Plenary talks

Maria Chudnovsky

Princeton University

Induced subgraphs and logarithmic tree width

Tree decompositions are a powerful tool in structural graph theory; they are traditionally used in the context of forbidden graph minors. Connecting tree decompositions and forbidden induced subgraphs has until recently remained out of reach.

Tree decompositions are closely related to the existence of "laminar collections of separations" in a graph, which roughly means that the separations in the collection “cooperate” with each other, and the pieces that are obtained when the graph is simultaneously decomposed by all the separations in the collection “line up” to form a tree structure. Such collections of separations come up naturally in the context of forbidden minors.

In the case of families where induced subgraphs are excluded, while there are often natural separations, they are usually very far from forming a laminar collection. However, under certain circumstances, these collections of natural separations can be partitioned into a small number of laminar collections (in this context "small" means either constant or logarithmic in the number of vertices of the graph). This in turn allows us to obtain a wide variety of structural and algorithmic results, which we will discuss in this talk.

Sudhir Ghorpade

Indian Institute of Technology Bombay

Let n, k be positive integers with $k \leq n$, and let C be a q-ary linear code of length n and dimension k, i.e., let C be a k-dimensional subspace of $\mathbb{F}_q^n$. Basic parameters of C include the minimum distance $d(C)$ and, more generally, the generalized Hamming weights (GHW) $d_1(C) < \cdots < d_k(C)$. There is a natural notion of a dual $C^\perp$ of C, and a beautiful relationship between the GHW of C and the GHW of $C^\perp$; this relationship was observed by Wei (1991) and is known as Wei duality.

One can associate to a C a (vector) matroid and in turn, a simplicial complex Δ_C. In 2012, Britz, Johnsen, Mayhew, and Shiromoto extended Wei duality to arbitrary matroids and even more general structures called demi-matroids. In another development, Johnsen and Verdure (2013) associated to C a fine set of invariants, called Betti numbers, which determine completely the GHW of C. These are, in fact, the Betti numbers of the graded minimal free resolutions of the Stanley-Reisner ring, say R_C, of the simplicial complex Δ_C. A basic fact here is that the ring R_C is Cohen-Macaulay, which follows from the classical combinatorial result that matroid complexes such as Δ_C are shellable.

In the recent past, there has been growing interest in rank metric codes. The study of rank metric codes goes back to Delsarte (1979) and Gabidulin (1985). We now understand the analogs of GHW for rank metric codes, and these are known as generalized rank weights (GRW). Moreover, Wei-type duality theorems are established for GRW of rank metric codes by Ducoat (2015) and Ravagnani (2016).

We will review these developments and then outline some newer notions and results. These include q-analogs of matroids that are relevant for the study of rank metric codes, notions of q-complexes, and their shellability, and the notion of Betti numbers of rank metric codes. An attempt will be made to keep the prerequisites at a minimum.

Parts of this talk are based on a joint work with T. Johnsen (2020) and also with R. Pratihar and T. H. Randrianarisoa (2021).

Christian Krattenthaler

University of Vienna

The (so-called) "Borwein Conjecture" arose around 1990 and states that the coefficients in the polynomial

$$(1-q)(1-q^2)(1-q^4)(1-q^5)\cdots(1-q^{3n-2})(1-q^{3n-1})$$

have the sign pattern $+--+--\dots$. This innocent looking prediction has withstood all proof attempts until two years ago when Chen Wang found a proof that combines asymptotic estimates with a computer verification for "small" n.

However, Borwein made actually in total three sign pattern conjectures of similar character - with the previously mentioned conjecture being just the first one -, and recently Wang discovered a further one. It seemed unlikely that Wang's proof could be adapted to work for these other conjectures since it crucially used identities that are only available for the "First Borwein Conjecture".

I shall start by presenting these conjectures and then review the history of the conjectures and the various attempts that have been made to prove them - as a matter of fact, these attempts concerned exclusively the "First Borwein Conjecture", while nobody had any idea how to attack the other conjectures.

I shall then outline a proof plan that is (in principle) applicable to all these conjectures. Indeed, this leads to a new proof of the "First Borwein Conjecture", the first proof of the "Second Borwein Conjecture", and to a proof of "two thirds" of Wang's conjecture. We are convinced that further work along these lines will lead to - at least - a partial proof of the "Third Borwein Conjecture".

I shall close with further open problems in the same spirit.

This is joint work with Chen Wang.

Contributed talks

Kristina Ago

University of Novi Sad

On MP-ratio for multiary words: well-definedness and upper bounds

In combinatorics on words, a number of ways to measure the degree of "palindromicity" of a given word have been proposed and researched in the literature. One such measure is the so-called *MP-ratio* (where the abbrevation MP stands for minimal-palindromic). An n-ary word is called *minimal-palindromic* if it does not contain palindromic subwords of length greater than $\lceil \frac{|w|}{n} \rceil$. The MP-ratio of a given n-ary word w is defined as the quotient $\frac{|rws|}{|w|}$, where r and s are words such that the word rws is minimal-palindromic and the length $|r| + |s|$ is minimal possible. The notion of MP-ratio was introduced by Holub and Saari for binary words, who proved that the MP-ratio is well-defined in that case and that it is bounded from above by 4, which is the best possible upper bound. For larger arities it is obvious what is the natural generalization of the notion of MP-ratio, but already the question whether such generalization is well-defined is much harder in comparison to the binary case.

The main result presented in this talk shows that the MP-ratio is well-defined for n-ary words for any n. The proof for $n > 3$ also provides an upper bound on the MP-ratio that grows exponentially with respect to n. Additionally, in the case $n = 3$, we show that the MP-ratio is bounded from above by 6. Since (as will also be seen in the talk) such an upper bound in general cannot be smaller than $2n$, the bound obtained in the ternary case is the best possible.

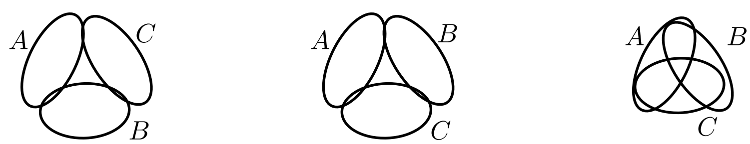
This is a joint work with B. Bašić.

Péter Ágoston

Eötvös Loránd University, Budapest

Orientation type of intersecting convex planar sets

Take a family of compact convex sets in the plane, whose members are pairwise intersecting (in the following, we simply call such a family an intersecting family). Several theorems exist about such families, such as the Helly theorem and its variants. For creating more such theorems, a better understanding of the structure of intersecting families seems to be useful. We thus defined the orientation of pairwise intersecting triples as roughly sketched in the following drawing.



$ABC = +1$ $\quad$ $ABC = -1$ $\quad$ $ABC = 0$

This notion of orientation can be defined more precisely thanks to Jobson et al. [1]. If no triple intersections occur, we call such a family a holey family and such a system of orientations a C-3OSET. If triple intersections are allowed, we call such a system of orientations a C-3POSET.

These orientation systems have the following properties (among others):

1) For any A, B and C from an intersecting family, $ABC = BCA = CAB = -CBA = -BAC = -ACB$.

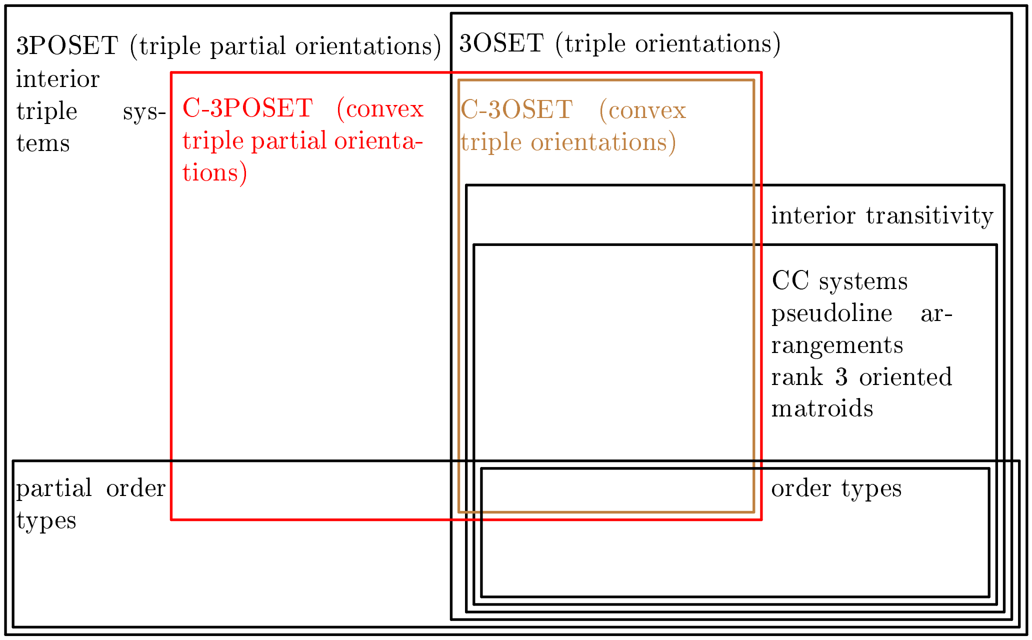
2) For any A, B, C and O from an intersecting family, $ABO = BCO = CAO = 1$ means $ABC = 1$ (similarly with -1 and with 0).

3) If $ABC = ABD = 0$, but $ACD \neq 0$ and $BCD \neq 0$, then $ACD = BCD$.

Furthermore, the following stronger property also holds.

3') $A_1A_2B_1B_2 = A_1A_2C_1C_2 = B_1B_2C_1C_2 = 0$, but $A_1B_1C_1 \neq 0$ and $A_2B_2C_2 \neq 0$, then $A_1B_1C_1 = A_2B_2C_2$.

The following diagram shows the relationship between C-3OSETS, C-3POSETS and other similar notions.



In our research, we tried to find further properties of these orientation systems and their relationship with the other orientation systems shown in the above table.

Joint work with Gábor Damásdi, Balázs Keszegh and Dömötör Pálvölgyi.

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Sara Ban

University of Rijeka, Faculty of Mathematics

Self-orthogonal $\mathbb{Z}_{2^k}$-codes constructed from bent functions

A Boolean function on n variables is a mapping $f : \mathbb{F}_2^n \to \mathbb{F}_2$. A bent function is a Boolean function f such that $W_f(v) = \sum_{x \in \mathbb{F}_2^n} (-1)^{f(x)+\langle v,x \rangle} = \pm 2^{\frac{n}{2}}$, for every $v \in \mathbb{F}_n^2$.
The subject of this talk is a construction of self-orthogonal codes over $\mathbb{Z}_{2^k}$ from bent functions.
First, we give a construction of a self-orthogonal $\mathbb{Z}_4$-code of length 2^{n+1} from a pair of bent functions on n variables. We prove that for $n \geq 4$ those codes can be extended to Type IV-II $\mathbb{Z}_4$-codes. From that family of Type IV-II $\mathbb{Z}_4$-codes, we construct a family of self-dual Type II binary codes by using the Gray map. We consider the weight distributions of the obtained codes. Furthermore, we construct a self-orthogonal $\mathbb{Z}_{2^k}$-code of length 2^{n+1} with all Euclidean weights divisible by 2^{k+2} from a pair of bent functions on n variables, for every $k \geq 3$.
This is joint work with Sanja Rukavina.

János Barát

Alfréd Rényi Institute of Mathematics

Saturated 1-plane and 2-plane drawings with few

A drawing of a graph is k-plane if every edge contains at most k crossings. A k-plane drawing is saturated if we cannot add any edge so that the drawing remains k-plane. It is well-known that saturated 0-plane drawings, that is, maximal plane graphs, of n vertices have exactly $3n-6$ edges. For $k>0$, the number of edges of saturated n-vertex k-plane graphs can take many different values.

Brandenburg et al. showed there are maximal 1-planar graphs with only $\frac{45}{17}n+O(1)\approx 2.647n$ edges and maximal 1-plane graphs with only $\frac{7}{3}n+O(1)\approx 2.33n$ edges. On the other hand, they showed that any maximal 1-planar graph has at least $\frac{28}{13}n-O(1)\approx 2.15n-O(1)$ edges, and a maximal 1-plane graph has at least $2.1n-O(1)$ edges.

We improved both lower bounds to $\frac{20n}{9}\approx 2.22n$.

For 2-plane graphs, a drawing is *l-simple* if any two edges have at most l points in common. Let $s_k^l(n)$ be the minimum number of edges of a saturated l-simple k-plane drawing. Klute and Parada showed that $s_2^1(n)\geq \frac{n}{2}$, $\frac{4n}{5}\geq s_2^2(n)\geq \frac{n}{2}$ and $\frac{2n}{3}\geq s_2^3(n)\geq \frac{n}{2}$.

We make the following improvements:

(i) For any $n>0$, $s_2(n)\geq n-1$.

(ii) $s_2^2(3)=3$, and for $n\neq 3$ $\lfloor 3n/4\rfloor \geq s_2^2(n)\geq \lfloor 2n/3\rfloor$,

(iii) $s_2^3(3)=3$, and for $n\neq 3$ $s_2^3(n)=\lfloor 2n/3\rfloor$.

Some of these bounds are achieved by interesting constructions.

Bojan Bašić

Department of Mathematics and Informatics, University of Novi Sad

The Heesch number in $\mathbb{E}^d$ *is asymptotically unbounded for* $d \to \infty$

Given two figures that do not tesselate the Euclidean plane $\mathbb{E}^2$, the *Heesch number* (introduced by Heinrich Heesch in 1968) ranks them by their ability to "advance" toward a tesselation; the Heesch number of a given figure is a nonnegative integer such that, the larger it is, the figure can advance "further" toward a tessellation (and if a figure tessellates the plane, it is convenient to define the Heesch number of that figure to be infinite). Speaking somewhat informally, we define the Heesch number of a given figure T to be the maximal nonnegative integer n such that T can be completely surrounded by congruent copies of itself n times in total. The main open question concerning the Heesch number is whether there exists the largest possible finite Heesch number (to put it another way, whether the set of all nonnegative integers that appear as values of the Heesch number of some figure has an upper bound); this question is called *Heesch's problem.*

In this talk we treat Heesch's problem in more-dimensional spaces. The main result is as follows: no matter how large a given integer n is, there always exists a dimension d (possibly dependent on n) and a hypersolid in $\mathbb{E}^d$ whose Heesch number is greater than n and finite. This answers the asymptotical version of Heesch's problem: there does not exist an uniform upper bound on the set of all possible finite values of the Heesch number in $\mathbb{E}^d$ for $d \to \infty$.

At the end of the talk, another result from the near past will be presented. Namely, for almost twenty years, the largest known finite Heesch number in $\mathbb{E}^2$ had been 5, until this has recently been surpassed, when a figure whose Heesch number equals 6 has been constructed.

This is a joint work with A. Slivková.

Davi Castro-Silva

CWI - Netherlands

A recursive Lovász theta number for simplex-avoiding sets

The *Lovász theta number* is an important graph parameter in combinatorial optimization, which has found many applications in combinatorics and geometry. By the *sandwich theorem* the theta number $\vartheta(G)$ of a finite graph G satisfies $\alpha(G) \leq \vartheta(G) \leq \chi(\overline{G})$, where $\alpha(G)$ is the independence number of G and $\chi(\overline{G})$ is the chromatic number of the complement of G (i.e. the edges of $\overline{G}$ are the non-edges of G and vice versa); even though it is sandwiched between two NP-hard graph parameters, the theta number can be computed efficiently using semidefinite programming.

In this talk we will recursively extend the Lovász theta number to geometric hypergraphs on the unit sphere and on Euclidean space, obtaining an upper bound for the independence ratio of these hypergraphs. As an application we reprove a result in Euclidean Ramsey theory in the measurable setting, namely that every k-simplex is exponentially Ramsey, and we improve existing bounds for the base of the exponential.

This is joint work with Fernando Oliveira, Lucas Slot and Frank Vallentin.

Nancy Clarke

Acadia University (Canada)

Rainbow Dominating Sets

Given a k-colouring of (the vertices of) a graph G, a dominating set D of G is said to be a rainbow (or achromatic) dominating set if every vertex of D has a different colour. Our parameter of interest is the rainbow dominating number $\rho(G)$, defined to be the minimum number of colours such that, for any $\rho(G)$-colouring of G, there exists a rainbow dominating set. In this talk, we present a variety of results including exact values of our parameter for several classes of graphs, as well as more general bounds. In particular, we consider graphs according to diameter, as well as lexicographic products.
This is joint work with Ruth Haas.

Francesc Comellas

Universitat Politecnica de Catalunya

On the resilience of hierarchical graphs.

Many graphs associated with complex real-life systems are small-world (with both a large local clustering coefficient and a small average distance) and scale-free (the degrees are distributed according to a power law) and very often, when the systems are modular, they are also hierarchical and include vertices with relatively high degree (hubs). In `doi:10.1088/1751-8113/49/22/225202` we introduced a generic family of deterministic hierarchical small-world scale-free graphs and determined some relevant topological properties.

In this talk we present a study on the resilience of this, and other classical graphs families, under a standard vertex cascading failure model (`doi:10.1016/j.ssci.2009.02.002`). Other graph families considered are: random Erdős-Rényi, small-world Watts-Strogatz, scale-free Barabasi-Albert, cluster power law, random regular, connected caveman, random geometric, geographical threshold, random partition and random regular sequence. We show that our family of hierarchical graphs is the most resilient under different categories of initial failing vertices for this cascading failure model. We contrast our results with those of `doi.org/10.1007/s41109-021-00404-4` which show a lack of robustness of hierarchical graphs to random and targeted removals of vertices.

This is joint work with Shima Aflatounian.

Radu-Cristian Curticapean

IT University of Copenhagen

The complexity of immanants

Immanants are matrix functionals that generalize determinants and permanents. Given an irreducible character χ_λ of S_n for some partition λ of n, the immanant associated with λ is a sum-product over permutations π in S_n, much like the determinant, but with $\chi_\lambda(\pi)$ playing the role of $sgn(n)$.

Hartmann showed in 1985 that immanants can be evaluated in polynomial time for sign-ish characters. More precisely, for a partition λ of n with s parts, let $b(\lambda) := n - s$ count the boxes to the right of the first column in the Young diagram of λ. The immanant associated with λ can be evaluated in $n^{O}(b(\lambda))$ time.

Since this initial result, complementing hardness results have been obtained for several families of immanants derived from partitions with unbounded $b(\lambda)$. This includes permanents, immanants associated with hook characters, and other classes. In this talk, we complete the picture of hard immanant families: Under a standard assumption from parameterized complexity, we rule out polynomial-time algorithms for (well-behaved) immanant families with unbounded $b(\lambda)$. For immanant families in which $b(\lambda)$ even grows polynomially, we establish hardness for $\#P$ and VNP.

Joel Danielsson

Department of statistics, Lund University School of Economics and Management

Entropy counting of combinatorial 3-spheres

An informal definition of a combinatorial 2-sphere is that it is obtained by gluing triangular pieces of paper together along their edges, so that the resulting surface forms a 2-sphere. One dimension higher, a combinatorial 3-sphere consists of tetrahedra glued together along their triangular faces to form a 3-sphere, and so on. The formal definition is that a combinatorial d-sphere is a simplicial complex homeomorphic to a d-sphere. We are interested in the asymptotics of how many distinct vertex-labeled combinatorial d-spheres there are, as a function of the number of vertices (n) and faces (m). This is known for d=2, where the geometry is simpler. Even for d=3, the number of spheres has only been determined as a function of either m or n. In both cases the count is dominated by spheres with many more faces than vertices, i.e. n=o(m).

Our main result is an improved upper bound on the number of 3-spheres with few faces compared to vertices. We study the entropy of a randomly picked 3-sphere. This is fundamentally an application of Shannon's encoding theorem, but more specifically we are using it in the form of a handy lemma from a recent paper by Palmer and Pálvölgyi. By applying this lemma, the problem is transformed into a new one: upper bounding the expected number of percolation clusters in a kind of site percolation on 2-spheres.

Mrinmoy Datta

Indian Institute of Technology Hyderabad

Footprint bounds and their applications to Reed-Muller type codes

Footprint bounds have been studied extensively to determine the important parameters such as minimum weights, generalized Hamming weights, and relative generalized Hamming weights of Reed-Muller type codes. In this talk, we will explain the notion of the footprint bounds and demonstrate their applications to determining parameters of affine Cartesian codes.

Sebastian Debus

UiT The Arctic University of Norway

A bidominance order and Specht ideals for the signed symmetric group

In this talk we consider Specht ideals and their varieties for the signed symmetric group B_n. The Specht polynomials of B_n span the irreducible representations which correspond to bipartitions. The ideals that are generated by all Specht polynomials of a given shape are called Specht ideals. We introduce a bidominance order on the set of bipartitions and characterize the covering cases in this poset. Furthermore, we prove an equivalence between bidominance of bipartitions, inclusion of the Specht ideals and their varieties. We present a notion of B_n orbit types and prove a set partition of the Specht varieties using orbit types and the bidominance order.

This is joint work together with Philippe Moustrou, Hugues Verdure and Cordian Riener

Danai Deligeorgaki

KTH

Unconditional Equivalence for DAG models

We study directed acyclic graph (DAG) models up to unconditional equivalence. Two DAGs are unconditionally equivalent if they encode the same set of unconditional d-separation statements. It turns out that each unconditional equivalence class (UEC) of DAGs can be uniquely represented with an undirected graph whose clique structure encodes the members of the class. Via this structure, we can understand the v-structures of all maximal DAGs in the class, which are shown to lie in the same Markov Equivalence Class, and provide a transformational characterization of unconditional equivalence. Combining these results, we introduce a hybrid algorithm for learning DAG models from observational data, called Greedy Unconditional Equivalence Search (GUES), which first estimates the UEC of the data using independence tests and then greedily searches the UEC for the optimal DAG.
This is joint work with Alex Markham, Pratik Misra and Liam Solus.

Blas Fernández

University of Primorska, Slovenia

On the trivial T-module of a graph

Let Γ denote a finite, simple and connected graph. Fix a vertex x of Γ and let $T = T(x)$ denote the Terwilliger algebra of Γ with respect to x. That is, a matrix subalgebra generated by the adjacency matrix of Γ together with certain diagonal matrices containing local information about the structure of Γ with respect to x.

It turns out that there exists a unique irreducible T-module with endpoint 0, the trivial T-module. Fiol and Garriga [2] introduced the concept of *pseudo-distance-regularity* around vertex x, which is based on giving to the vertices of the graph some weights which correspond to the entries of the (normalized) positive eigenvector. They showed that the unique irreducible T-module with endpoint 0 is thin if and only if Γ is pseudo-distance-regular around x (see also [1, Theorem 3.1]).

We study the trivial T-module under the assumption that it is thin. The main result of the talk is a purely combinatorial characterization of the property, that the unique irreducible T-module with endpoint 0 is thin and so, that Γ is pseudo distance-regular around x.

This is joint work with Štefko Miklavič.

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Luis Ferroni

KTH Royal Institute of Technology

Ehrhart polynomials, flag Eulerian numbers and algebras of Veronese type

In this talk we will address the Ehrhart theory of a generalized version of hypersimplices. We will explain how to count the number of lattice points in dilations of certain slices of rectangular prisms. These polytopes are polypositroids and their Ehrhart polynomials have positive coefficients. Two applications regarding a generalization of the flag Eulerian numbers and an interpretation for the numerator of all algebras of Veronese type will be discussed.

Ragnar Freij-Hollanti

Aalto University

Derived matroids in combinatorics and linear algebra

Let M be an arbitrary matroid. Gian-Carlo Rota and Henry Crapo asked, partly independently and with various precise formulations, for a natural definition of a matroid δM that has as its ground set the collection of circuits of M. For represented matroids, this is straightforward, but the combinatorics of δM is in general not uniquely defined by that of M, unless M is projectively unique, for example binary. We propose a purely combinatorial construction of δM, defined via the rank function on M, and via operation that resembles a closure operation on the collection of dependent sets. We study some of the basic properties of δM, show that it equals the previous definition for binary matroids, and conjecture that our δM is in a precise sense the most generic of all possible derived matroids for a representable matroid M.

Nima Ghanbari

University of Bergen

Some results on the super domination and co-even domination number of a graph

Abstract

Let $G = (V, E)$ be a simple graph. A dominating set of G is a subset $D \subseteq V$ such that every vertex not in D is adjacent to at least one vertex in D. The cardinality of a smallest dominating set of G, denoted by $\gamma(G)$, is the domination number of G. A dominating set S is called a super dominating set of G, if for every vertex $u \in \overline{S} = V - S$, there exists $v \in S$ such that $N(v) \cap \overline{S} = \{u\}$. The cardinality of a smallest super dominating set of G, denoted by $\gamma_{sp}(G)$, is the super domination number of G. A dominating set D is called co-even dominating set if the degree of vertex v is even number for all $v \in V - D$. The cardinality of a smallest co-even dominating set of G, denoted by $\gamma_{ce}(G)$, is the co-even domination number of G. In this paper, we study the super domination number and co-even domination number of some unary and binary operations on graphs.

Keywords: domination number, super domination number, co-even dominating set, operations, counting

AMS Subj. Class.: 05C09, 05C12, 05C38, 05C69, 05C75 , 05C76, 05C92

Introduction

Let $G = (V, E)$ be a simple graph with n vertices. Throughout this paper we consider only simple graphs. A set $D \subseteq V(G)$ is a dominating set if every vertex in $V(G) - D$ is adjacent to at least one vertex in D. The domination number $\gamma(G)$ is the minimum cardinality of a dominating set in G. There are various domination numbers in the literature.

The concept of super domination number was introduced by Lemańska et al. in 2015 [7]. A dominating set S is called a super dominating set of G, if for every vertex $u \in \overline{S}$, there exists $v \in S$ such that $N(v) \cap \overline{S} = \{u\}$. The cardinality of a smallest super dominating set of G, denoted by $\gamma_{sp}(G)$, is the super domination number of G.

Recently, Shalaan et al. introduced the concept of co-even domination number [8]. By their definition, a dominating set D is called a co-even dominating set if the degree of vertex v is even number for all $v \in V - D$. The cardinality of a smallest co-even dominating set of G, denoted by $\gamma_{ce}(G)$, is the co-even domination number of G.

In this paper, we study the super domination number and co-even domination number of some unary and binary operations on graphs.

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Harald Gropp

Universitaet Heidelberg

On orbital matrices, also beyond the polar circle

Orbital matrices were introduced already 35 years ago, but have not been studied very much since then. There are polar bears, there is a polar circle, maybe the discussion of orbital matrices beyond the polar circle could be another input for future research. An orbital matrix is a generalization of the incidence matrix of a symmetric 2-design. It is a square matrix with non-negative integer entries with constant row and column sum such that $AA^t = (k + x - \lambda)I + \lambda J$. The existence problem of orbital matrices is discussed, especially for $\lambda \leq 3$. The theorem of Bruck-Ryser-Chowla holds also for orbital matrices. However, there remain a lot of cases where other techniques have to be used for deciding the existence or non-existence of such a matrix.

Michaela Hiller

RWTH Aachen University

Domination among Elimination Sequences

In 1991, it was shown by Favaron, Mahéo and Saclé that the residue, i.e. the number of zeros remaining when applying the Havel-Hakimi algorithm to a degree sequence, yields a lower bound on the independence number of any graph realising the sequence. In 1996, Triesch simplified and generalised the result by introducing elimination sequences. We now prove that for any given degree sequence the elimination sequence derived from the Havel-Havel algorithm dominates all other elimination sequences. Our result implies a conjecture posed by Michael Barrus in 2010: When iteratively laying off degrees from a graphic sequence until only a list of zeros remains, the number of zeros is at most the residue of this sequence

Marija Jelić Milutinović

University of Belgrade, Faculty of Mathematics

Perfect Matching Complexes of Honeycomb Graphs

The matching complex $M(G)$ of a graph G is the simplicial complex with the ground set given by edges of G and faces given by subsets of disjoint edges, i.e., matchings of G. Matchings arise in various applications. In chemistry, the Kekulé structure of an aromatic compound is a perfect matching of its carbon skeleton. Matchings are also used for transportation problems and other combinatorial optimization problems to find optimal assignments.

There is a long history of the study of matching complexes. Research on the topology of matching complexes focuses particularly on complete graphs and complete bipartite graphs; see for example, [**?**, **?**, **?**]. The study of the matching complexes for these graphs arose from algebraic group theory [**?**, **?**] and proved to be interesting in its own right. Also, these matching complexes and their joins are suitable candidates for configuration spaces in various combinatorial geometric problems [**?**, **?**]. While some aspects of the matching complexes of complete graphs and complete bipartite graphs are known, such as connectivity bounds and the existence of torsion in higher homology groups, our understanding of the topology is still incomplete. Beyond complete graphs and complete bipartite graphs, cycles, paths, forests, small grid graphs, tiling of polygons, and caterpillar graphs have been studied and the matching complexes of these families of graphs are not known to have torsion. Further, matching complexes of cycles, paths, forests, and caterpillar graphs are known to be contractible or homotopy equivalent to a wedge of spheres [**?**, **?**]. The literature points to a recurring question, which is a main motivation for this project: what graph properties give rise to torsion in matching complexes?

Since we know that matching complexes of forests are contractible or wedges of spheres, one may reason that cycles must contribute to the presence of torsion in matching complexes. Further, since matching complexes of cycles themselves are contractible or homotopy equivalent to a wedge of spheres, the relationship between cycles must be a factor. The tilings of hexagons, called honeycomb graphs, provide a sufficiently complicated family to be studied. Precisely, *honeycomb graph* $H = H_{k\times m\times n}$ is a hexagonal tiling whose congruent, opposite sides are of length k, m and n hexagons respectively (see honeycomb graph $2 \times 2 \times 2$ bellow).

It turns out that determining the homotopy type of the matching complex of a honeycomb graph is a challenging question, so we investigate a natural subcomplex of a matching complex. Precisely, we define the following new graph complex. First, a *perfect matching* on a graph G is a matching in which every vertex is incident to exactly one edge. *The perfect matching complex* of graph G, denoted by $\mathcal{M}_p(G)$, is the simplicial complex whose vertex set is the set of edges E, and whose facets correspond to perfect matchings on graph G. We will observe that for complete graphs, complete bipartite graphs, paths and cycles, perfect matching complexes are either easily determined or they are familiar complexes, and then we will focus on perfect matching complexes of honeycomb graphs.

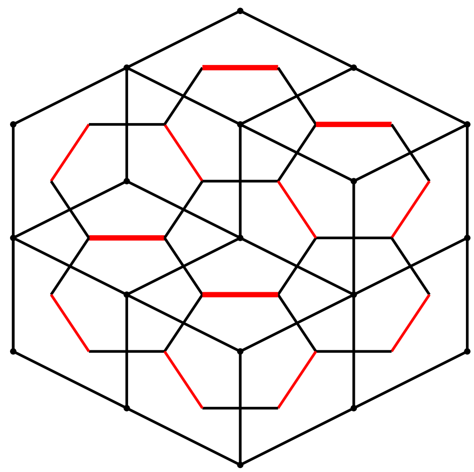

Figure 1: Example of a honeycomb graph $2 \times 2 \times 2$ and corresponding plane partition.

In order to determine these complexes, we will use the well known bijection between perfect matchings of the honeycomb graph $H_{k\times m\times n}$ and plane partitions $P_{k\times m\times n}$, as in [?]. This very interesting bijection will be explained in the talk. Also, we use discrete Morse theory. This theory, introduced by Forman [?], is a combinatorial technique which determines the homotopy type of a simplicial complex by pairing faces of the complex. These pairings correspond with a sequence of collapses on the complex, resulting in a homotopy equivalent cell complex. Our main results are the following homotopy types of the perfect matching complexes: $\mathcal{M}_p(H_{1\times 1\times n}) \simeq *$, $\mathcal{M}_p(H_{1\times 2\times n}) \simeq S^{n-1}$, then for $m, n \geq 3$ we have $\mathcal{M}_p(H_{1\times m\times n}) \simeq *$, and $\mathcal{M}_p(H_{2\times 2\times 2}) \simeq S^3 \vee S^3$. We will conclude with further directions and open questions.

Vedran Krčadinac

University of Zagreb

On t-designs with three intersection numbers

The *degree* of a t-(v, k, λ) design is the number d of distinct block intersection sizes. Designs of degree $d = 1$ are symmetric and of strength $t \leq 2$. Designs with $d = 2$ are known as quasi-symmetric and the strength is bounded by $t \leq 4$. Regarding designs of degree $d = 3$, it is known that $t \leq 5$ holds and the only examples with $t = 5$ are hypothesised to be the Witt 5-$(24, 8, 1)$ design and its complement [1].

We will report on designs with $d = 3$ and $t = 4$. In this case there are infinitely many feasible parameters. Designs with small parameters exist and are related to the quadratic residue codes. We will also give some preliminary results on designs with $d = 3$ and $t = 3$.

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Alexander Lazar

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Lexicographic Shellability Statistics

A theorem of Bjorner states that a pure simplicial complex Δ is the independence complex of a matroid if and only if, given any total order of the vertices, the induced lexicographic order of the facets of Δ is a shelling order. In this talk I will present preliminary results of work with Joseph Doolittle and Bennet Goeckner in which we study the topological and combinatorial properties of complexes with at least one such vertex order.

Tuomo Lehtilä

University of Turku, Finland

New bounds for total dominating identifying codes

A set of vertices S in graph G is a *dominating* set if each vertex is in S or adjacent to a vertex in S and it is a total dominating set if it is a dominating set and each vertex in S is adjacent to another vertex in S. Set C is an identifying code in graph G if C is a dominating set and each vertex has a unique closed neighbourhood within C. Moreover, C is a total dominating identifying code or a differentiating-total dominating set if C is an identifying code and a total dominating set. The cardinality of a smallest total dominating identifying code in the graph G is denoted by $\gamma_t^{ID}(G)$ or by $\gamma^{ID}(G)$ in the case of usual identifying codes. Identifying codes were introduced in 1998 by Karpovsky et al. and total dominating identifying codes by Haynes et al. in 2006. Since then especially the original identifying codes have been studied widely although there are also some recent advances on the total dominating variant. Two vertices are called twins if they have the same open or closed neighbourhoods and a graph without twins is called twin-free. In particular, when two vertices u and v are adjacent to the same set of vertices, at least one of them is in any identifying code. Hence, much of the study of these types of special dominating sets is aimed at twin-free graphs. Graph G has girth g if the smallest cycle in G has g vertices. Previously two different upper bounds for total dominating identifying codes in trees have been offered. Based on these two upper bounds we get a new corollary for twin-free trees: Let T be a twin-free tree on n vertices, then $\gamma_t^{ID}(T) \leq 3n/4$. The bound is tight and we characterize every twin-free tree attaining this value. After that we show, that this upper bound actually holds for every twin-free graph G which has girth at least 5 and the bound is tight also for other graphs than trees, such as, the 8-cycle C_8. Besides (total dominating) identifying codes, there exist related concepts such as locating-dominating sets. We denote the cardinality of a smallest such set in graph G by $\gamma^L(G)$. In particular, it is known that $\gamma^L(G) \leq \gamma^{ID}(G) \leq 2\gamma^L(G)$. Inspired by these types of results we show a tight bound $\gamma^{ID}(G) \leq \gamma_t^{ID}(G) \leq 2\gamma^{ID}(G) - 2$ and that $\gamma^L(G) \leq \gamma_t^{ID}(G) \leq 3\gamma^L(G) - log_2(\gamma^L(G) + 1)$. Also the latter bound is at least almost tight as we give graphs with $\gamma^L(G) = 2^k - 1$ and $\gamma_t^{ID}(G) = 3 \cdot 2^k - 2k - 3 = 3\gamma^L(G) - 2log_2(\gamma^L(G) + 1)$ for every integer $k \geq 2$. Finally, we show that if G is a connected graph of order at least $n \geq 4$ and has an identifying code, then $\gamma_t^{ID}(G) \leq n - 1$. Moreover, we give an exact and non-trivial characterization for the extremal graphs attaining this upper bound.

Anna Margarethe Limbach

RWTH Aachen University

Clique dynamics of locally cyclic graphs with $\delta \geq 6$

We prove that the clique graph operator k is divergent on a locally cyclic graph G (i. e. $N_G(v)$ is a circle) with minimum degree $\delta(G) = 6$ if and only if G is 6-regular. The clique graph kG of a graph G has the maximal complete subgraphs of G as vertices, and the edges are given by non-empty intersections. If all iterated clique graphs of G are pairwise non-isomorphic, the graph G is k-divergent; otherwise, it is k-convergent. To prove our claim, we explicitly construct the iterated clique graphs of those infinite locally cyclic graphs with $\delta \geq 6$ which induce simply connected simplicial surfaces. These graphs are k-convergent if the size of triangular-shaped subgraphs of a specific type is bounded from above. We apply this criterion by using the universal cover of the triangular complex of an arbitrary finite locally cyclic graph with $\delta = 6$, which shows our divergence characterisation.
This is joint work with Markus Baumeister.

Pratik Misra

KTH Royal Institute of Technology

Symmetrically colored Gaussian graphical models with toric vanishing ideal

Gaussian graphical models are semi-algebraic subsets of the cone of positive definite covariance matrices. They are widely used throughout natural sciences, computational biology and many other fields. Computing the vanishing ideal of the model gives us an implicit description of the model. Now, introducing colors in a graph gives rise to new symmetries in the model. In this talk, I will characterize those graphs and color patterns for which the vanishing ideal of the model is generated in degree 1 and 2. These turn out to be colored Gaussian graphical models whose ideals are toric and the resulting colored graphs are RCOP block graphs.

Guillermo Nunez Ponasso

Worcester Polytechnic Institute

The maximal determinant problem over the third roots of unity

We survey the maximal determinant problem for +-1 matrices, and consider a generalisation to roots of unity. In particular we will give an overview of our results over the third roots of unity, including both upper and lower bounds for the determinant.

Silvia Pagani

Università Cattolica del Sacro Cuore, Brescia - Italy

Power sum polynomials and the ghosts behind them

Given a point P in $\mathrm{PG}(n,q)$, its Rédei factor is the linear polynomial in $n+1$ variables, whose coefficients are the point coordinates. The power sum polynomial G^S associated to a multi-subset S of the projective plane $\mathrm{PG}(2,q)$ is the sum of the $(q-1)$-th powers of the Rédei factors of the points of S. The classification of multi-subsets having the same power sum polynomial bases on the determination of those multi-subsets associated to the zero polynomial, called ghosts. In fact, two multi-subsets S_1 and S_2 such that $G^{S_1} = G^{S_2}$ "differ" by a ghost Z, namely, $S_2 = S_1 \uplus_p Z$, where $\uplus_p$ is the multiset sum modulo p (the field characteristic).

In this talk we investigate the space of ghosts, compute its dimension and characterize some classes of ghosts. Moreover, we explicitly enumerate ghosts for planes of small order. The present talk is based on joint work with Marco Della Vedova (Università di Bergamo) and Silvia Pianta (Università Cattolica del Sacro Cuore).

Keywords: Ghost; multiset sum; power sum polynomial; projective plane
MSC classification: 51E15, 51E22, 11T06, 05B25

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Péter Pál Pach

Budapest University of Technology and Economics

The Alon-Jaeger-Tarsi conjecture via group ring identities

The Alon-Jaeger-Tarsi conjecture states that for any finite field F of size at least 4 and any nonsingular matrix M over F there exists a vector x such that neither x nor Mx has a 0 component. In joint work with János Nagy we proved this conjecture when the size of the field is sufficiently large, namely, when $61 < |F| \neq 79$. In this talk we will discuss this result.

Marta Pavelka

University of Miami

Four non-Hamiltonian simplicial complexes

We present four higher-dimensional counterexamples to "obvious" generalizations of graph-theoretical results:
(a) A maximal non-weakly-Hamiltonian d-complex that has no (tight, loose, or weak) Hamiltonian paths.
(b) A self-complementary complex that has no (tight, loose, or weak) Hamiltonian paths.
(c) A Dirac-type complex that has no (tight, loose, or weak) Hamiltonian cycles.
(d) A strongly connected, 2-connected 2-complex whose square has no (tight, loose, or weak) Hamiltonian cycles.

1 Introduction

In 1952 Dirac [1] gave a simple degree condition sufficient to guarantee the existence of Hamiltonian cycles in a graph: namely, he proved that any graph with $n \geq 3$ vertices is Hamiltonian if every vertex has at least $\frac{n}{2}$ neighbors. Later Pósa [2] and Chvátal [3] refined this result introducing the notion of *degree sequence*, a vector that lists all vertex degrees in weakly-increasing order. Chvátal characterized the degree sequences that force a graph to have Hamiltonian paths or cycles. Thanks to his argument, we now know that all self-complementary graphs are traceable (that is, they all admit a Hamiltonian path). Further sufficient conditions for Hamiltonian cycles were found in the next decade: In 1974 Fleischner [4] proved that for every 2-connected graph G, the graph G^2 has a Hamiltonian cycle. Later Chen, Chang and Chang [5] showed that every 2-connected unit-interval graph G has Hamiltonian cycles.

Considerable efforts have been made to extend these results to hypergraphs or simplicial complexes. Many have studied "tight-Hamiltonian" and "loose-Hamiltonian" paths in d-dimensional simplicial complexes; for $d = 1$ all these notions boil down to ordinary Hamiltonian paths. Katona–Kierstead [6] proved a Dirac-type theorem assuming all the ridge degrees to be at least $\binom{n-3}{d}+d$. Others [7, 8, 9] has extended Dirac's theorem for d-complexes with a very large number of vertices. Benedetti–Seccia–Varbaro [10] introduced "weak-Hamiltonian" paths and extended Chen–Chang–Chang to all dimensions, proving that a unit-interval d-complex that remains strongly connected after the removal of d or less vertices, has tight Hamiltonian paths.

Here we present four higher-dimensional counterexamples to "obvious" generalizations of graph-theoretical results:

(a) A maximal non-weakly-Hamiltonian d-complex that has no (tight, loose, or weak) Hamiltonian paths.
(b) A self-complementary complex that has no (tight, loose, or weak) Hamiltonian paths.
(c) A Dirac-type complex that has no (tight, loose, or weak) Hamiltonian cycles.
(d) A strongly connected, 2-connected 2-complex whose square has no (tight, loose, or weak) Hamiltonian cycles.

A maximal non-weakly-Hamiltonian d-complex that has no Hamiltonian paths

Let $d \geq 1$. Consider the d-complex W^d obtained by joining the $(d-1)$-dimensional simplex with a disjoint union of 3 points. W^d does not have (tight, loose or weak) Hamiltonian path [BSV]. We claim that for any $d \geq 2$, W^d is inclusion-maximal with respect to the property of not having a weak Hamiltonian path. In fact, let us label the vertices of the $(d-1)$-simplex by $1, \ldots, d$ and let us call "apices" the remaining three vertices, labeled by $d+1, d+2, d+3$. Let F be a subset of $[d+3]$ size $d+1 \geq 3$. There are three cases:

(i) If F contains exactly one apex, then F already belongs to W^d.

(ii) If F contains exactly two apices, then up to permuting the labels of the first d vertices and the labels of the last three we can assume that $F = [1, 2, \ldots, d-1, d+2, d+3]$. Hence $W^d \cup F$ admits a weak Hamiltonian cycle formed by the three faces H_1, H_{d+3}, F.

(iii) If F contains exactly three apices, then up to permuting the labels of the first d vertices and the labels of the last three we can assume that $F = [1, 2, \ldots, d-2, d+1, d+2, d+3]$. (In case $d = 2$, F is simply $[d+1, d+2, d+3]$.) Then $W^d \cup F$ admits the weak Hamiltonian cycle H_1, H_{d+3}, F.

So the claim is proven. In contrast, in dimension one, any inclusion-maximal non-Hamiltonian graph is traceable, a result which is used in the proof of Chvátal's theorem.

A self-complementary 2-complex without Hamiltonian paths

Consider the 2-dimensional complex

$$A = 135, 234, 124, 136, 125, 235, 145, 123, 126, 134.$$

This A is self-complementary: it is isomorphic to its complement

$$A^c = 146, 156, 236, 245, 246, 256, 345, 346, 356, 456$$

via the map $1 \to 6$, $2 \to 5$, $3 \to 4$, $4 \to 2$, $5 \to 3$, $6 \to 1$. Yet A is neither traceable nor weakly-traceable by our software [11]. Note that A is weakly-Hamiltonian but not weakly-traceable, and under-closed but not chordal. In contrast, all under-closed 1-complexes (i.e. all interval graphs) are chordal, and all weakly-Hamiltonian 1-complexes are weakly-traceable.

A Dirac-type 2-complex without Hamiltonian cycles

Consider the complex

$$B = 126, 136, 146, 156, 236, 246, 356, 456, 256, 346.$$

Vertices 1, 2, 3, 4, 5 are in exactly four facets; vertex 6 is present in all ten facets. So if by "degree" of a vertex we mean the number of facets containing it, then every vertex of B has degree larger than half the number of vertices. However, B does not have (tight, loose, or weak) Hamiltonian cycles. Moreover, B is inclusion-maximal with respect to the last property: Adding any triangle would turn B into a weakly-Hamiltonian complex. Furthermore, B also does not have (tight, loose, or weak) Hamiltonian paths.

A strongly connected (2-connected) 2-complex whose square has no Hamiltonian cycles

Given a d-dimensional simplicial complex C, we denote by C^2 the d-dimensional simplicial complex obtained from C by performing all the bistellar flips (or Pachner moves) that do not add vertices. This notion boils down for $d = 1$ to a square of a graph. Consider now the 2-dimensional complex

$$C = 123, 124, 125, 156, 148, 137.$$

Then C^2 is obtained by adding the following triangles to C:

$$145, 245, 134, 234, 135, 235, 126, 256, 127, 237, 128, 248.$$

C^2 does not have (tight, loose, or weak) Hamiltonian cycles. In contrast, in dimension one, square of any 2-connected graph has a Hamiltonian cycle.

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Ignacio M. Pelayo

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Location in Pseudotrees

Location problems consist of determining a reference set of vertices S in a graph such that at least every vertex not in S is univocally associated to a set of "coordinates" that localize it. Since the vertex set of a graph is, not always but very often, a reference set, the question is not to prove its existence but to find the minimum one.

There are mainly two types of locations: metric location and neighbor location. By metric location we refer to those locations in which the usual distance between vertices in graphs plays a crucial role, meanwhile in the neighbor location the role of the graph distance is substituted by the relation of adjacency between vertices.

Let G be a nontrivial connected graph and S a proper subset of its vertices. The S is called metric-locating (resp. neighbor-locating) if for every pair of distinct vertices $x, y \in V(G) - S$, $d_G(x, v) \neq d_G(y, v)$, for some vertex $v \in S$ (resp. $N(x) \cap S \neq N(y) \cap S$).

During the last two decades a wide variety of both, metric and neighbor, location paramaters, has been introduced and studied, such as: metric dimension, strong metric dimension, identifying code number, location-domination number, etc.

In this talk, we present a number of both known and new results related to these parameters, for the class of pseudotrees, i,e., the family of connected graphs of order n and size m such that $n - 1 \leq m \leq n$.

Sven Polak

Centrum Wiskunde & Informatica

New lower bounds on crossing numbers of $K_{m,n}$ from permutation modules and semidefinite programming

In this talk we explain how to use semidefinite programming and representation theory to compute new lower bounds on the crossing number of the complete bipartite graph $K_{m,n}$, extending a method from de Klerk et al. [SIAM J. Discrete Math. 20 (2006), 189–202] and extending the subsequent reduction by De Klerk, Pasechnik and Schrijver [Math. Prog. Ser. A and B, 109 (2007) 613–624].

We exploit the full symmetry of the problem by developing a block-diagonalization of the underlying matrix algebra and use it to improve bounds on several concrete instances. Our results imply that $\mathrm{cr}(K_{10,n}) \geq 4.87057n^2 - 10n$, $\mathrm{cr}(K_{11,n}) \geq 5.99939n^2 - 12.5n$, $\mathrm{cr}(K_{12,n}) \geq 7.25579n^2 - 15n$, $\mathrm{cr}(K_{13,n}) \geq 8.65675n^2 - 18n$ for all n. The latter three bounds are computed using a relaxation of the original semidefinite programming bound, by only requiring one small matrix block to be positive semidefinite. Our lower bound on $K_{13,n}$ implies that for each fixed $m \geq 13$, $\lim_{n\to\infty} \mathrm{cr}(K_{m,n})/Z(m,n) \geq 0.8878m/(m-1)$. Here $Z(m,n)$ denotes the *Zarankiewicz* number: the conjectured crossing number of $K_{m,n}$.

This talk is based on joint work with Daniel Brosch.

Sanja Rukavina

University of Rijeka

On some recent results on biplanes and triplanes

Fundamental problems of design theory are those of existence and classification of designs with certain parameter set. In this talk we are interested in biplanes and triplanes, i.e., in 2-$(v, k, 2)$ and 2-$(v, k, 3)$ symmetric designs.

The existence of a biplane with parameters $(121, 16, 2)$ is an open problem. Recently, it has been proved by Alavi, Daneshkhah and Praeger that the order of an automorphism group of a possible biplane $\mathcal{D}$ of order 14 divide $2^7 \cdot 3^2 \cdot 5 \cdot 7 \cdot 11 \cdot 13$. We show that such a biplane do not have an automorphism of order 11 or 13, and thereby establish that $|Aut(\mathcal{D})|$ divides $2^7 \cdot 3^2 \cdot 5 \cdot 7$. Further, we exclude a possible action of some small groups of order divisible by five or seven, on a biplane with parameters $(121, 16, 2)$.

Triplanes of order 12, i.e. symmetric $(71, 15, 3)$ designs, have the greatest number of points among all known triplanes and it is not known if a triplane $(v, k, 3)$ exists for $v > 71$. We give the first example of a triplane of order 12 that does not admit an automorphism of order 3, obtained by using binary linear codes.

Eero Räty

Umeå Universitet

Inequalities on Projected Volumes

Given $2^n - 1$ real numbers x_A indexed by the non-empty subsets $A \subset \{1, .., n\}$, is it possible to construct a body $T \subset \mathbb{R}^n$ such that $x_A = |T_A|$ where $|T_A|$ is the $|A|$-dimensional volume of the projection of T onto the subspace spanned by the axes in A? As it is more convenient to take logarithms we denote by ψ_n the set of all vectors x for which there is a body T such that $x_A = \log |T_A|$ for all A. Bollobás and Thomason showed that ψ_n is contained in the polyhedral cone defined by the class of 'uniform cover inequalities'. Tan and Zeng conjectured that the convex hull $\mathrm{conv}(\psi_n)$ is equal to the cone given by the uniform cover inequalities.

We prove that this conjecture is nearly right: the closed convex hull $\overline{\mathrm{conv}}(\psi_n)$ is equal to the cone given by the uniform cover inequalities.

Joint work with Imre Leader and Zarko Randelovic.

Andrés David Santamaría-Galvis

UP - FAMNIT (Slovenia)

Partitioning the projective plane and the dunce hat

The faces of a simplicial complex induce a partial order by inclusion in a natural way. We say that the complex is partitionable if its poset can be partitioned into Boolean intervals, with a maximal face at the top of each.

In this work we show that all the triangulations of the real projective plane, the dunce hat, and the open Möbius strip are partitionable. To prove that, we introduce simple yet useful gluing tools that allow us to reduce the discussion about partitionability of a given complex in terms of smaller constituent relative subcomplexes. The gluing process generates partitioning schemes with a distinctive shelling-like flavor.

Benjamin Schröter

KTH Royal Institute of Technology

On Merino–Welsh conjecture for split matroids

Merino and Welsh conjectured that either the number of acyclic orientations or the number of totally cyclic orientations of a graph G is larger than its number of spanning trees. All three numbers are evaluations of the Tutte polynomial of G. Thus it is natural to extend the conjecture to matroids which are a generalization of graphs which allows a notion of Tutte polynomials. Furthermore, this conjecture was strengthened by Conde and Merino who posted an additive and multiplicative version of this conjecture. In this talk we discuss the above conjectures for the large case of split matroids.

This talks is based on joint work with Luis Ferroni

Tim Seynnaeve

University of Bern

Maximum likelihood degrees of diagonal linear covariance models

The linear covariance model associated to a linear space of symmetric matrices is the collection of all Gaussian probability distributions whose covariance matrix belongs to said linear space. If the space consists of diagonal matrices, we can associate to it a representable matroid. We show that the maximum likelihood degree of a diagonal linear covariance model is a matroid invariant, and express it in terms of the characteristic polynomial.

This talk is based on joint work with Christopher Eur, Tara Fife, and Jose Samper

Anna Slivková

University of Novi Sad

On the existence of m-morphic, σ-morphic and m-anisohedral tiles for some variations of the notion of tiling

A tile (a simply connected, closed and bounded region) in the Euclidean plane $\mathbb{E}^2$ is called *m-morphic* if it tiles the plane in exactly m noncongruent ways. It is an open question (asked in 1977 by Grünbaum and Shephard) whether for each positive integer m there exists an m-morphic tile (the largest possible m for which such a tile has been found is $m = 11$, by Myers). It is also unknown whether there exists a tile that tiles the plane in infinitely many ways, but only countably many (such a tile is called *σ-morphic*).

A somewhat related notion is the *isohedral number*. The isohedral number of a given tiling is defined as the number of different orbits into which the tiles are partitioned under the action of the symmetry group of the tiling. The isohedral number of a tile is the smallest possible isohedral number among all the possible tilings admitted by the considered tile. It is an open question (asked by Berglund in 1993) whether for each positive integer m there exists a tile whose isohedral number is m, also called an m-anisohedral tile (the largest possible m for which such a tile has been found is $m = 10$, also by Myers).

Although the mentioned problems are open in the stated formulations, some of them are solved for some variations of the notion of tiling. Such variations are: i) tilings in $\mathbb{E}^d$ for some d, $d \geq 3$; ii) tilings where tiles do not have to be connected; iii) tilings with sets of more tiles; iv) tilings where tiles have colored edges (which imposes some restrictions on how two tiles can be matched together). It is known that, in the case iii), m-morphic sets of tiles exist for each m (Harborth, 1977) as well as that a σ-morphic set of tiles exists (Schmitt, 1986). It is also known that tiles with isohedral number m exist for all m in all the cases i) (in $\mathbb{E}^d$ for any d, $d \geq 3$), ii) and iv) (all by Socolar, 2007).

That makes 5 out of 12 possible combinations solved. In this talk we present solutions for 6 out of the remaining 7 questions (where the questions posed in $\mathbb{E}^d$ are solved for all d, $d \geq 3$). All these questions are solved in the affirmative. The only problem left open is whether there exists a σ-morphic disconnected tile.

This is a joint work with B. Bašić and A. Džuklevski.

Liam Solus

KTH

Discrete Geometry in Causal Structure Learning

Causal inference, the problem of inferring cause-effect systems and estimating causal effects from data, is a fundamental problem in modern data science and artificial intelligence. A key step in the causal inference pipeline is to find a good solution to the problem of causal discovery, in which we want to learn a directed acyclic graph (DAG) representing the cause-effect relations amongst the variables in the data-generating distribution. The most commonly used algorithms for addressing this problem rely on combinatorial moves between DAGs that are used to greedily search for an optimal scoring system. An alternative approach treats causal discovery as a linear optimization problem. However, a lack of a complete H-representation for the feasible region of this program makes this approach difficult in general. In this talk, we will consider the lower-dimensional faces of this convex polytope and find that its edges encode all moves typically used by the popular greedy search algorithms. We suggest a push for a complete combinatorial characterization of the edges of this convex polytope as a means to more reliable greedy causal discovery algorithms that can out-perform the state-of-the-art. This talk is based on joint work with Svante Linusson and Petter Restadh.

Klara Stokes

Umeå University

Rigidity of rod configurations

A rod configuration is a geometric realization of a hypergraph in terms of points and line segments of Euclidean space, together with a notion of motion that treats the line segments as rigid bodies (rods). We give combinatorial criteria for when a rod configuration is rigid in the plane, thereby extending previous results by Whiteley (applicable only for a certain family of "independent" configurations), and by Jackson and Jordán (applicable only for configurations coming from 2-regular hypergraphs). This is joint work with Signe Lundqvist and Lars-Daniel Öhman.

Kristijan Tabak

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Normalized difference sets tiling - generalizations

A difference set tiling of an abelian group G is a collection of (v, k, λ) difference sets $D_i,\ i \in I$, which also make a partition of $G \setminus \{1\}$. It was conjectured by Ćustić, Krčadinac and Zhou that in such case $\prod_{g \in D_i} g = 1$ for all $i \in I$ (normalized difference set). This was named a 'normalized tiling conjecture' or NTC. Using the character theory it has been recently proved that NTC is true if v is odd and difference set has a multiplier. In this talk we shall present some generalizations of results on NTC.

Lorenzo Venturello

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Gorenstein algebras from simplicial complexes

Gorenstein algebras form an intriguing class of objects which often show up in combinatorics and geometry. In this talk I will present a construction which associates to every pure simplicial complex a standard graded Gorenstein algebra. We describe a combinatorial presentation of this algebra as a polynomial ring modulo an ideal generated by monomials and pure binomials. When the simplicial complex is flag, i.e., it is the clique complex of its graph, our main results establish equivalences between well studied properties of the complex (being S_2, Cohen-Macaulay, Shellable) with those of the algebra (being quadratic, Koszul, having a quadratic GB). Finally, we study the h-vector of the Gorenstein algebras in our construction and answer a question of Peeva and Stillman by showing that it is very often not gamma-positive. This is joint work with Alessio D'Alì.

Zelealem Yilma

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The minimum number of spanning trees in regular multigraphs

In a recent article, Bogdanowicz determines the minimum number of spanning trees a connected cubic multigraph on a fixed number of vertices can have and identifies the unique graph that attains this minimum value. He conjectures that a generalized form of this construction, which we here call a *padded paddle graph*, would be extremal for d-regular multigraphs where $d \geq 5$ is odd.

We prove that, indeed, the padded paddle minimises the number of spanning trees, but this is true only when the number of vertices, n, is greater than $\frac{9d+6}{8}$. We show that a different graph, which we call the *padded cycle*, is optimal for $n < \frac{9d+6}{8}$. This fully determines the d-regular multi-graphs minimising the number of spanning trees for odd values of d.

The main tools we use are mathematical induction, concavity of the determinant, and a *lifting* operation similar to one used by Ok and Thomassen. We employ this approach to also consider and completely solve the even-degree case. Here, the parity of n plays a major role and we show that, apart from a handful of irregular cases when both d and n are small, the unique extremal graphs are padded cycles when n is even and a different family, which we call *fish graphs*, when n is odd.