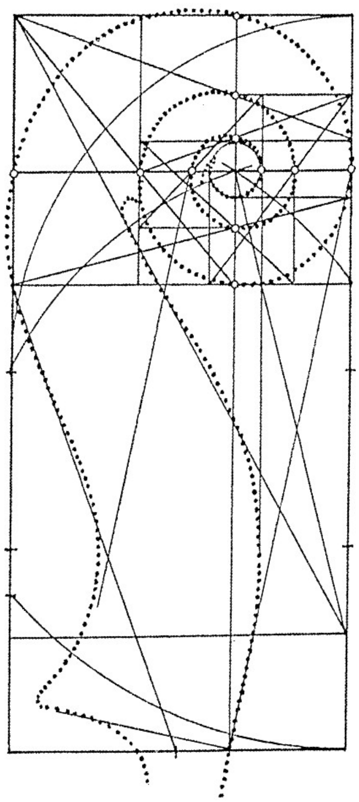
Fig. III: 12



20 Additional comments on possible proportioning of the soundhole contour and the notches, cf. Lomnäs 1997.

IV

Proportioning of the scroll and the pegbox[21]



21 This section is essentially a summary of Lomnäs 1998. (Extensive lists of intermediate analysing results are excluded.)

Historical background

In iconographical objects dating from as early as ca. 1470 are displayed (medieval) fiddles and rebecs furnished with scrolls (Geiser 1974: 109, 115f.), however, "die schöne, elegant gestochene Schnecke [...] liess sich nur in Bildern nach 1600 nachweisen".[22] Scrolls exhibiting different "transitional" forms are found from the whole of the 16th century and a good part of the 17th (Geiser 1974: passim).

Early scroll and pegbox forms may have developed from scroll-formed ornaments occurring on Italian coats-of-arms already in the Middle Ages and/or from the ornamental conventionalized paper scroll (cartouche). However, later forms suggest influences from antique Roman wall decorations (Dreyer 1959: 7), adopted and developed with Rafael and his school (so-called grotesqueries). There is always a tendency of coiling, at the same time found in the Ionian volute and the S-shaped double spiral, similar to the contours of the soundholes of the violins and the side contours of the bellies, as well.

From preserved objects, however, is evident that a common convention as to the form of the scroll and the pegbox was missing until ca. 1600 (cf. above, Section III).

Source material

The general aspects on sources mentioned above, Section I, are perfectly valid concerning scrolls and pegboxes. Hence, the same kind of source material as was used in Section II, with the bellies, now has to be sought after. The requirements as to being representative and the quality of the photographic reproductions are identical with those in Section II. Altogether 256 reproductions of scrolls and pegboxes are selected and individually listed in Lomnäs 1998, Appendices I-III, including all data used in the study. In the present report they are summarized in tables and graphics.

Images:

Photos of 10 violoncelli in Musée instrumental du Conservatoire national, Paris.

Illustrations in:

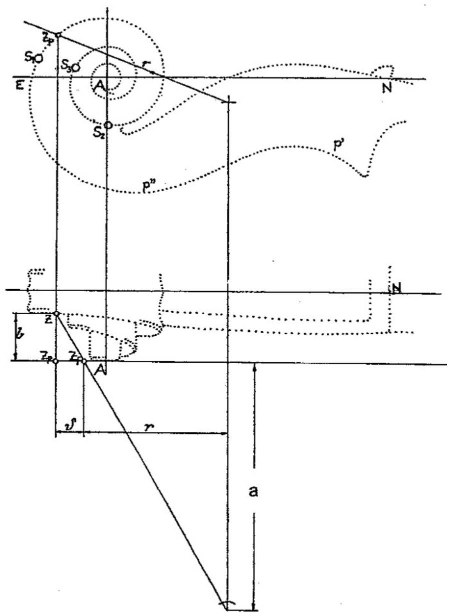
1. Goodkind, Herbert K. (1972): *Violin Iconography of Antonio Stradivari*. Larchmond, New York: published by the author.
2. Hamma, Walter (1986): *Geigenbauer der deutschen Schule des 17. bis 19. Jahrhunderts*. Vol. I-II. Tutzing: Hans Schneider.
3. Hamma, Walter (1993): *Meister italienischer Geigenbaukunst.* Wilhelmshaven. 8th ed.
4. Pilař, Vladimír, & Šrámek, František (1989): *Umění houslařů*. Prague: Panton.
5. Sacconi, Simone F. (1972): *I 'segreti' di Stradivari*. Cremona: Libreria del Convegno.
6. Senn, Walter, & Roy, Karl (1986): *Jacob Stainer. Leben und Werk des tiroler Meisters 1617–1683*. Frankfurt/M: Erwin Bochinsky.
7. *The Strad*. 1982-1998.

22 Out of the earliest images in detail displaying "complete" scrolls and pegboxes are paintings by Michelangelo Merisi (ca. 1573-1610) named "Caravaggio" from his birthplace, situated in the region where some of the earliest known violin instruments were made (cf. Geiser 1974: ills. 102, 111 and 157). A violin showing fully developed scroll and pegbox was painted by him 1590-92 in a picture called "The Lute Player" (Hermitage, S:t Petersburg).

In Lomnäs 1998, Addendum I-II, are considered (and discussed in detail) "uncertainties" (cf. above, Section I), including the so-called parallax, i.e. perspective circumstances in the images.

The significance of "parallax" is illustrated below, Fig. IV: 1. If the optic angle (90° in the figure), the distance of the object **a** (in the figure much shortened) and the depth positions **r** and **b** are measured in the source picture, the latter may be adjusted by the measure ***δ*** in all points of interest.

Fig. IV: 1



In this study the total of "uncertainties" ("tolerances") is in accordance to the decisions in Sections II and III above settled to ca. ±0.4 mm (violin/viola) and ca. ±0.6 mm (cello) (both assuming image scale 1:1); proportions are settled to ca. ±0.025.

Proportioning

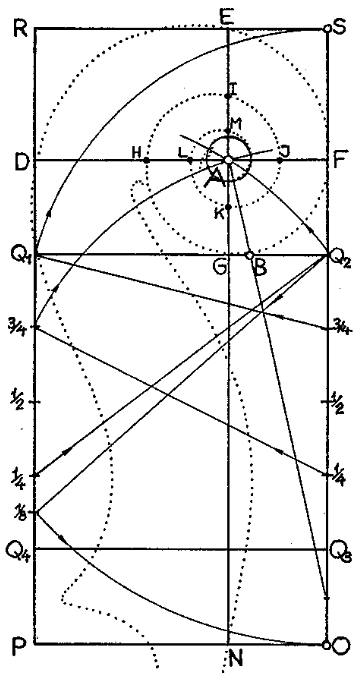
Usually scrolls of the period in question are not "symmetrical" in a modern sense of the word when observed from the left and right sides, or from the dorsal and frontal directions, or from the head end of the scroll. This is not astonishing since "harmony" (sometimes called *"symmetria"*) was a more central ambition, meaning the right balance of parts and of the entirety.

It seems reasonable that a supposed proportioning of scroll and pegbox had to limit the intentions to a few points on one of the side views (projections) observed at right angles, as shown in Fig. IV: 1 above. The other side could be reproduced later on (as a reflected picture). Below the scroll and the pegbox will be regarded as mainly "symmetrical" (in modern sense) on both sides of a symmetry plane, cf. Fig. IV: 1. The analysis will be limited to one projection only. (Besides, often the source images reproduce only one side.)

Analysis and evaluation

Initially a side projection of each scroll/pegbox (with regard to "uncertainties" and parallax) has to be inscribed in a proportioning network according to the description in Section I above. In the complex three-dimensional double spiral only two readily definable points may be discerned: the position of the centre of the ear of the scroll and the end of the pegbox, i.e. the points **A** and **N** in Fig. IV: 2 below. It is reasonably enough to accept the (prolonged) right line AN as the "axis" in a proportioning rectangle.

Fig. IV: 2



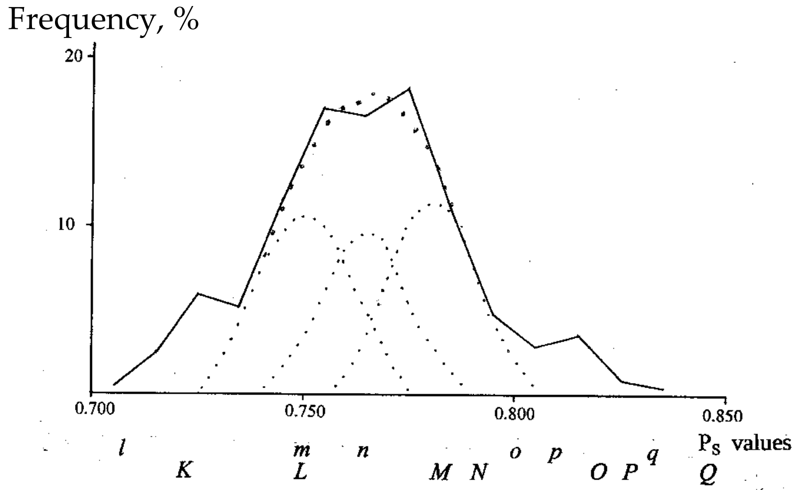
The sides of this rectangle (parallel to the axis) have to intersect the contour of the scroll in the points where the contour crosses the axis AN, i.e. in point E, and its "transversal axis" in D and F in Fig. IV: 2. From a transversal line through point G, the second intersection of the axis and the scroll contour, a square $Q_{1\text{-}4}$ may be drawn, cf. Fig. I: 4. Thus, the size of the square is defined by the length of the "transversal axis" DF. The rectangle's fourth side is drawn through the N point.

Each object of this investigation now has to be analysed concerning the positions of the points S, O and A, the diameter of the scroll ear, the intersection points H-M, cf. Fig IV: 2, the side projection of the pegbox, finally at least some positions and proportions related to the third dimension of the objects. (All data reported in Lomnäs 1998, cf. above.)

The S position

may be defined by the method described in Section I, Fig. I: 4 and the table, Fig. I: 5. The proportion of the "length" of the scroll, $\mathbf{P_s}$, is $SQ_2 : Q_2Q_3$. Measured P_s values may be grouped (intervals 0.010), the number of P_s values within each group counted and visualized in a graph, Fig. IV: 3.

Fig. IV: 3



In the figure the dividing points are designated according to the table, Fig. I: 5 (cf. Fig IV: 5 below). As suggested in the figure, the central part of the curve may be regarded as the result of three overlapping statistical bell-formed curves (width 0.050, owing to "uncertainties" mentioned above) at ***m*/*L*,** ***n*** and ***M*.** Possible dividing points are positioned relatively tight, for which reason a separation of individual P_s values is impossible. However, obviously all P_s values can be "explained" by the proportioning method used. In fact, two proportioning points (***m*/*M*** and ***n*/*N*,** i.e. 3/4 and 2x*minor*) cover next to all registered Ps values.

The O position
may be defined with the method used above, however with both circle centres and starting points chosen from the complete list, cf. Fig. IV: 5 further below. Measured P_o values are grouped (interval 0.020) and visualized in a graph, Fig. IV: 4 (next page).

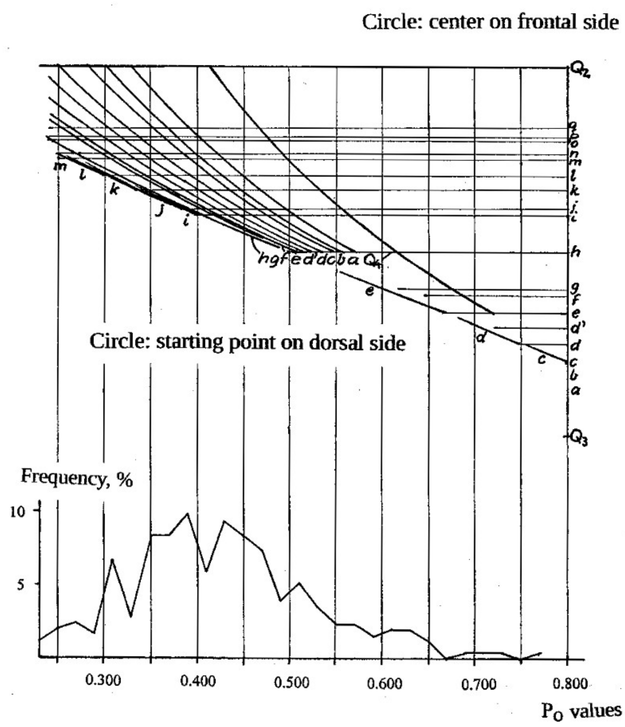
Nearly all P_o values may (like the P_s values) be "explained" even by using a few of the dividing points at hand, and the results may overlap each other. Probably, however, simple methods should have been preferred to more complex and time-consuming ones. Hence, as few as possible dividing points were probably preferred and used when positioning both S, O and A (cf. further below).

It is evident (in an individual scroll/pegbox) that the centre of the P_s circle may have been used as centre of the P_o circle in 86 % of all sources. In 41 % of the cases also Q_2 may have been used as centre, and in 25 % of the cases the point ***h*,** i.e. the middle point of the square (10 % in cases where neither the P_s centre nor Q_2 may be used). Thus, the mentioned few alternatives may together cover nearly all cases.

In addition the starting point of the P_o circle line in 26 % of the cases may be on the same level as the centre, although on the opposite side of the square. More common is, in 76 % of the cases, that the starting point of the P_o circle on the dorsal side may be derived from the centre of the P_s circle, e.g.: P_s centre = 3/4 is followed by P_o starting point = 1/4 or 1/8. (Below, this kind of "consequence" will be named "reflection".)

Furthermore, the point Q_4 may be used as starting point of the P_o circle in 19 % of the cases. Thus, the alternatives altogether cover all cases.

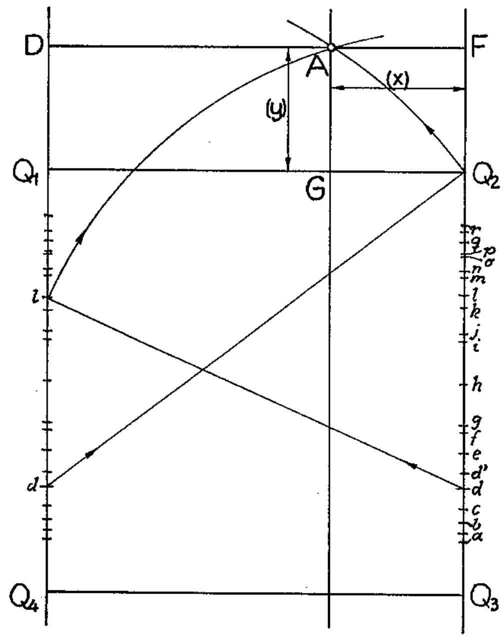
Fig. IV: 4



The A position

This position may be settled by using two crossing circle lines, cf. Section I, Fig. I: 4, and below, Fig. IV: 5, where all probable centre and starting points are marked.

Fig. IV: 5



Principally all plausible circles should be tested for all 256 scrolls of which the coordinates (x) and (y) have been measured and registered. However, since the dividing points are situated rather closely, even the circles will be drawn close together. As a consequence, all (x)/(y) values may easily be "explained", without differentiation. More interesting is, if the dividing points used earlier (when settling the S and O points), or some other logically "related" point, may be used also when settling the A position, i.e. (x) and (y).

Counting of test circles reveals that starting points of P_o-circles may "explain" 67 % of centres of "D-circles" (= circles with centres on the dorsal side of the scroll), "reflected" P_s-circle centre of additional 22 % and the point Q_4 of 9 %, i.e. altogether 98 %. In all cases the starting point of the "D-circle" was Q_2. Since a more simple "explanation" of the "D-circle" drawing hardly is possible, further alternatives are unnecessary.

In a similar way is revealed that the centres of the "F-circles" (= centres on the frontal scroll side) in 97 % of all cases are positioned on the same levels (dividing points) as the centre of the "D-circle". Further cases may be "explained" in similar ways.

The starting point of the "F-circle" may in all cases be positioned on the same level as the centre of the P_s-circle (on the opposite side of the square).

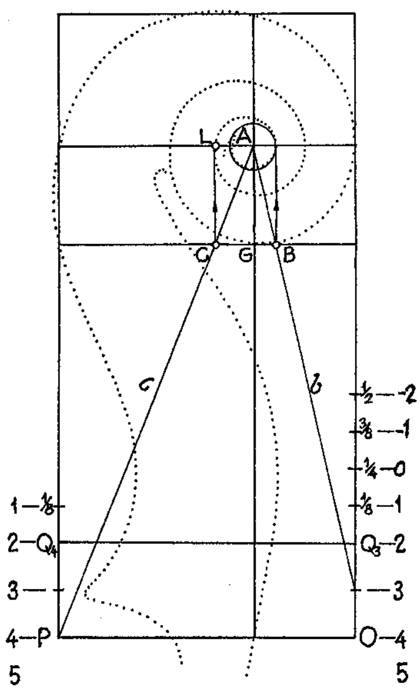
Obviously all dividing points required for the proportioning of the scroll so far in most cases (76 % or more) may be derived from the position of the centre of the P_s-circle.

By the addition of the corners $Q_{2\text{-}4}$ of the proportioning square and the middle point **h** of the square side, practically all cases are covered.

In Fig. IV: 2 above is demonstrated in which way very few dividing points are sufficient for the proportioning of the positions of S, O and A.

The radius/diameter of the scroll ear

Fig. IV: 6



It is possible to execute proportioning of the volutions by applying similar triangles in the proportioning network drawn around the scroll, in order to settle the points where the volution contour intersects the "axis", i.e. the points H-M, cf. Fig. IV: 2. However, at first the end of the coil has to be settled, i.e. the radius or diameter of the ear.

Fig. IV: 6 above demonstrates a hypothetical method intended for the proportioning of the radius of the ear. Here the pegbox is included, and so is the inner coil of the volution, too, i.e. its crossing of the "axis" DF.

The endpoints of the right lines ***b*** and ***c*** have to be registered of all 256 objects; their frequencies (in %) are displayed in Fig. IV: 7 (line ***b***) and Fig. IV: 8 (line ***c***).

Fig. IV: 7

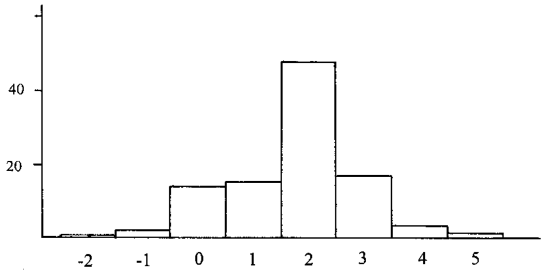
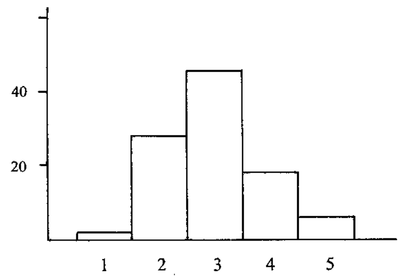


Fig. IV: 8



According to Fig. IV: 7 the radius of the ear may have been settled by line ***b*** in 80 % of all cases; starting point is 1, 2 or 3, especially 2, i.e. the square corner Q_3.

As to line ***c***, Fig. IV: 8, a similar result is found: 92 % of all cases are covered by the points 2-4. (The position of point L of the volution may be settled in another way as well, cf. below.)

The intersection points H-M of the volution contour

A pre-requisite for proportioning with triangles are projections of the ear radius on the sides of the proportioning network around the scroll, cf. Fig. IV: 9-14 below. Now the intersection points H-M may be settled in the following manner: all possible diagonal lines (within the proportioning network) that may have been used in order to settle the intersecting points have to be tested, defined and registered.

Fig. IV: 9

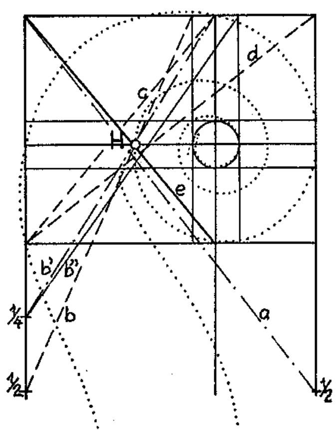


Fig. IV: 10

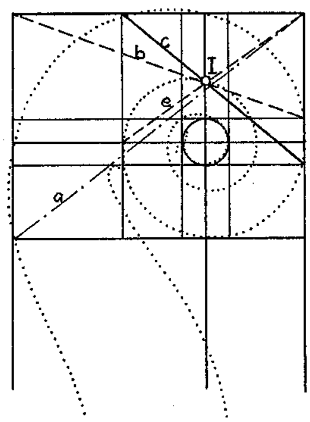


Fig. IV: 11

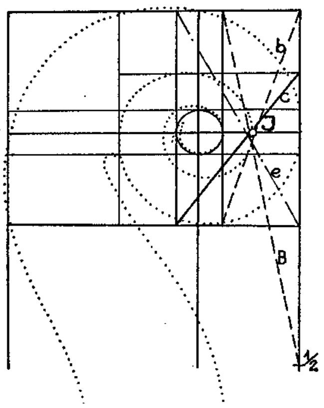


Fig. IV: 12

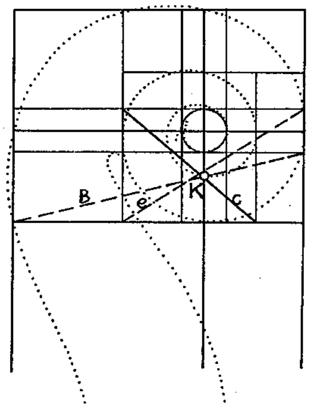


Fig. IV: 13

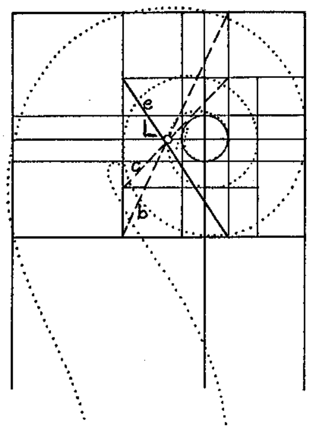
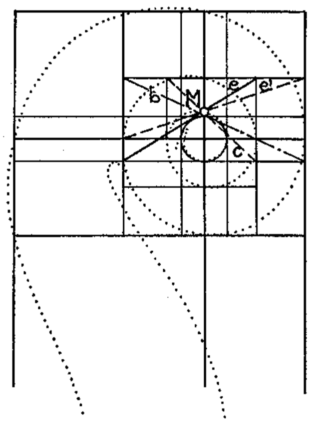


Fig. IV: 14



Concerning point H the number of possible lines is limited. However, as soon as this and following points have been settled their positions may all be used for proportioning further diagonals. Principally all possible diagonals should be tested. Yet their number has to be limited: they are many, and several of them are completely senseless. Instead, for all 256 items in this study only the plausible diagonals have to be, and have been, registered.

The letter designations applied on the proportioning diagonal lines in Fig. IV: 10-14 are inspired by the designations chosen in Fig. IV: 9. Identical letters indicate that the proportioning principle is similar, meaning that the diagonal starts and ends in points positioned similarly in relation to the intersection point, *or* "rotated" ca. 90° clockwise *or* "reflected" in relation to the "axis" DF or EG; yet, one of the end points is always positioned on an "axis" or one of the ear circle tangents.

At first hand the "rotated" alternatives will be registered, in small letters; "reflected" alternatives are noted in capitals. For example: in Fig. IV: 11 line **b** may be noted **E**, line **e** also **C**; in Fig. IV: 12 line **e** likewise **C**, in Fig. IV: 13 the line **e** either **C** and **d**, line **b** also **c**, in Fig. IV: 14 the line **b** alternatively **c**, line **e** also **c** or **C** or **d**. The exchangeability demonstrates the near relations between the proportioning principles. Below, in Fig. IV: 15 are registered not all, but the most common, of the proportioning lines applicable to the points H-M.

Fig. IV: 15

Intersection point	Proportioning line, frequency (%)									
	B	a	b	b′	b″	c	d	e	e′	x
H	-	33	25	15	20	1	25	50	-	8
I	-	30	43	-	-	40	-	39	-	14
J	56	-	61	-	-	27	-	44	-	9
K	49	-	1	-	-	43	-	12	-	30
L	-	-	41	-	-	39	-	34	-	28
M	-	-	32	-	-	35	-	42	50	11

To each intersection point are as a rule several proportioning lines related, the most common of them listed above. However, their frequencies are big enough to demonstrate that a summing up of them results in more than 100 % for all points H-M. In addition, the total of unusual proportioning lines are noted in column x of the table. It is evident that just a few nearly related principles are sufficient to "explain" the positions of all intersection points.

The side projection of the pegbox

Concerning the pegbox no proportioning hypothesis has been developed. However, the contours seem to adhere fairy well to certain "supporting" lines that may be derived from the proportioning rectangle RSOP, cf. Fig. IV: 16 below. An inventory of all 256 items of this study results in the table, Fig. IV: 17. The widths of the "tunnels" (the "supporting" lines) in the figure are settled to ±2.5 % of the distance AF.

Fig. IV: 16

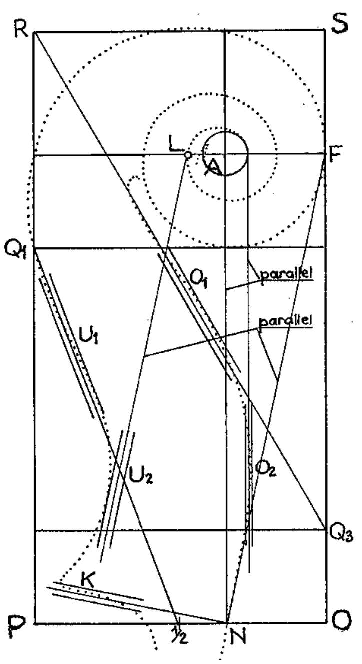


Fig. IV: 17

Part of the contour	Adheres to "supporting" line
U_1	78 %
U_2	62 %
O_1	64 %
O_2	65 %
K	56 %

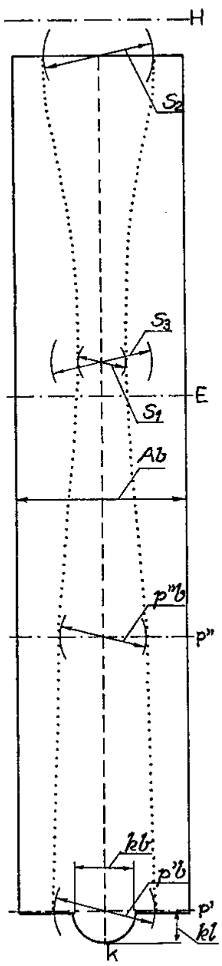
Obviously the proposed "supporting" lines may have been used in a majority of the cases. The dorsal parts of the contour, U_1 and U_2, adhere fairly well while the frontal parts, O_1 and O_2, diverge more. However, this should not be named "proportioning".

Widths: positions and possible "proportions"

In contrast to the side contour of the scroll and pegbox that easily may be reproduced as a parallel projection on a flat surface, the three-dimensional design of particularly the scroll has to be treated in another way. Proportioning constructions founded on the flat proportioning square can't be transferred directly to the curved surface of neither the scroll nor the pegbox. Instead, they have to be proportioned separately on a flat surface, an "unrolling" of the curved surface, cf. Fig. IV: 18. However, it is difficult, if not impossible, to find a reasonable way of application of the classical proportioning method hitherto applied in the present investigation. Instead will be tested if the essential widths may have been derived from distances in the side projections, and transferred as distances, not as intended proportions.

(The "unrolled" scroll and pegbox in Fig. IV: 18 certainly is inspired by Antonio Stradivari's models, cf. Sacconi 1972: 21, 122, 213.)

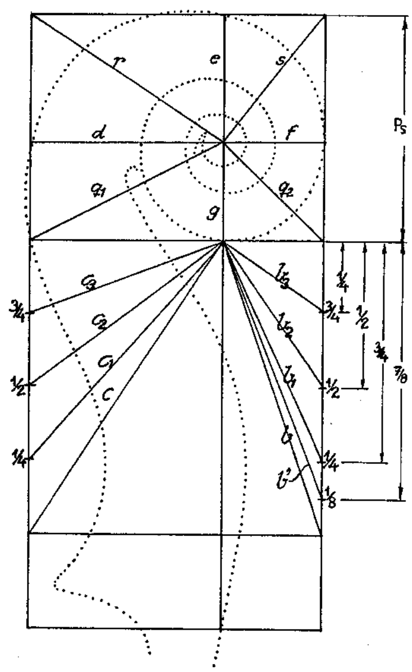
Fig. IV: 18



Preparatory studies (and the result reported below) apparently indicate that the widths at several points in the model may have been settled by applying distances in the side projection, cf. Fig. IV: 19 (next page). The model (Fig. IV: 18) begins (down below) in point k (the "heel" of the pegbox) and ends above in S_2 (the width at the bottom of the "throat" close to point H in the side projection).

Widths indicated in the model:
A maximum width at the point *p´* (cf. Fig. IV: 18): the distance *p´b* is the width of the pegbox at the lower edge, *Nb* the width at the upper edge, i.e. at the nut). A (relative) maximum width *p"b* at the point *p"* (ca. at the beginning of the "throat"). A minimum width S_1, just above point E (of the side projection) and a (relative) maximum width S_2 ca. at point H. In addition to the widths shown in Fig. IV: 18 there is as a rule another (relative) minimum S_3 at the inner spiral and then a more or less vigorously accelerating increase of the width to the end point *Ab* (width of the model in Fig. IV: 18), i.e. the width at the ear.

Fig. IV: 19



The figure displays a number of easily defined distances available in the side projection of a scroll and pegbox. In addition the distances may have been divided in parts (1/2, 1/3 etc.) in order to supply further alternatives. In the present study, however, they will be limited to the most frequent ones. (The source material adds special difficulties: frontal and dorsal views often are in different scales and perspectives, hard to compare with the side projections.)

With respect to each object in this study, the widths indicated in the model, Fig. IV: 18, may be compared to distances chosen from the side projection of the object. Similars are counted and the result registered in a table, Fig. IV: 20, in percentage of all objects. The total may exceed 100 since many widths may be "explained" in more than one way. In column **y** the total of additional possible "explained" widths is registered (widths taken from the side projection but not indicated in Fig. IV: 19).

Fig. IV: 20

Width in Fig. IV: 18	**Distances in Fig. IV: 19, frequencies (%)**						**y (%)**
Nb	1/2: 38	b/2: 36	b_2: 11			s: 20	18
kb	1/3: 5 3/8: 11		$b_1/2$: 9	g: 12			21
p´b	1/2: 19	b/2: 18		e: 8	q_2: 11		21
p"b	3/8: 17		$b_1/2+b_3$: 35		q_2: 19		12
S_1	3/16+1/5: 16 1/4: 17		$b_1/4+b_3/2$: 24		$q_2/2$: 21		12
S_2	1/2: 20	b/2: 20			q_2: 8	s: 18	21
S_3	1/2: 18		$b_1/2+b_3$: 16	e: 19	q_2: 19		18
Ab	P_s: 6 7/8: 12 3/4: 14		b_1: 20	c_2: 13 c_3: 9	q_1: 12	r: 17	10

Final comments

Of the present investigation is evident that the application of old proportioning methods described in Section I may "explain" the geometrical design of classical violin instruments, at least of the belly (Section II), the soundholes (Section III) and the scroll and pegbox (Section IV).

The geometrical proportioning constructions being most in vogue evidently were, as a rule, the simplest and most time-saving to the instrument maker. Thus, there was no need for neither models nor forms (like the well known Stradivari *forma*). Instead the design of each instrument copy could be more or less individual. In this study are not two "identical" instruments to be found.

In spite of the simplicity of the common proportioning methods demonstrated above, this *investigation*, however, with the proposal to find *evidence* for the methods and the use of them, had to be rigorous, and demanding a comprehensive source material. As a consequence this text maybe tends to be less attractive to a reader.

Hence, there still exists an important task to be solved: the realization of a simple, instructive manual, summarizing the principles and mostly used applications of the proportioning methods, "The Rules of the Old Masters", a project similar to the one Giovanni Marchi was unwilling to undertake, however this time to be executed by a modern, experienced and pedagogically gifted luthier. And now Giovanni Iviglia's sigh, "siamo alle solite", might perhaps be interpreted more positively, as "back to basics", in the sense that violin making would keep firm to the old "secret" principles of design as an expression of *"beauty in proportions"*.

Bibliography

Source material

Section II: Sources are 234 individual illustrations taken from literature listed on p. 16.

Section III: Sources are 124 f-hole models reproduced in Edler, Hans: *Geigen-F-Modelle nach den Originalen alter Meister*. Siegburg/Rhld 1976: Franz Schmitt OHG. 2nd ed.

Section IV: Sources are 256 pictures: a) 10 photos of instruments in Musée instrumental du Conservatoire national, Paris; b) 244 illustrations chosen from

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APPENDIX A

#	Maker	Residence	Period	Instr.	L/W
1	Amati A	Cremona	a	vl	n
2	Amati N	Cremona	b	vl	g
3	Stradivari A	Cremona	c	va	d
4	Stradivari A	Cremona	c	va	k
5	Stradivari A	Cremona	c	vl	k
6	Stradivari A	Cremona	c	b	i
7	Stradivari A	Cremona	d	vc	j
8	Stradivari A	Cremona	d	vc	j
9	Stradivari A	Cremona	e	vc	k
10	Stradivari F, O	Cremona	e	vc	l
11	Deconet DG	Venedig	f	vl	i
12	Deconet M	Venedig	f	va	g
13	Deconet M	Venedig	ghi	vc	hij
14	Busan D	Venedig	h	vl	e
15	Bellosius A	Venedig	g	vl	l
16	Testore PA	Mailand	f	vc	h
17	Testore PA	Mailand	f	vc	h
18	Grancino G I	Mailand	cd	va	k
19	Grancino G I	Mailand	c	vc	e
20	Grancino G II	Mailand	cd	b/vc	bc
21	Grancino GB I	Mailand	cd	vc	k
22	Landolfi CF	Mailand	f	vc	h
23	Landolfi PA	Mailand	fg	vc	h
24	Mantegazza PJ	Mailand	ghi	vl	l
25	Mantegazza F	Mailand	ghi	va	k
26	Farotti C	Mailand	m	vc	g
27	Carcassi L&T	Florenz	fg	vc	f
28	Carcassi T	Florenz	f	vc	n
29	Carcassi V	Florenz	ij	vc	g
30	Carcassi V	Florenz	ij	vc	l
31	Gabbrielli GB	Florenz	f	va	ef
32	Gabbrielli GB	Florenz	f	vc	ef

33	Gabbrielli GB	Florenz	fgh	vc	m
34	Gabbrielli GB	Florenz	f	vc	m
35	Gabbrielli GB	Florenz	ghi	vc	d
36	Malvolti PA	Florenz	ef	vc	e
37	Arcangioli L	Florenz	lm	vc	h
38	Castello P	Genua	fg	vl	l
39	Cavaleri J	Genua	ef	vl	k
40	Cordanus JPh	Genua	g	vl	l
41	Rocca E	Genua	m	vl	e
42	Rocca E	Genua	m	vl	i
43	Scarampella G	Genua	lm	va	l
44	Bergonzi MA	Cremona	ef	vl	m
45	Bergonzi N	Cremona	ghi	vl	o
46	Bergonzi N	Cremona	fg	vc	g
47	Bergonzi Z	Cremona	g-j	vc	fg
48	Ceruti GB	Cremona	j	vl	k
49	Storioni L	Cremona	h	vl	i
50	Storioni L	Cremona	f	vc	j
51	Gagliano F	Neapel	g	vl	e
52	Gagliano F	Neapel	gh	vl	k
53	Gagliano F	Neapel	gh	vl	gh
54	Gagliano J	Neapel	ghi	vl	k
55	Gagliano J	Neapel	ij	vl	k
56	Gagliano J	Neapel	ij	vc	e
57	Gagliano J,A	Neapel	h	vl	m
58	Gagliano Werkst.	Neapel	ij	vl	e
59	Gagliano Werkst.	Neapel	jk	vl	d
60	Gagliano R&A	Neapel	lm	vl	h
61	Gagliano R&A	Neapel	lm	vc	l
62	Gagliano R&A	Neapel	m	vc	i
63	Ventapane V	Neapel	hi	vl	l
64	Ventapane L	Neapel	k	vl	j
65	Ventapane L	Neapel	h-k	vc	m
66	Ventapane L	Neapel	kl	vc	l
67	Vinaccia A	Neapel	gh	vl	l

68	Vinaccia Werkst.	Neapel	ij	vc	f
69	Gragnani A	Livorno	hi	vl	m
70	Guadagnini JB	Parma	g	vc	i
71	Guadagnini JB	Turin	h	vl	k
72	Guadagnini G	Pavia	j	vl	k
73	Guadagnini G	Pavia	ij	vl	m
74	Guadagnini G	Mailand	i	va	g
75	Guadagnini G	Turin	ghi	vc	g
76	Guadagnini G	Turin	hi	vl	i
77	Guadagnini G	Turin	ij	vl	k
78	Pressenda JF	Turin	l	vl	j
79	Pressenda JF	Turin	l	vl	h
80	Pressenda JF	Turin	m	vc	h
81	D´Espine A	Turin	l	vl	i
82	Panormo VT	Paris	h	vl	j
83	Panormo VT	London	j	vl	k
84	Panormo J, GL	London	j	vl	k
85	Pique FL	Paris	j	vl	ij
86	Lupot N	Orléans	i	vl	jk
87	Lupot N	Paris	i	vl	i
88	Lupot N	Paris	kl	vl	i
89	Aldric JF	Paris	kl	vc	h
90	Gand ChF	Paris	k	vl	j
91	Gand ChF	Paris	l	vc	hi
92	Widhalm VA	Regensburg	gh	vc	h
93	Widhalm ML	Nürnberg	ij	vl	l
94	Geissenhof F	Wien	i	vc	f
95	Stradivari A	Cremona	e	vl	g
96	Stradivari A	Cremona	e	vl	k
97	Rogeri GB	Brescia	d	vl	k
98	Mantegazza F/C	Milano	f-i	va	de
99	Beretta F	Como, Parma	fgh	vl	n
100	Ceruti GB (JB)	Cremona	i	va	e
101	Ceruti GB	Cremona	h	vl	o
102	Ceruti GB	Cremona	k	vl	hi

103	D´Espine A	Turin	j	vl	ij
104	Mantegazza PG	Mailand	fgh	va	d
105	Mantegazza PG	Mailand	g	vl	m
106	Mantegazza F	Mailand	h	va	i
107	Marchi JA	Bologna	fgh	vl	h
108	Marchi JA	Bologna	g	vl	n
109	Marconcini G	Ferrara	j	vl	l
110	Storioni L	Cremona	hi	vl	l
111	Storioni L	Cremona	g	vl	k
112	Storioni L	Cremona	g	vl	l
113	Storioni L	Cremona	f-i	vl	g
114	Ceruti GB (JB)	Cremona	j	va	h
115	Buchstetter J	Regensburg	g	vl	mn
116	Dalinger S	Wien	g	vl	k
117	Dalinger S	Wien	h	vl	l
118	Dihl M	Mainz	g	vl	i
119	Dihl M	Mainz	h	vc	j
120	Eberll JU	Prag	f	vl	k
121	Ergele JC	Freiburg	i	va	h
122	Ficker ChS	Markneukirchen	j	vl	k
123	Ficker JCh	Neukirchen	i	vl	n
124	Fiker JCh	Markneukirchen	i	vl	i
125	Fischer J	Regensburg	j	vl	m
126	Fischer J	Regensburg	j	va	l
127	Fischer J	Regensburg	j	va	e
128	Fischer PhJ	Würzburg	h	va	m
129	Fischer Z	Würzburg	i	vc	e
130	Gedler JB	Füssen	h	vl	k
131	Gedler JB	Füssen	j	vl	f
132	Geissenhof F	Wien	h	vl	k
133	Geissenhof F	Wien	k	vl	i
134	Geissenhof F	Wien	k	vl	j
135	Geissenhof F	Wien	j	vl	h
136	Glass CFA I	Klingenthal	gh	vl	l
137	Hamm JG	Neukirchen	h	vl	j

138	Hamm JG	Markneukirchen	g-k	vl	n
139	Hammig JCh	Markneukirchen	i	vc	hi
140	Hornstainer A	Mittenwald	g	vl	l
141	Havelka SJ	Linz	i	vl	kl
142	Helmer CJ	Prag	g	vc	c
143	Helmer CJ	Prag	i	vl	k
144	Hellmer C	Prag	g-j	vl	k
145	Hollmayr J	Neuburg a.d. D.	g	vl	n
146	Hopf D	Klingenthal	kl	vl	l
147	Hornstainer F	Mittenwald	i	vl	k
148	Hornsteiner II.	Mittenwald	i	vl	i
149	Hornsteiner J	Mittenwald	k	vl	l
150	Hornstainerdax	Mittenwald	g	vl	h
151	Hornsteiner P	Mittenwald	j	vl	h
152	Hunger ChF	Leipzig	g	va	k
153	Jais A	Mittenwald	i	vl	h
154	Karner B	Mittenwald	g	vl	h
155	Karner B	Mittenwald	h	vl	j
156	Karner JG	Enns	jk	vl	k
157	Kerkovics F	Pressburg	k	va	g
158	Kloz Ae	Mittenwald	gh	vl	i
159	Kloz Ae	Mittenwald	h	vl	i
160	Kloz G	Mittenwald	g	vl	i
161	Kloz G	Mittenwald	g	vl	k
162	Kloz S	Mittenwald	f	vl	g
163	Kloz S	Mittenwald	f	vl	e
164	Klotz F	Mittenwald	i	vc	i
165	Knitl J	Mittenwald	g	vl	i
166	Knitl J	Mittenwald	h	vl	g
167	Koster FA	Regensburg	g	va	m
168	Krausch GA	Wien	j	va	g
169	Leeb JG	Pressburg	i	vl	e
170	Leidolff JF	Wien	g	va	i
171	Lippold CF	Neukirchen	ij	vl	k
172	Neiner M	Mittenwald	h	vl	k

173	Neiner M	Mittenwald	hi	vl	i
174	Niggeli S	Füssen	g	va	h
175	Partl JCh	Wien	h	vl	j
176	Pfretzschner CF	Markneukirchen	h	vl	e
177	Psenner JG	Innsbruck	g	vl	i
178	Radeck J	Wien	i	vl	h
179	Rief D	Vils	i	vl	m
180	Schoenfelder JG	Neukirchen	i	vl	i
181	Schönfelder JG	Neukirchen	gh	vl	i
182	Schuster JCh	Markneukirchen	g	vl	m
183	Simon F	Salzburg	i	va	l
184	Simon F	Salzburg	i	vc	l
185	Stadlmann M I	Wien	h	vl	k
186	Stainer J	Absam	b	vl	g
187	Stainer J	Absam	c	vl	i
188	Stainer J	Absam	c	vl	j
189	Staininger J	Frankfurt	i	vl	h
190	Staininger F	Aschaffenburg	i	vl	d
191	Storck JF	Augsburg	j	vc	n
192	Stoss FA	Füssen	g	vl	g
193	Stoss M	Wien	k	va	(e)
194	Straub J	Röthenbach	jkl	vl	i
195	Thir A	Pressburg	i	vl	n
196	Thir JG	Wien	g	va	g
197	Trumhart GA	Straubing	l	vl	m
198	Wagner B	Ellwangen	g	vl	l
199	Wagner S	Meersburg	i	vl	m
200	Widhalm L	Nürnberg	g	va	j
201	Widhalm L	Nürnberg	gh	vl	n
202	Widhalm L	Nürnberg	h	va	j
203	Willer J	Prag	l	vl	k
204	Wutzlhoffer S	Brünn	g	vl	i
205	Eberle JO	Praha	g	vl	o
206	Eberle JO	Praha	efg	vl	o
207	Edlinger JJ	Praha	f	vl	k

208	Hellmer JJ	Praha	f	vl	m
209	Hellmer JJ	Praha	e	vc	e
210	Hellmer KJ	Praha	hij	vl	j
211	Hulinský TO	Praha	g	va	n
212	Hulinský TO	Praha	h	va	o
213	Hulinský TO	Praha	gh	va	l
214	Klor FA	Kolín n. L.	e	va	i
215	Klor FA	Praha	f	va	l
216	Lang S	Praha	f	vl	g
217	Laske JA	Praha	g	vl	m
218	Muschl JJ	Praha	f-i	vl	j
219	Rauch Jan	Chomutov	e	vl	m
220	Rauch Josef II	Chomutov	h	vl	l
221	Rauch Josef IV	Chomutov	i	vl	o
222	Rauch KJ	Praha	gh	vl	l
223	Strnad K	Praha	i	vl	l
224	Strnad K	Praha	j	vl	l
225	Strnad K	Praha	j	vl	o
226	Strnad K	Praha	ij	vl	o
227	Šembera KV	Praha	j	vl	o
228	Wild IA	Brno	j	vl	o
229	Willer JM	Praha	i	vl	o
230	Willer JM	Praha	i	vl	n
231	Willer JM	Praha	ij	vl	o
232	Willer JM	Praha	j	va	m
233	Wutzelhofer B	Brno	kl	vl	k
234	Wutzelhofer S II	Brno	i	vl	k

APPENDIX B

# 98	**va L/W group I**
C	HKt MaGt YDs
D circle	AZcs LBct
D	CLr HLt ILs YTLA ZPt
P circle	ADct LDcs
P	DRLA FLt
W	ABr AKKV AZt Y
M	BKt IPs ZPr
E	ADt ALr IFr IJt MaZt
F	BMt CDs CKt MaDt
G	APws CBr EBt HDt IKs J MaFs ZFr
Cw	BDw DI HF
Dw	BCw DC PF
Pw	SL
Mw	EH JD PP´
Ew	CK EM MaY VC YG
Fw	a c i t EwZ 2 5 6

# 32	**vc L/W group II**
C	IFt MaBs MaFr WDs YKt
D circle	AMacr AZcs LBct LTc
D	BPs CPt ELr HLt HTLA ILs ZLr
P circle	ADct LTcs Ly´c
P	KLs KRLA
W	ACs AFr AJs AKKU AMat
M	EPr JDt WPt
E	ALr APs IMr WFt WKs
F	CKt EDs GKs HPt JDr YLr
G	CBs EFs MaFt MaMs YDt ZMr
Cw	BDw EH GW HE JY KI VB ZH
Dw	BCw DC
Pw	Py´ SL
Mw	GD
Ew	GD IB PMa YG ZM
Fw	DB

# 186	**vl L/W group III**
C	ALr AP HKt MaBt MaFs MaMr YMt
D circle	AJct AMacr AZcs LTc
D	BPs ELr HLt HTLA ILs ZLr
P circle	ADct AKcr Ly´c
P	FLt
W	ABr AKKU ALKy AMat
M	BKt YPt
E	HMt IFr YDr YFt YKs Z
F	BMr CDs EDs ZDs
G	APws HDt IKs J MaBt MaFs MaMr
Cw	BDw CH DI ED
Dw	BCw GH MP (SK)
Pw	MW z´L
Mw	CY WK
Ew	EM ZM
Fw	a c 2 5 6

# 170	**va L/W group IV**
C	HPr MaGt YDs YMt
D circle	AIc AMacr LTc
D	CPt FPr HLt ILs
P circle	LTcs
P	DRLA FLt
W	AGs AZt
M	EPr JDt WPt
E	HDs HMt IBs IFr IGt WMs YFt (Z)
F	GDr HPt
G	APws IMt YDt
Cw	BDw DS ED YF ZD
Dw	BCw FI KMa KP (PF) SK
Pw	DMa JV MW Ph´ TL
Mw	GD PP´ VK
Ew	(CK) EM IB MaF MP PMa YG
Fw	n 2 5 6

# 218	vc L/W group V
C	APs HDs HKt IBs MaJs
D circle	AJct LBct
D	FPs HLt ILs JPt ZLr
P circle	ADct Ly´c
P	By´LA
W	ACs AZs Y
M	BKt GDt
E	ADt YBt YFt
F	CKs CMt MaKt
G circle	AFcr
G	ALr HPr MaBr YDs YKt
Cw	DI FDw HB IR ZD
Dw	FCw DG KMa MaV
Pw	j´L JV
Mw	KG MC PB YK
Ew	EW EY HJ
Fw	i IB z

# 132	vl L/W group VI
C	APs HDs IBt IKr WDr YKt
D circle	AJcs
D	Aj´s ASLA Wj´r WLt Zj´LA
P circle	ADct AKc Ly´c
P	DRLA Ih´LA ITr Jj´s Kj´r Ky´LA Yj´t
W	AE- AZ-
M	Aj´r ALt BDr HLs (K)
E	HKt IFr MaJr (Z)
F	EDs HPt ZDs
G circle	AMct
G	APws IKt MaFt ZFs
Cw	BDw BW CD CH FP IK
Dw	BCw BW GH
Pw	DE DZ TL
Mw	BD JW PF PP´
Ew	MaF PMa WG YG
Fw	a b k BJ r 5 6

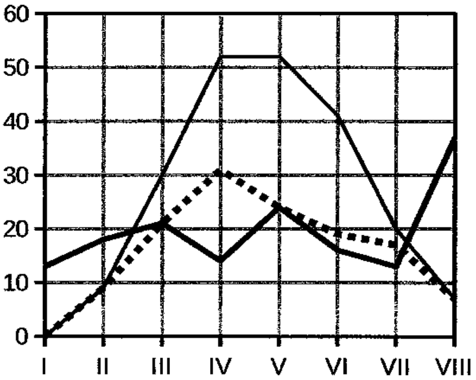
# 112	vl L/W group VII
C	APs WKt ZGs
D circle	ACcr LTc
D	Aj´s ASLA BPs FPr Wj´r Yj´t YLt
P circle	ADcs DLcs
P	Bz´r Dy´LA IJ´t
W	ABr ACs AJs AKKU
M	ALt BDr K WPt
E	HKt YDr YFt
F	EDs JDr MaDt
G circle	-
G	APws IDr
Cw	CD EH IK KI WF
Dw	JK
Pw	DZ SL
Mw	BD FZ PF
Ew	MaF ZW
Fw	c r CI EwJ 5 6

# 65	vc L/W group VIII
C	ALr HDt MaBt MaFr
D circle	AMac Lj´cs
D	CPt HTLA JPt WLt miLt 2miPt
P circle	-
P	DLs Dy´LA
W	ABr ACs A2mis AZt HIs mi
M	ALt CDt K miPt WPt
E	APs MaBr MaGs miKt WKt
F	JDs 2miDs
G circle	AFcs AKct
G	APt APws EFr IDs MaKt miDt WDt ZFt ZKs
Cw	FI IK VB 2miY
Dw	CH PK
Pw	FY RL
Mw	BD ZH
Ew	EK EM FMa MaY 2miI
Fw	c BE s JK u x 4

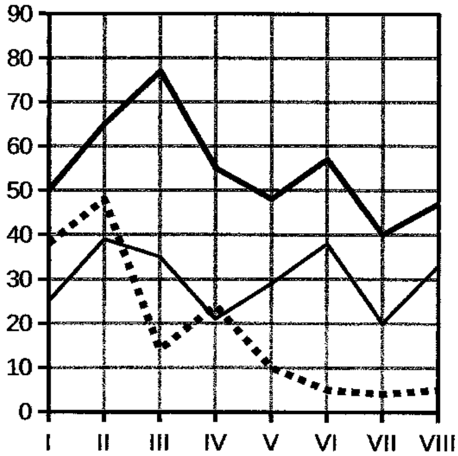
APPENDIX C

(Cf. important remarks on p. 27)

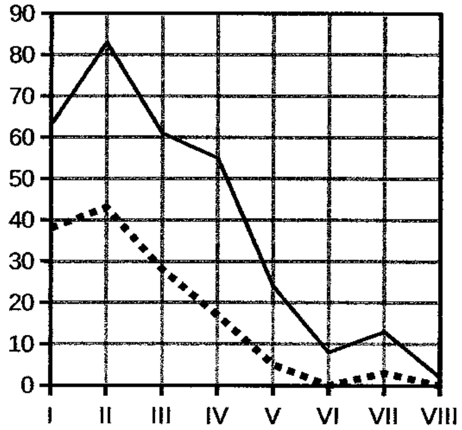
Level C



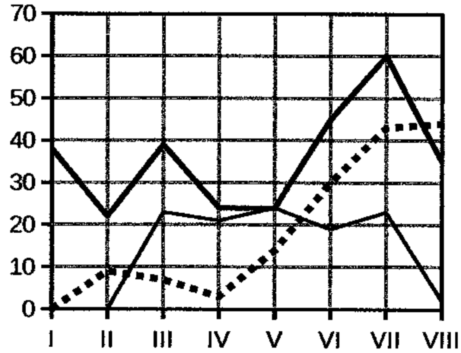
Dotted line: AKKAc
Thin line: ALr
Thick line: APs



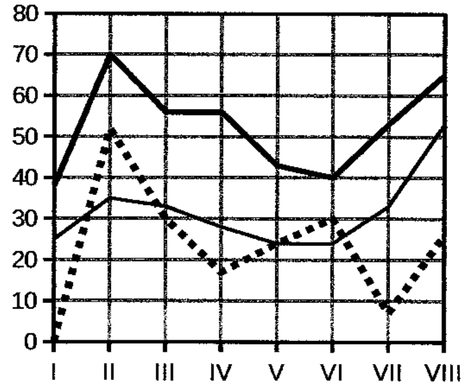
Dotted line: MaJt
Thin line: MaBr/s
Thick line: MaBr/s+MaGs/t



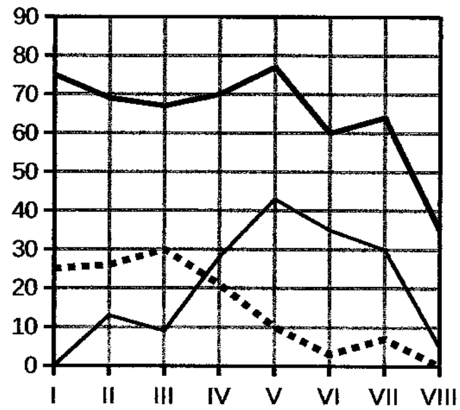
Dotted line: HKt
Thin line: HKt+HPr



Dotted line: ZGr/s/t
Thin line: WMt
Thick line: WMt+WKt

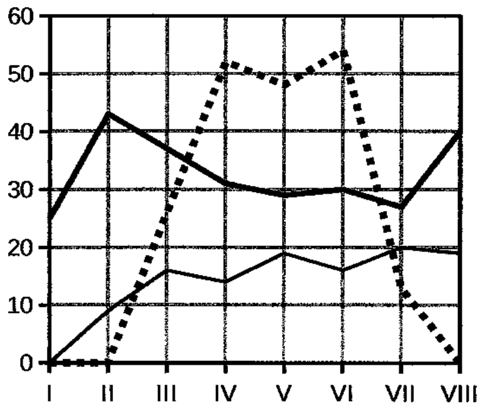


Dotted line: IFs/t
Thin line: IKr/s
Thick line: IKr/s+IBt

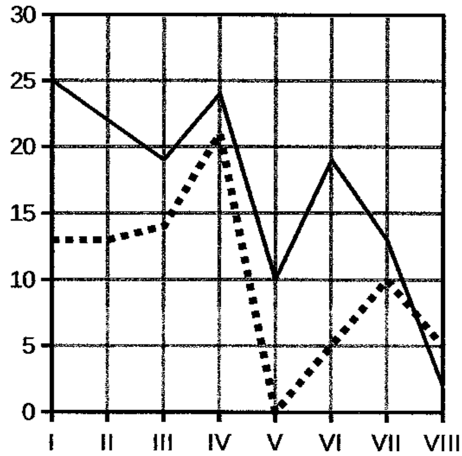


Dotted line: YMt
Thin line: Ykt
Thick line: YMt+YKt+YDs

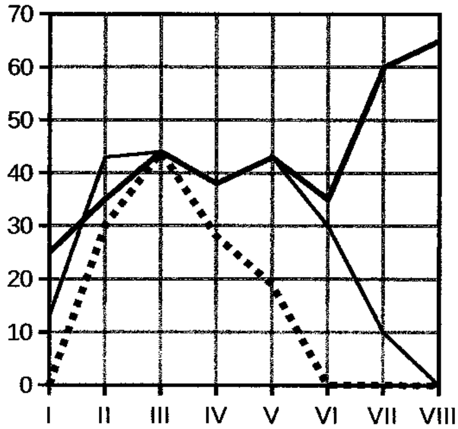
Level D



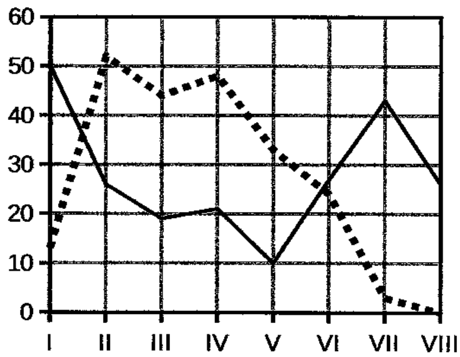
Dotted line: AIc
Thin line: ASLA
Thick line: BPs



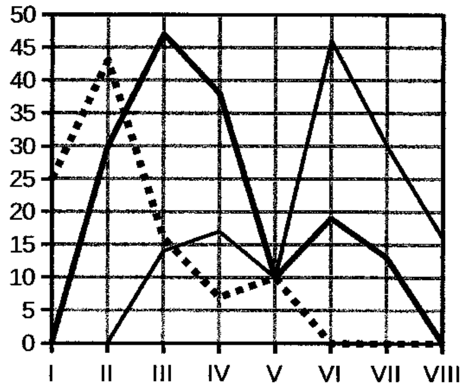
Dotted line: CLr
Thin line: CPt



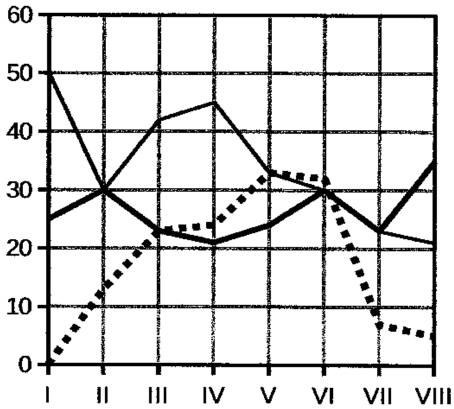
Dotted line: AMacr
Thin line: AZcr/s
Thick line: AJcs/t



Dotted line: HLt
Thin line: HTLA

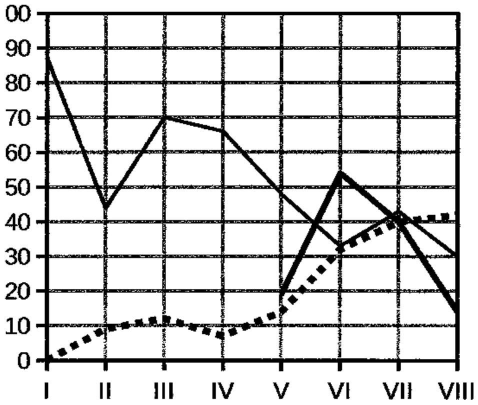


Dotted line: LBct
Thin line: LKcr
Thick line: LTc

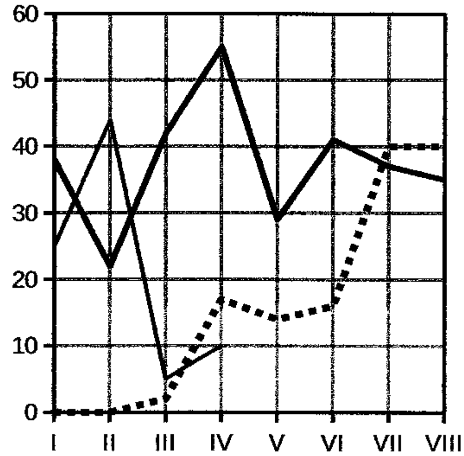


Dotted line: JPt
Thin line: ILs
Thick line: ZLr

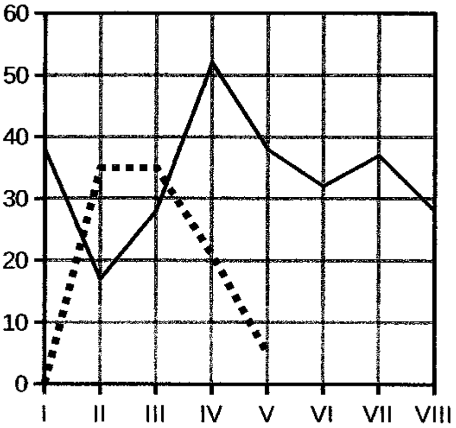
Level P



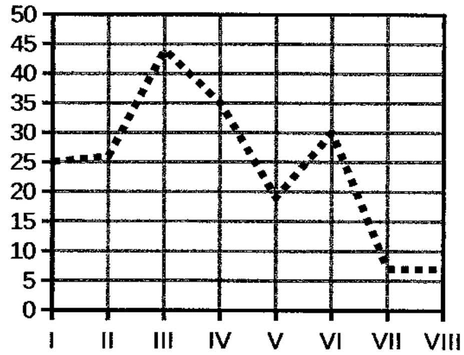
Dotted line: ADcs
Thin line: ADct
Thick line: AKc



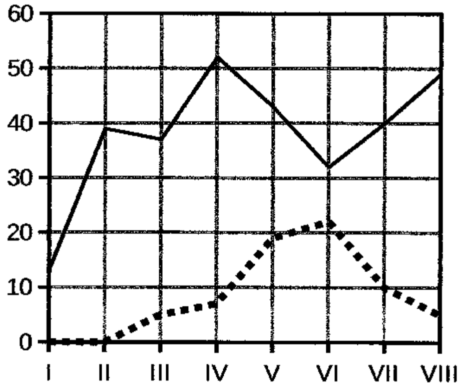
Dotted line: DLr
Thin line: BLt
Thick line: DRLA



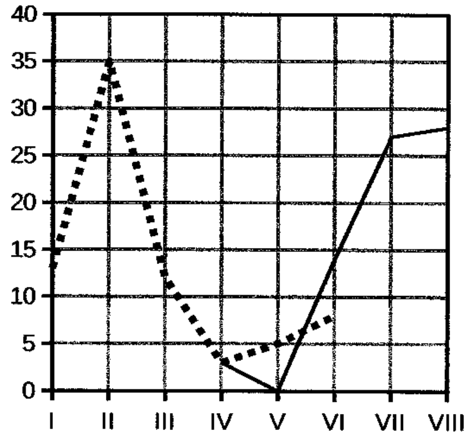
Dotted line: LTcs
Thin line: LDcs



Dotted line: FL

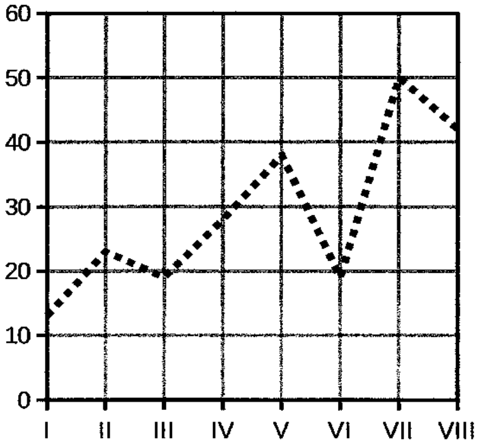


Dotted line: Lz´ct
Thin line: Ly´c

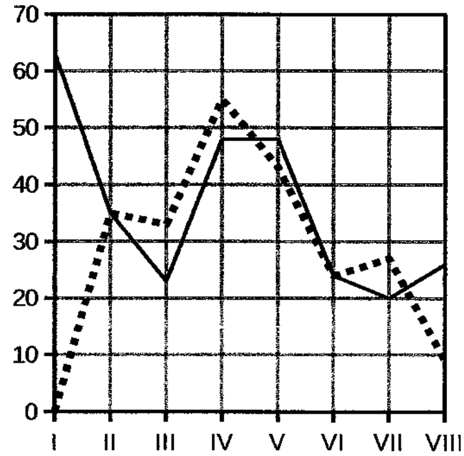


Dotted line: KLs
Thin line: KLt

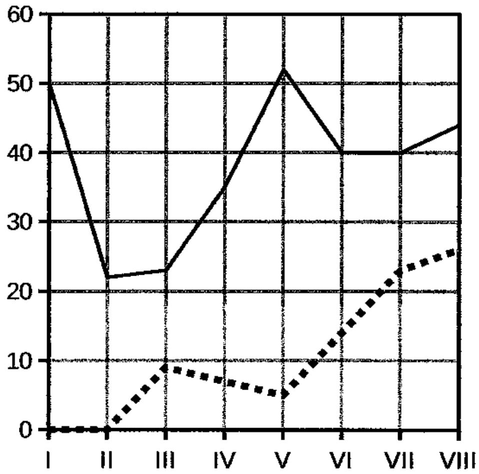
Level W



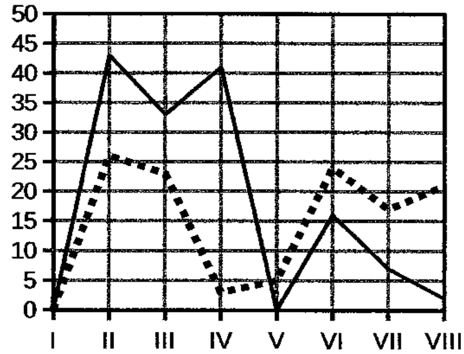
Dotted line: ABr



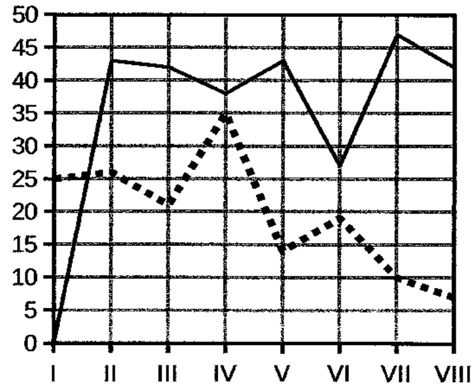
Dotted line: AKKU
Thin line: ALKy



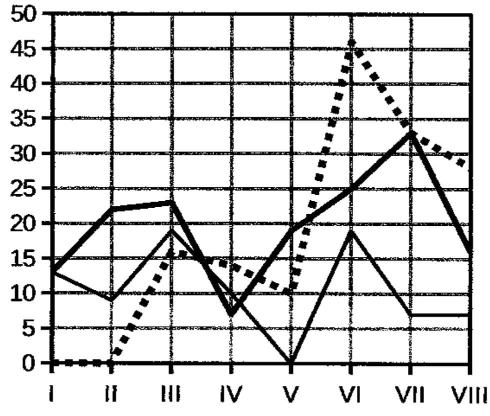
Dotted line: AEt
Thin line: ACs



Dotted line: AGs
Thin line: AMat

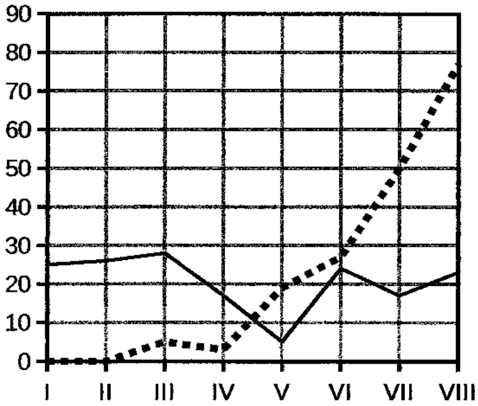


Dotted line: AFr
Thin line: AJs

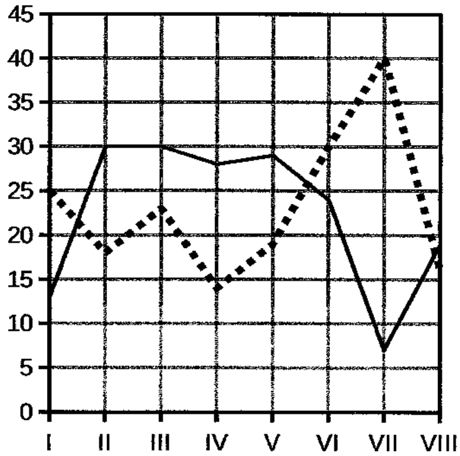


Dotted line: AZt
Thin line: HIr
Thick line: HIs

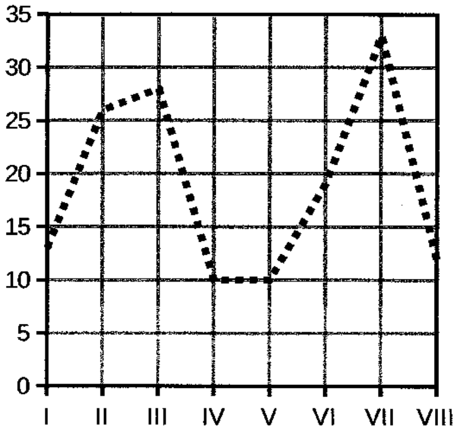
Level M



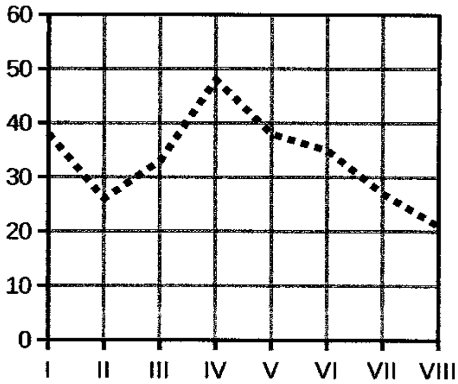
Dotted line: K (= ALt)
Thin line: BDr/s



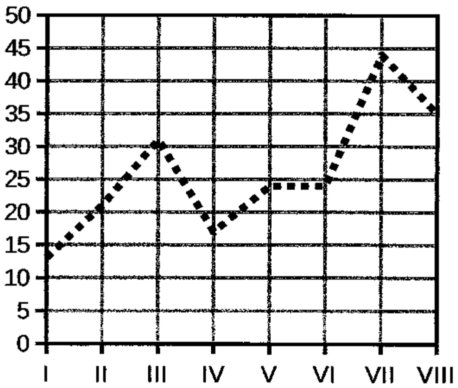
Dotted line: HLs
Thin line: JDt



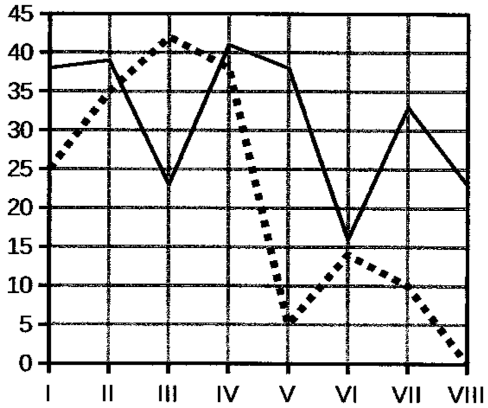
Dotted line: CDt



Dotted line: WPt

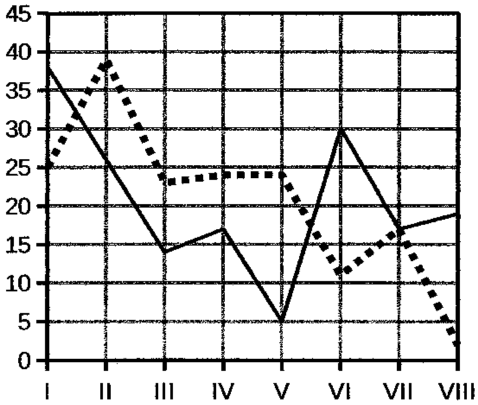


Dotted line: GDs/t

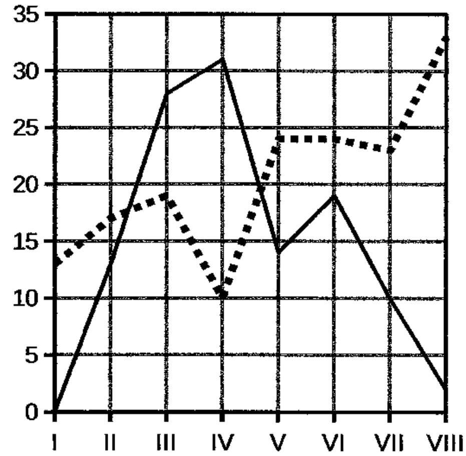


Dotted line: YPt
Thin line: ZPr

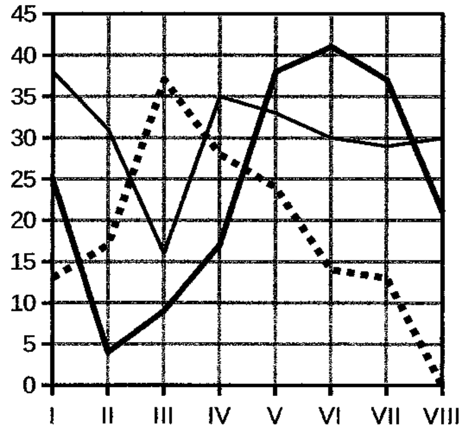
Level E



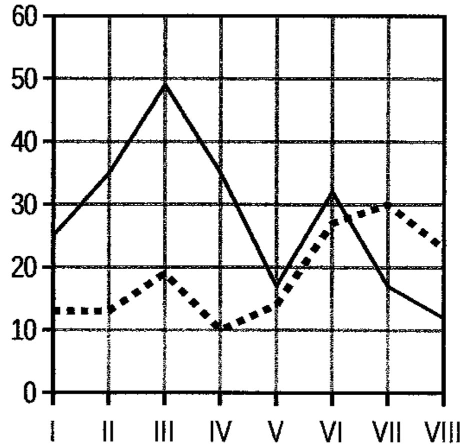
Dotted line: ADt
Thin line: APs



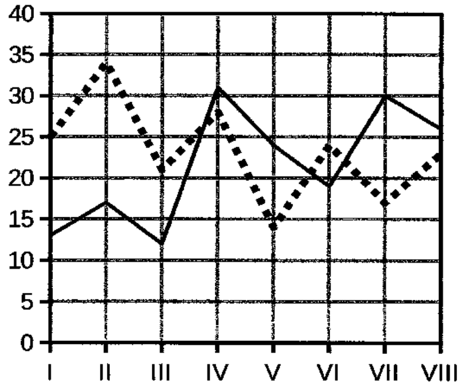
Dotted line: WFt
Thin line: WMs



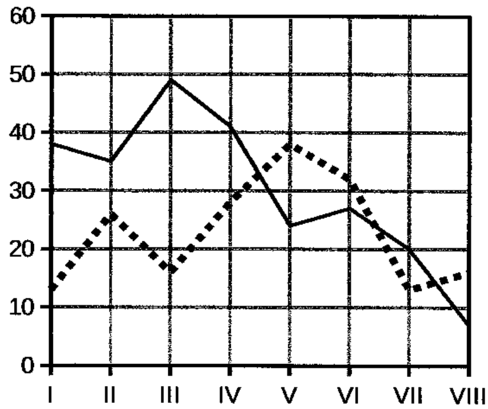
Dotted line: HMt
Thin line: HDs
Thick line: HKt



Dotted line: YDr
Thin linje: YKs

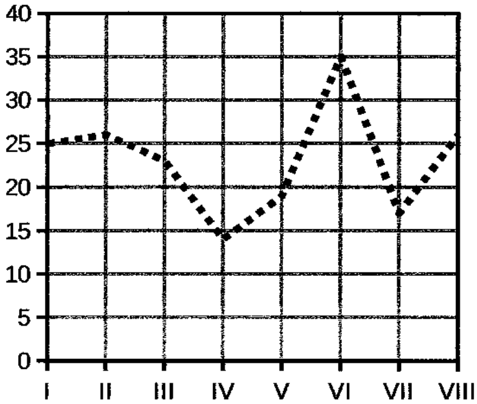


Dotted line: IBs
Thin line: IFr/s

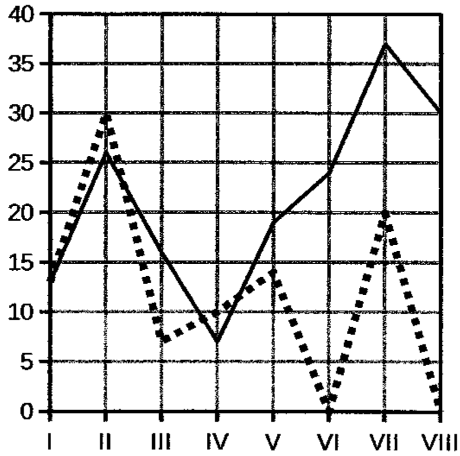


Dotted line: YFt
Thin line: Z

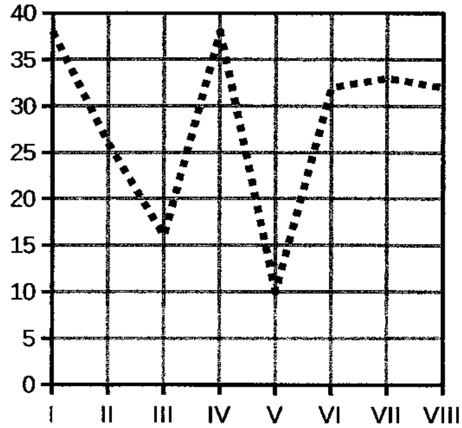
Level F



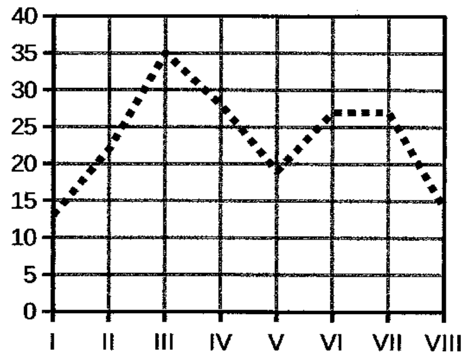
Dotted line: CDs



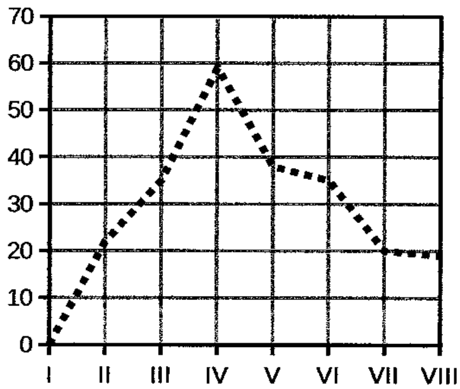
Dotted line: JKt
Thin line: JDr/s



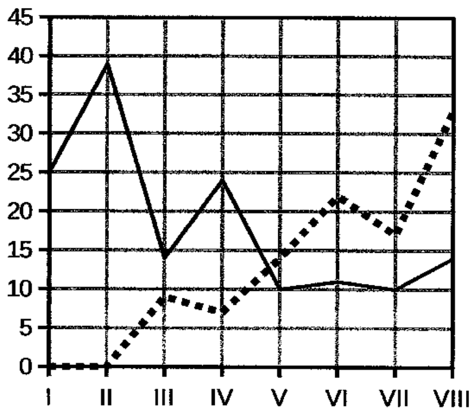
Dotted line: EDs/t



Dotted line: MaDt

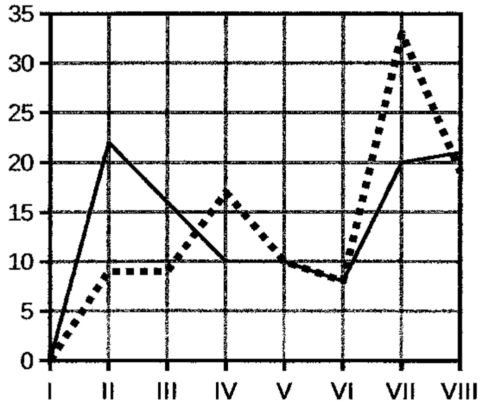


Dotted line: HPt

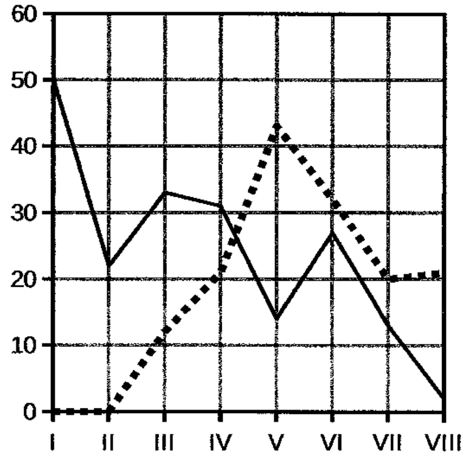


Dotted line: ZDt
Thin line: ZDs

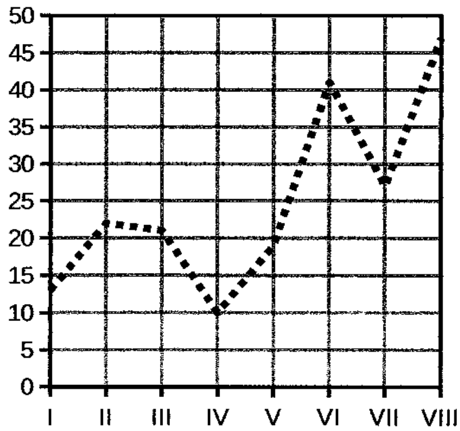
Level G



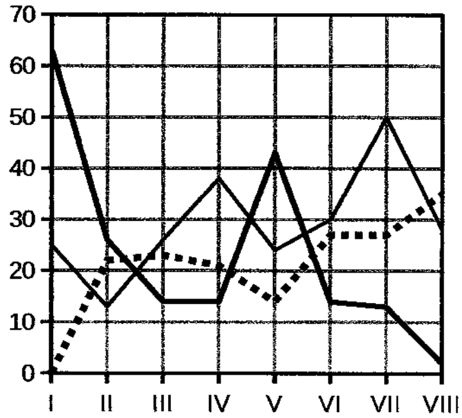
Dotted line: AKct
Thin line: ABcr



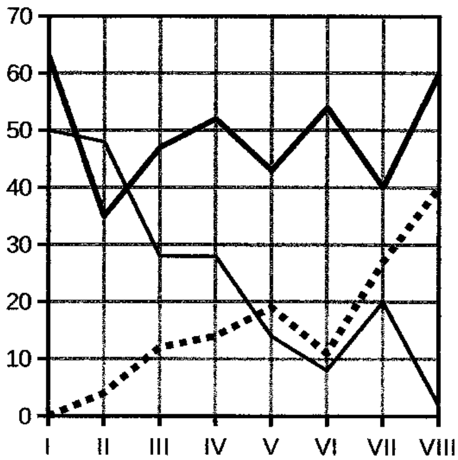
Dotted line: IKt
Thin line: IMt



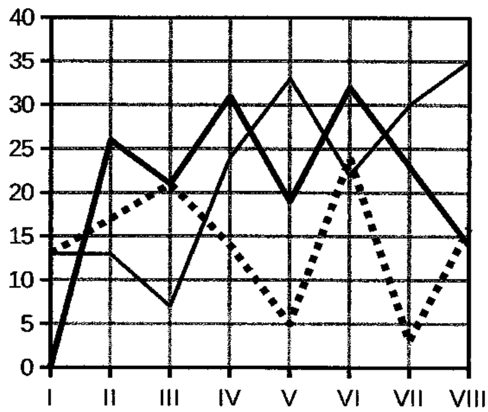
Dotted line: AFcs



Dotted line: MaKr/s
Thin line: MaFs/t
Thick line: MaMr/s

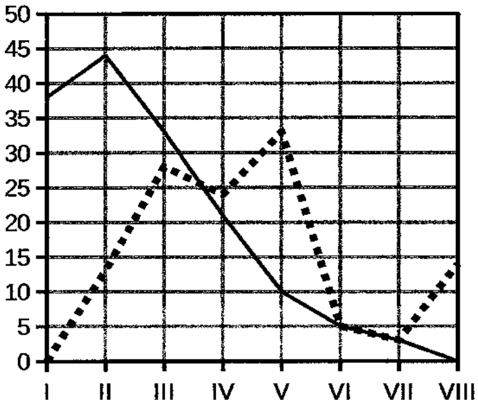


Dotted line: APt
Thin line: HDt
Thick line: APws

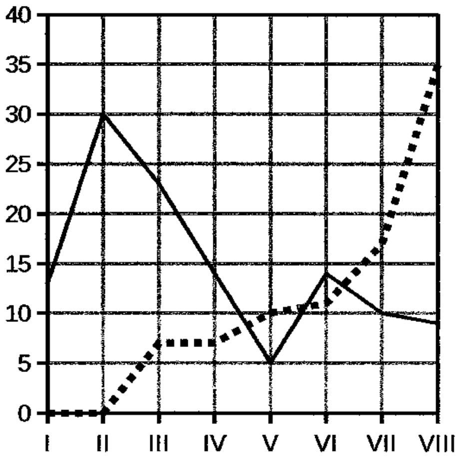


Dotted line: ZBt
Thin line: YDt
Thick line: YPr

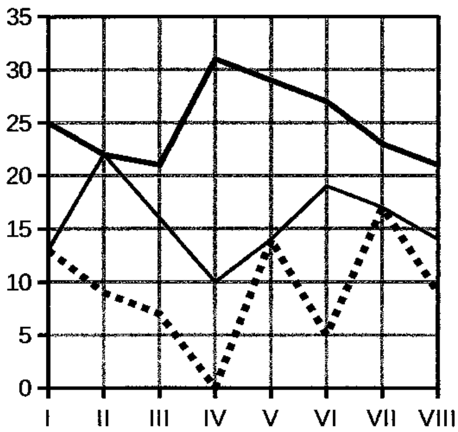
Width Cw



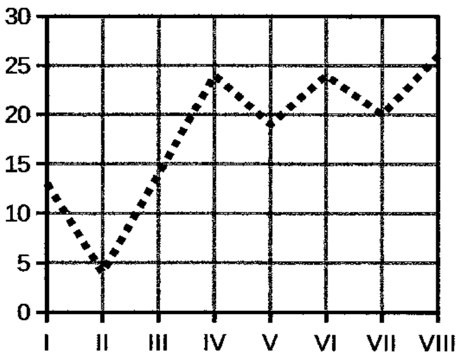
Dotted line: DS
Thin line: DI



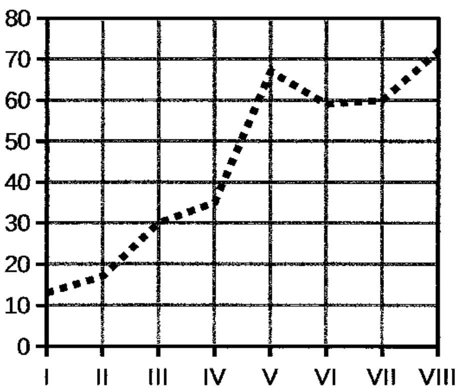
Dotted line: PMa
Thin line: VB



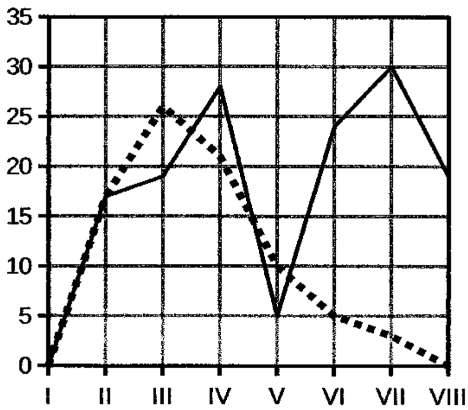
Dotted line: GW
Thin line: GY
Thick line: ED



Dotted line: WF

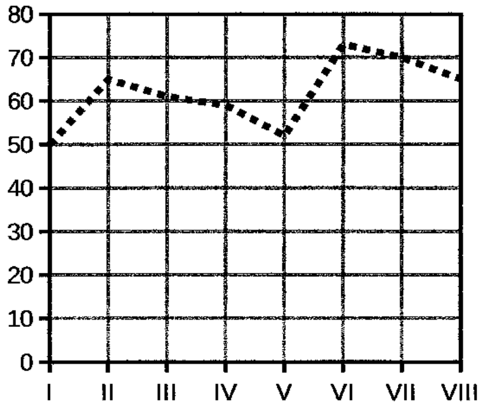


Dotted line: IK/V

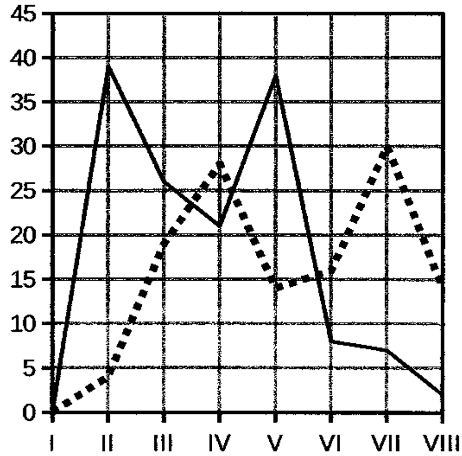


Dotted line: ZD
Thin line: YF

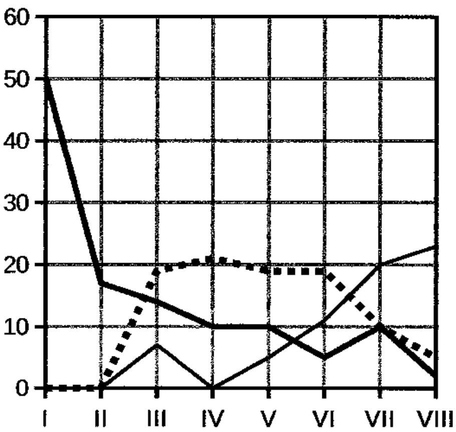
Width Dw



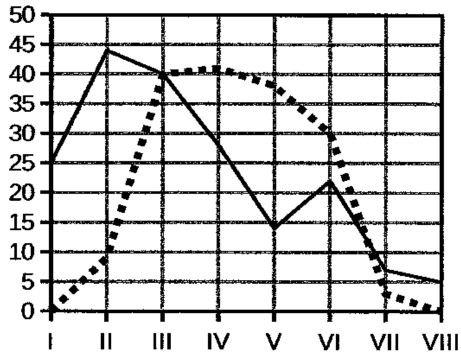
Dotted line: BCw



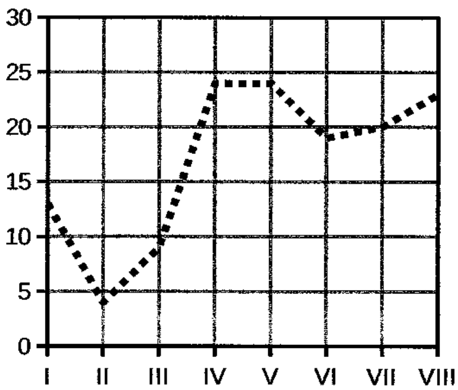
Dotted line: KP
Thin line: KMa



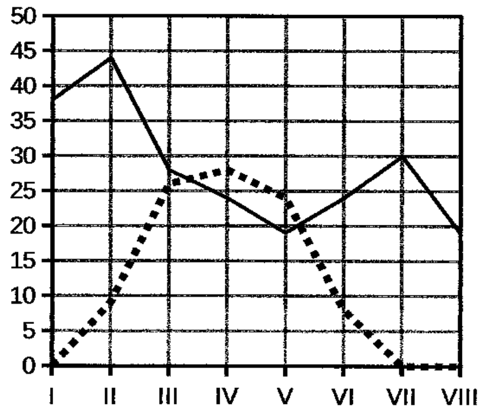
Dotted line: DJ
Thin line: DG
Thick line: DC



Dotted line: MaV
Thin line: MP

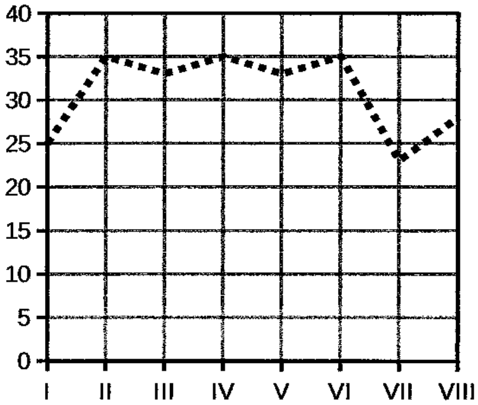


Dotted line: FI

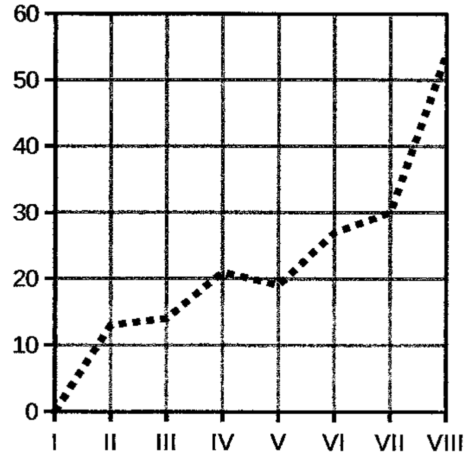


Dotted line: SK
Thin line: PF

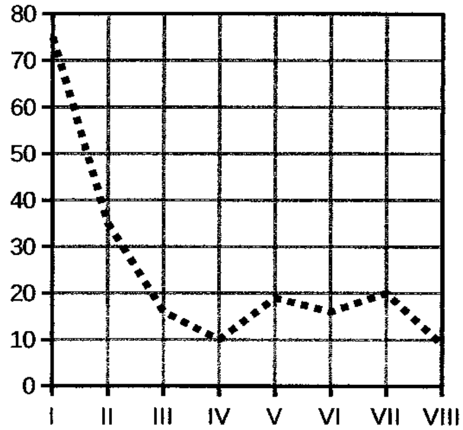
Width Pw



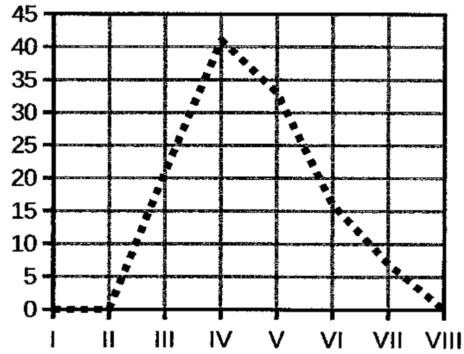
Dotted line: Ph´



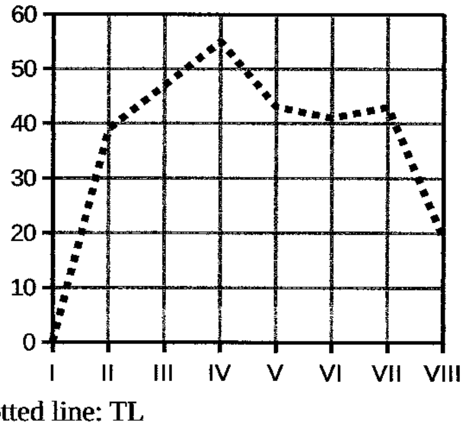
Dotted line: z´L



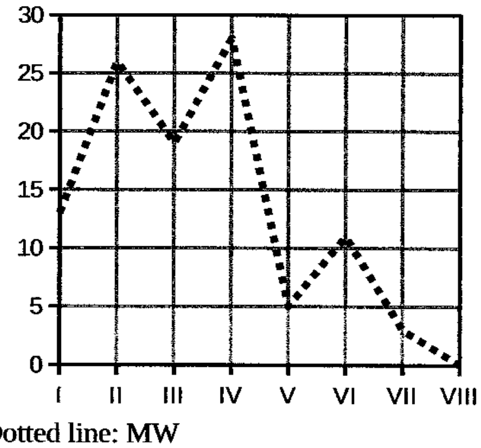
Dotted line: SL



Dotted line: JW

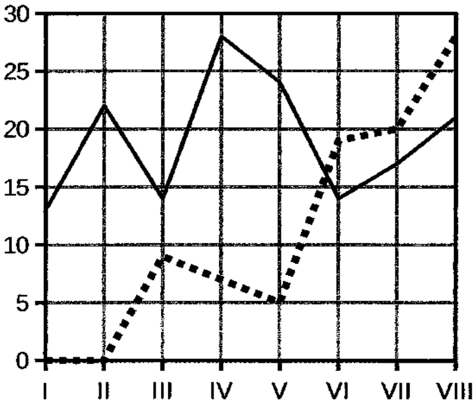


Dotted line: TL

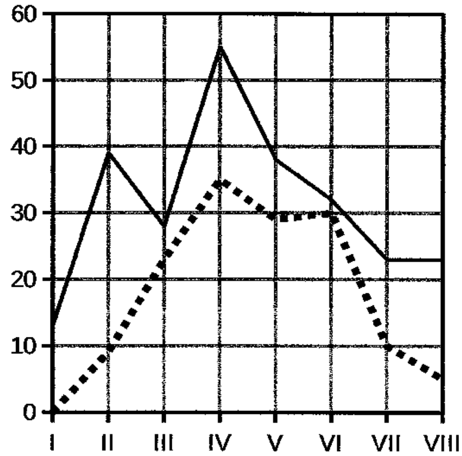


Dotted line: MW

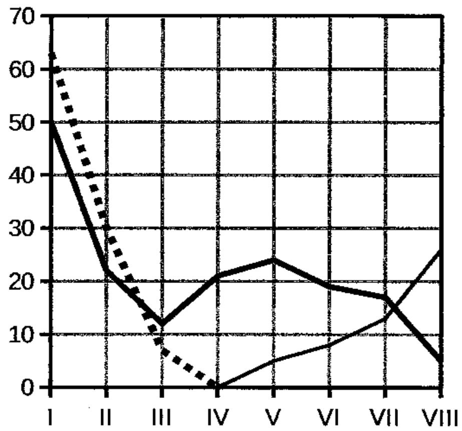
Width Mw



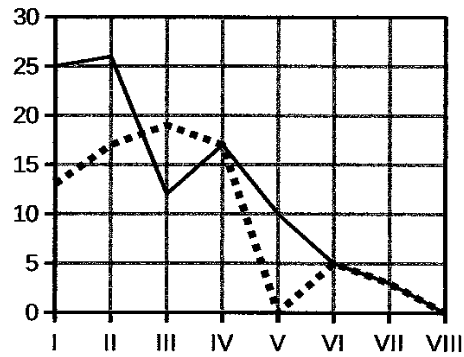
Dotted line: BD
Thin line: CW



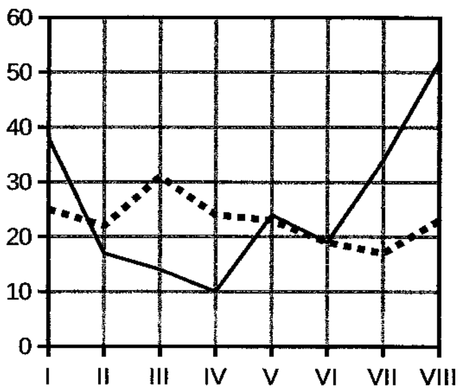
Dotted line: VK
Thin line: PP´



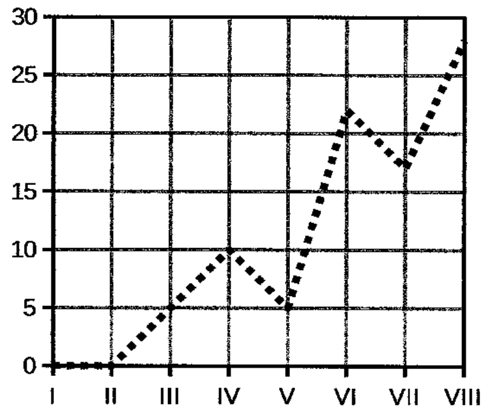
Dotted line: JD
Thin line: JW
Thick line: GD



Dotted line: WK
Thin line: YK

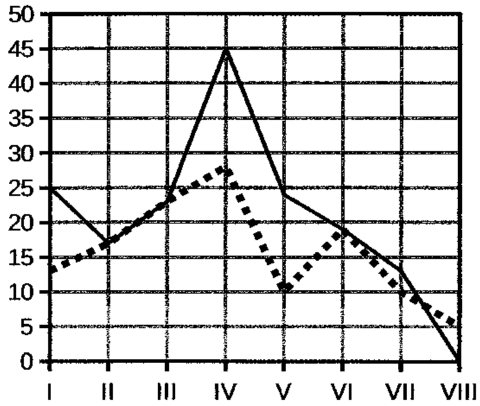


Dotted line: MC+ME/Z
Thin line: KC+KJ

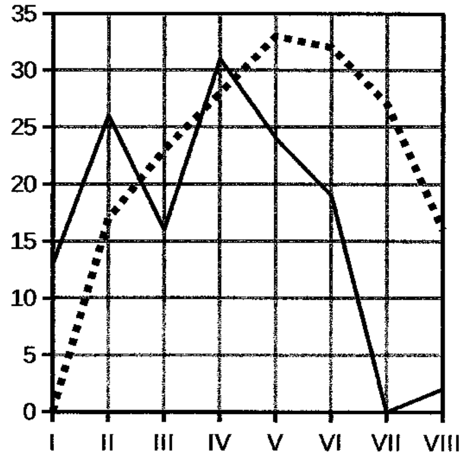


Dotted line: ZH

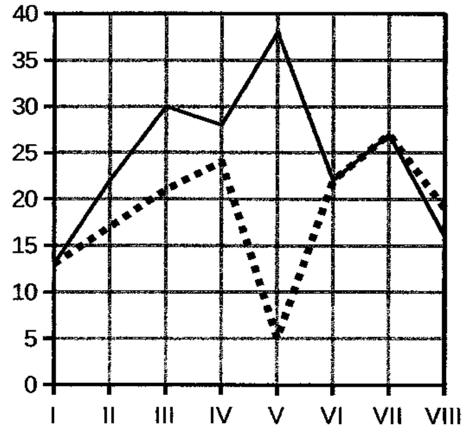
Width Ew



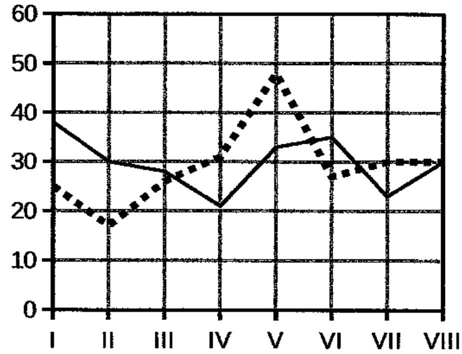
Dotted line: BI
Thin line: CK



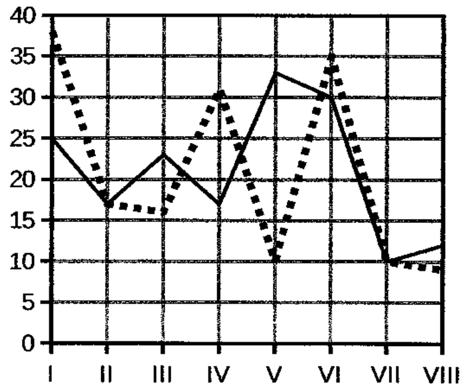
Dotted line: PMa
Thin line: MP



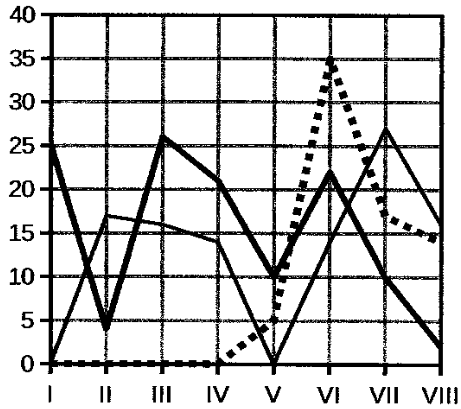
Dotted line: EW
Thin line: EM



Dotted line: WG
Thin line: YG

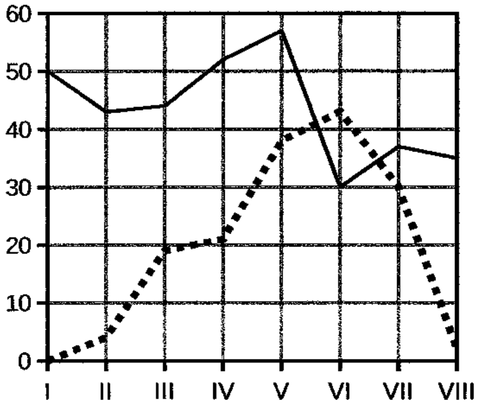


Dotted line: MaF
Thin line: MaY

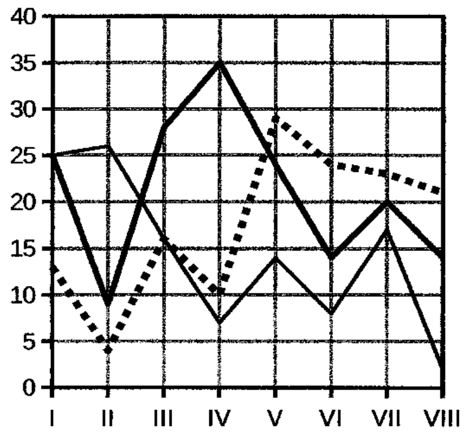


Dotted line: ZF
Thin line: ZW
Thick line: ZM

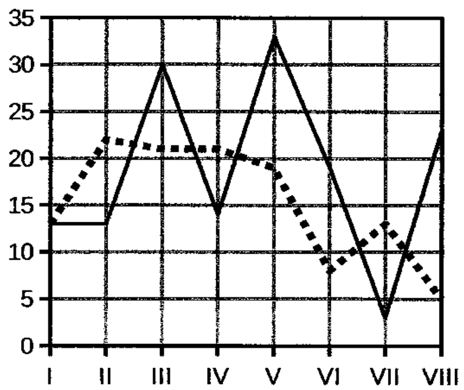
Width Fw



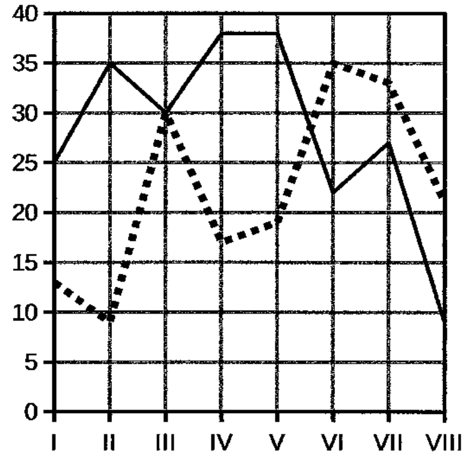
Dotted line: BK
Thin line: F´Fw=E´Ew (a)



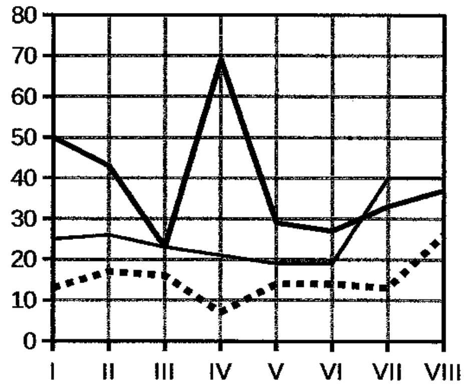
Dotted line: CI
Thin line: CM (t)
Thick line: GM (p)



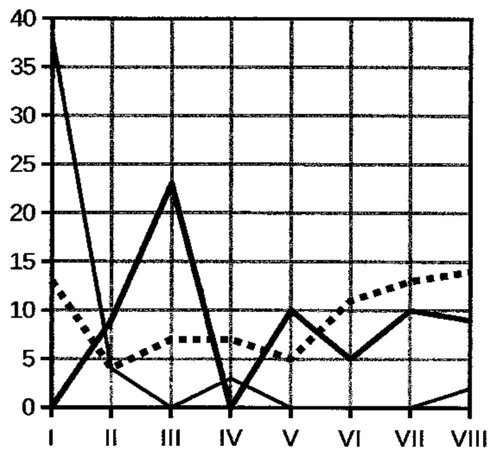
Dotted line: JM
Thin line: FG (i)



Dotted line: EwC (5)
Thin line: KB (c)



Dotted line: EwG (4)
Thin line: EwE (6)
Thick line: EwM (2)



Dotted line: EEw (x)
Thin line: EEw=FwFw (y)
Thick line: EEw=EFw=EwFw (z)