Nonequilibrium materials science
with a twist

Michael Sentef

Joint Theory Colloquium, DESY & Uni Hamburg

Hamburg, October 16, 2019

Funded through DFG Emmy Noether Programme (SE 2558/2-1)

Max Planck Institute for the Structure and Dynamics of Matter

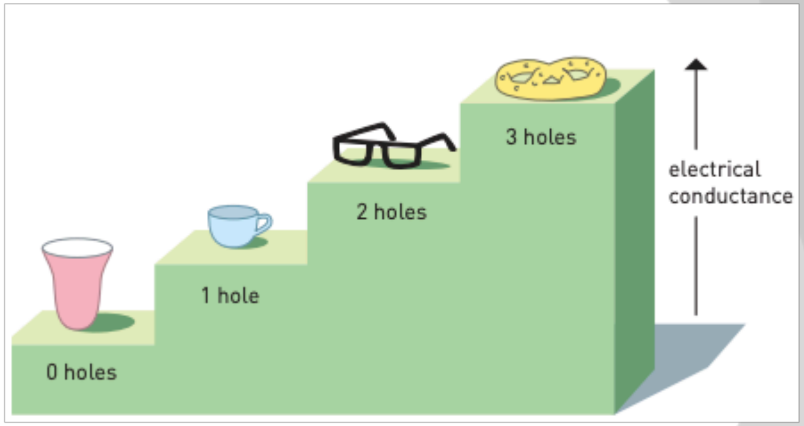
Unifying themes in physics

mpsd

Physics Nobel Prize 2019

universe = coffee mug

Physics Nobel Prize 2016



material = coffee mug

THE NOBEL PRIZE IN PHYSICS 2016, THE ROYAL SWEDISH ACADEMY OF SCIENCES, HTTP://KVA.SE

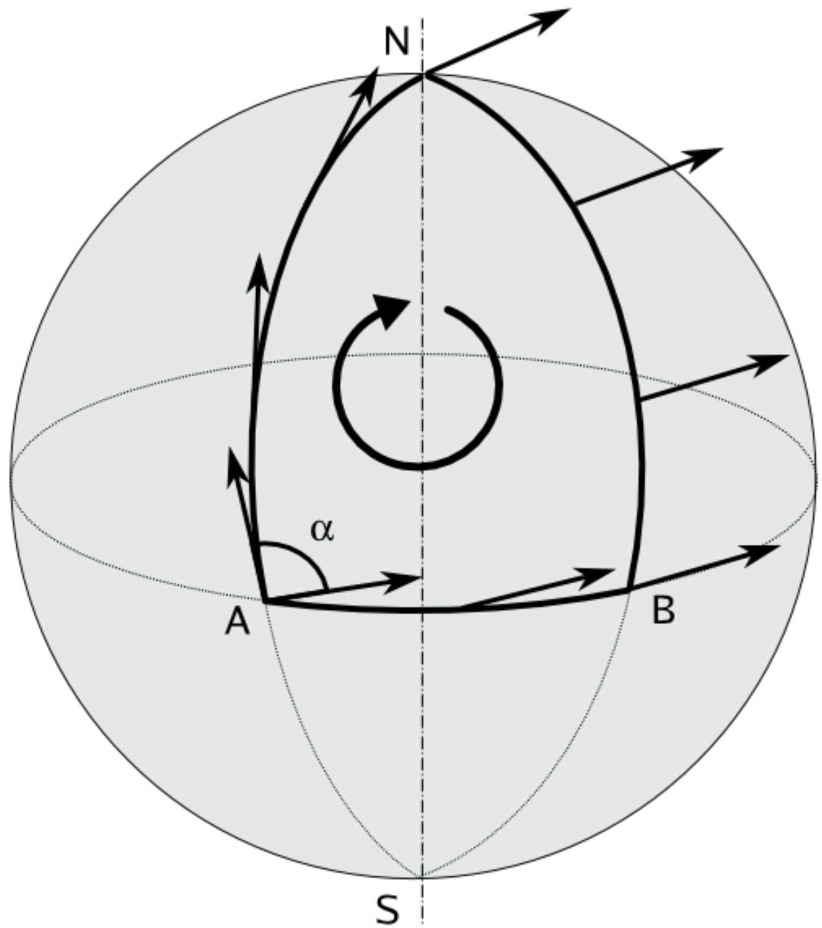
Why material = coffee mug?
Can we use light to change topology of a material?

Outline

1. Topology in materials
2. Floquet states
3. Light-induced Hall effect in graphene (2D Dirac)
4. Optical control of Majoranas (2D chiral superconductor)

① Topological states of matter

Global Change without Local Change *illustrates Berry's Phase*



Topological states of matter

$$H(R(t))|\psi(t)\rangle = i\hbar\frac{\partial}{\partial t}|\psi(t)\rangle$$

M. V. Berry, Proc. R. Soc. A 392, 45 (1984)

$$H(R(t))|n(R(t))\rangle = E_n(R(t))|n(R(t))\rangle$$

$$|\psi(0)\rangle = |n(R(0))\rangle$$

Start system in the n^{th} eigenstate

$$|\psi(t)\rangle = e^{i\phi_n}|n(R(t))\rangle$$

Adiabatic theorem tells us that we stay in the n^{th} eigenstate, but we can pick up a phase that does not affect the physical state.

$$\theta_n(t) = -\frac{1}{\hbar}\int_0^t E_n(t')dt'$$

$$\phi_n(t) = \theta_n(t) + \gamma_n(t)$$

Dynamical phase, but an additional phase is also allowed (this is called the Berry phase γ).

Topological states of matter

mpsd

$$|\psi(t)\rangle = e^{i\phi_n}|n(R(t))\rangle$$

$$\phi_n(t) = \theta_n(t) + \gamma_n(t)$$

$$H(R(t))|\psi(t)\rangle = i\hbar\frac{\partial}{\partial t}|\psi(t)\rangle$$

$$\frac{\partial}{\partial t}|n(R)\rangle + i\frac{d}{dt}\gamma_n(t)|n(R)\rangle = 0$$

Equation for Berry's phase

$$\frac{d}{dt}\gamma_n(t) = i\langle n(R)|\frac{\partial}{\partial t}|n(R)\rangle$$

Operate with bra on l.h.s.

$$\frac{d}{dt}\gamma_n(t) = i\langle n(R)|\nabla_R|n(R)\rangle\frac{dR}{dt}$$

$$\gamma_n(t) = i\int_{R_i}^{R_f}\langle n(R)|\nabla_R|n(R)\rangle dR$$

Dynamical phase, but an additional phase is also allowed (this is called the **Berry phase** γ).

Topological states of matter

mpsd

$$\gamma_n(t) = i \int_{R_i}^{R_f} \langle n(R) | \nabla_R | n(R) \rangle dR$$

If we now consider cyclic evolutions around a closed circuit C in a time T such that $R(0) = R(T)$ then the Berry phase looks like the following

$$\gamma_n(C) = i \oint_C \langle n(R) | \nabla_R | n(R) \rangle dR$$

Berry phase, related to changes of the eigenstate when moved along path in parameter space.

$$\nabla_R \langle n | n \rangle = 0$$

$$\langle \nabla_R n | n \rangle + \langle n | \nabla_R n \rangle = \langle n | \nabla_R n \rangle^* + \langle n | \nabla_R n \rangle = 0$$

$$2 \cdot \Re e \langle n | \nabla_R n \rangle = 0$$

Berry phase is real.

Topological states of matter

Berry connection as a gauge potential.

$$\gamma_n(C) = \oint_C A_n dR$$

$$A_n(R) = i\langle n(R)|\nabla_R|n(R)\rangle$$

$$|n(R)\rangle \rightarrow |n(R)\rangle' = e^{i\xi_n(R)}|n(R)\rangle$$

Under gauge transformation.

$$A_n(R) \rightarrow A'_n(R) = A_n(R) - \nabla_R \xi_n(R)$$

$$\gamma_n(R) \rightarrow \gamma'_n(R) = \gamma_n(R)$$

Gives no change to the Berry phase.

Berry phase is gauge invariant and can be measured, e.g. Aharonov-Bohm effect.

Topological states of matter

C. Kane, "Topological band theory and the Z2 invariant", Chapter 1 in "Topological Insulators", Elsevier (2013)

mpsd

Topological band theory of solids

$$H(\mathbf{k}) = e^{i\mathbf{k}\cdot\mathbf{r}} H e^{-i\mathbf{k}\cdot\mathbf{r}}$$

eigenvalues $E_n(\mathbf{k})$ and eigenvectors $|u_n(\mathbf{k})\rangle$

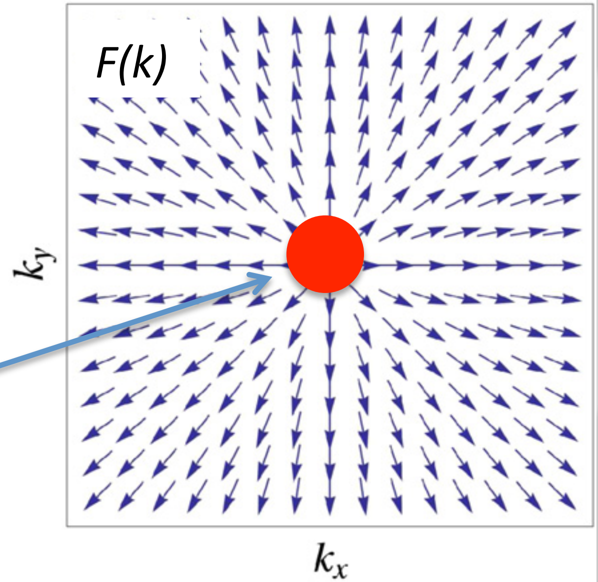
Bloch state under gauge transformation $|u(\mathbf{k})\rangle \to e^{i\phi(\mathbf{k})}|u(\mathbf{k})\rangle$

Berry connection $\mathbf{A} = -i\langle u(\mathbf{k})|\nabla_{\mathbf{k}}|u(\mathbf{k})\rangle \longrightarrow \mathbf{A} \to \mathbf{A} + \nabla_{\mathbf{k}}\phi(\mathbf{k})$

Berry phase $\gamma_C = \oint_C \mathbf{A}\cdot d\mathbf{k} = \int_S \mathcal{F} d^2\mathbf{k}$

$\mathcal{F} = \nabla \times \mathbf{A}$ defines the Berry curvature

closed surface S $\quad n = \frac{1}{2\pi}\int_S \mathcal{F} d^2\mathbf{k}$



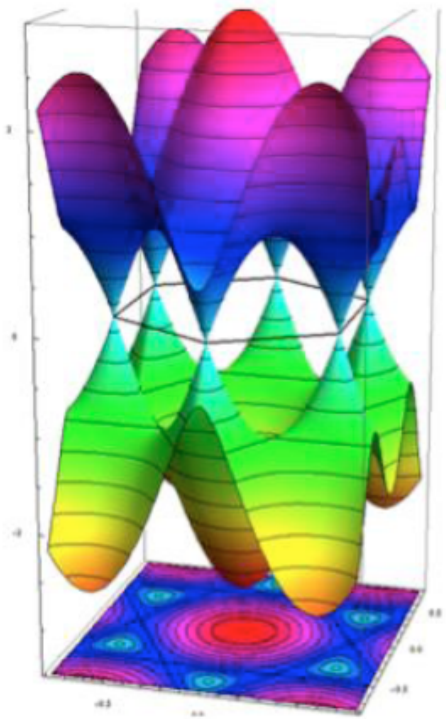
Chern number = topological invariant
= number of Dirac monopoles inside the surface

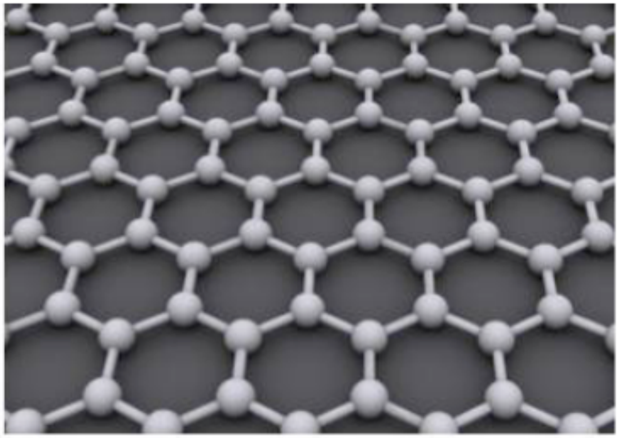
Topological states of matter

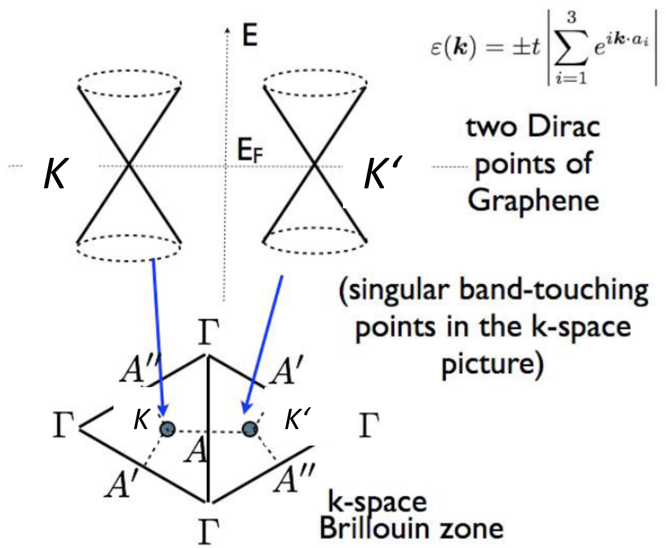
mpsd

2D Graphene:

- Dirac points (2 valleys)

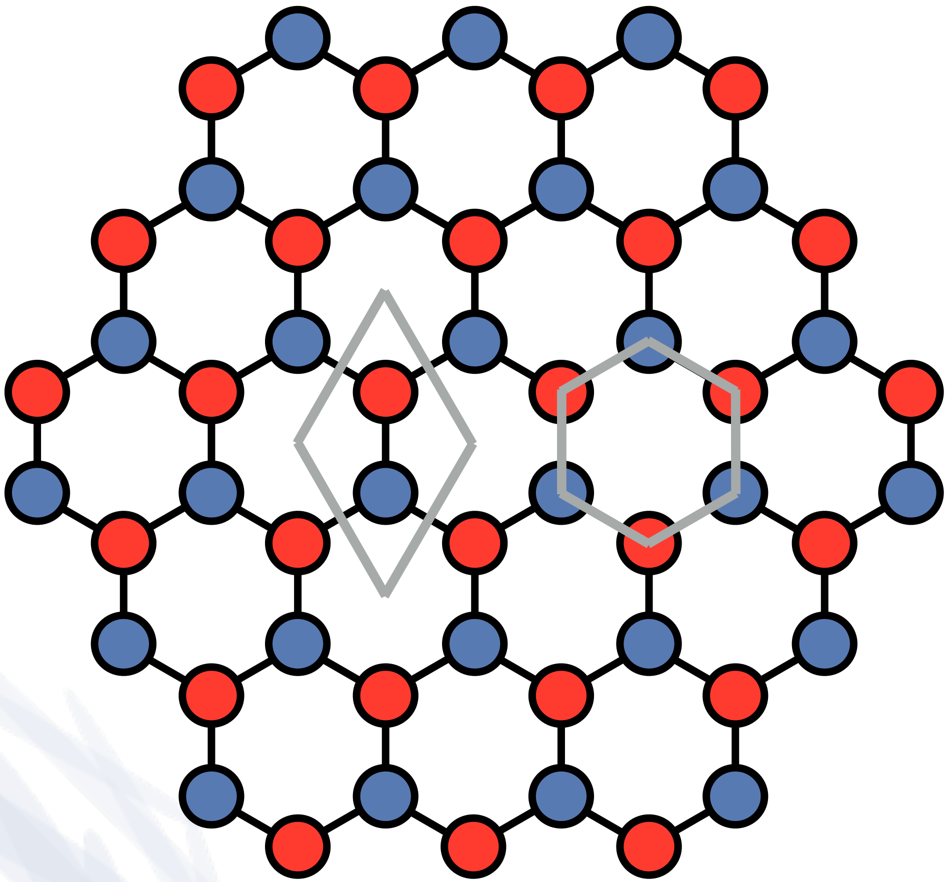






Honeycomb lattice

Two triangular sublattices bonded together



Two atoms in unit cell:

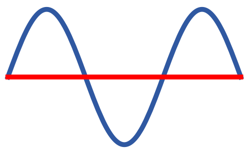
A B

Electrons always in a superposition of A- and B-sublattice states

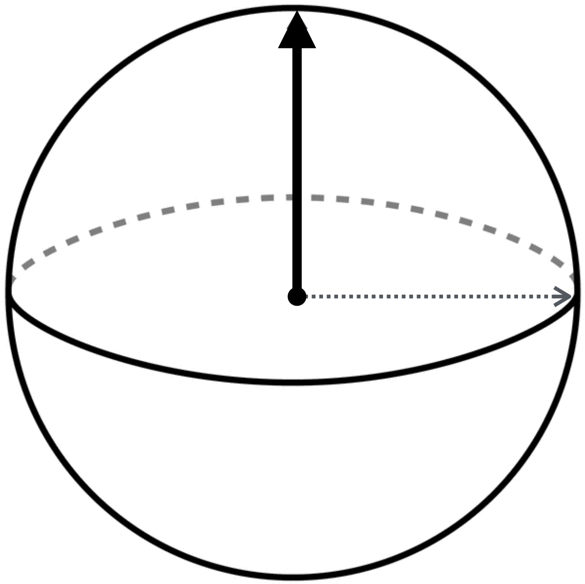
Bloch sphere

Visualizes states in a two-level system

$|A\rangle$

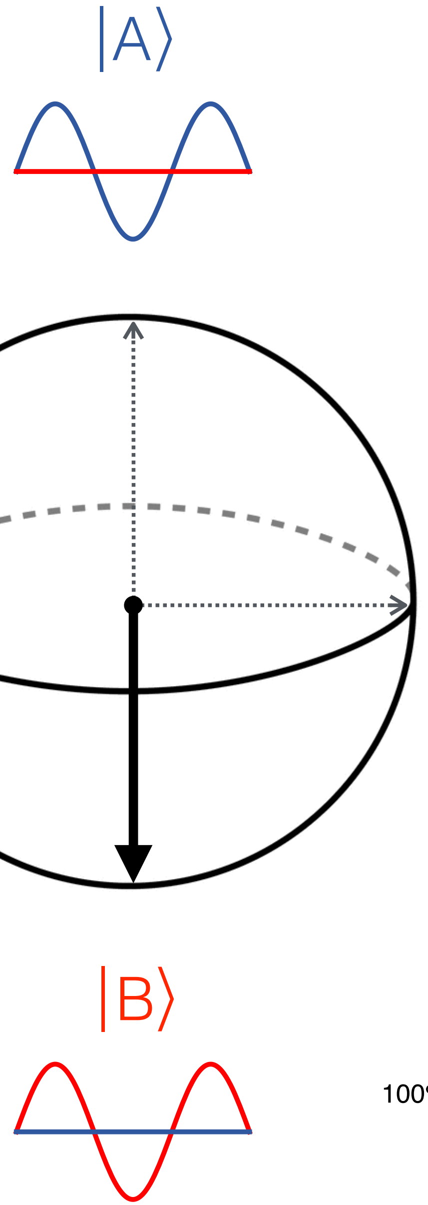


100% A-sublattice character



Bloch sphere

Visualizes states in a two-level system

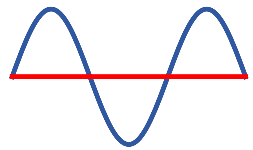


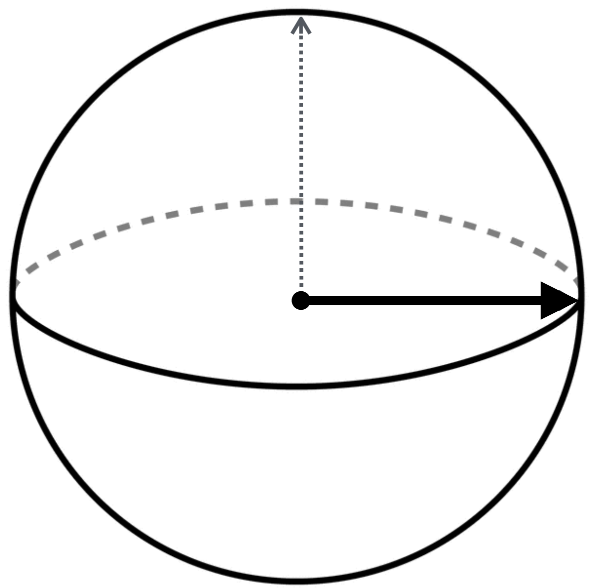
100% B-sublattice character

Bloch sphere

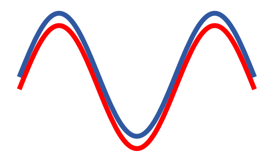
Visualizes states in a two-level system

$|A\rangle$



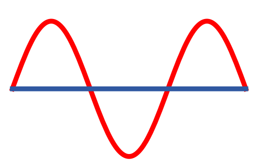


$|A\rangle + e^{i\varphi}|B\rangle$



On the equator we have equal amplitudes on the A and B sublattices, but there can be a phase difference between them

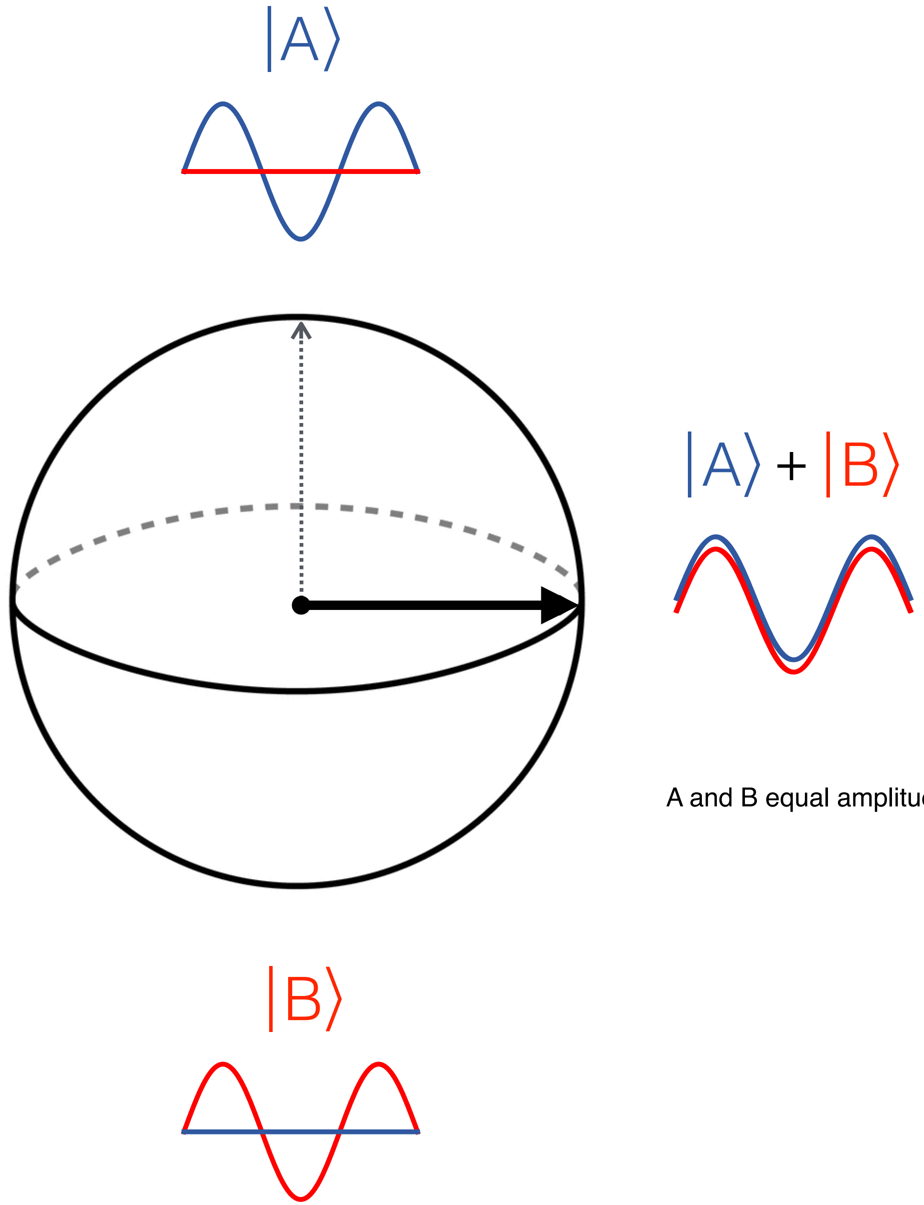
$|B\rangle$



James McIver

Bloch sphere

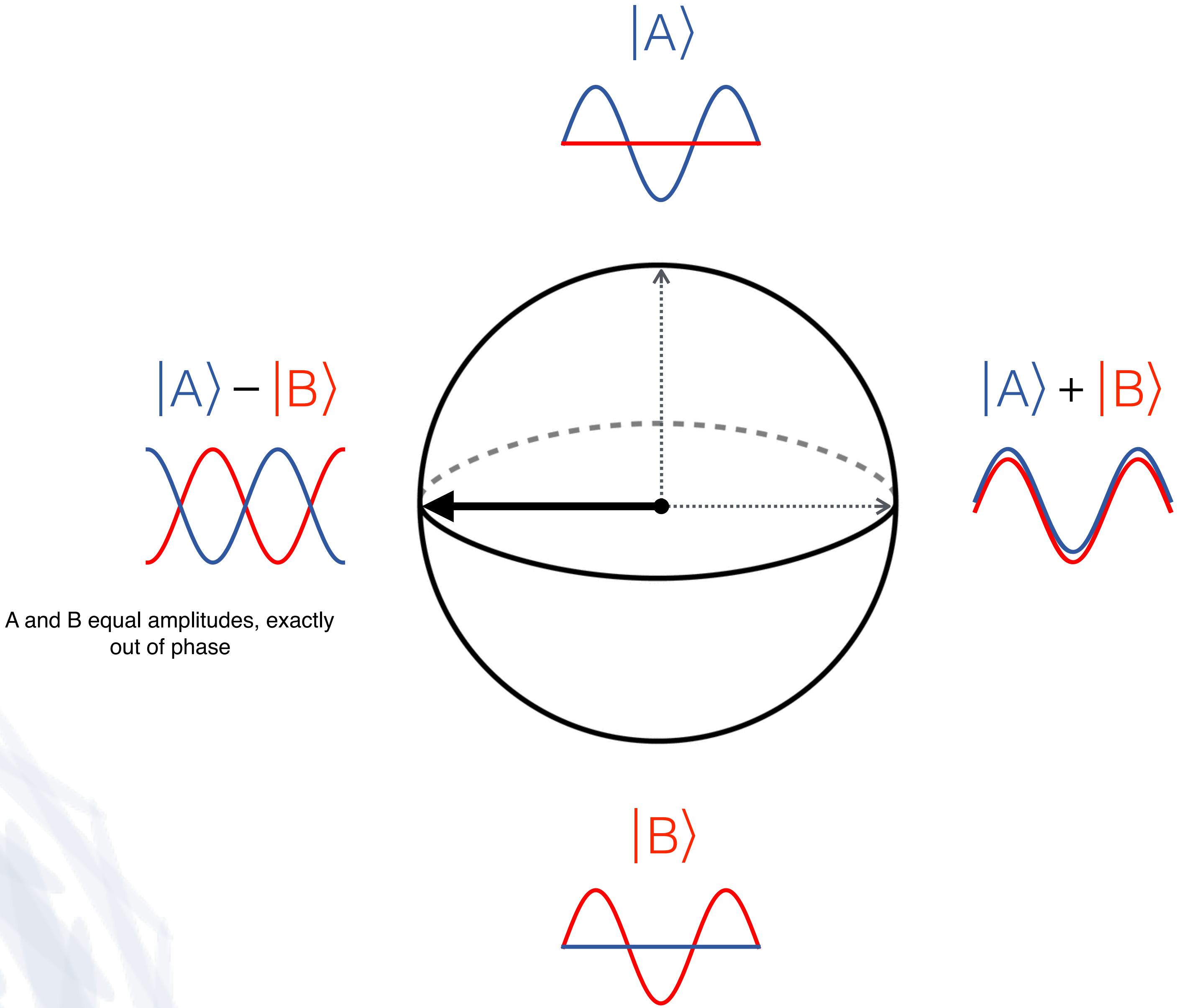
Visualizes states in a two-level system



A and B equal amplitudes, in phase

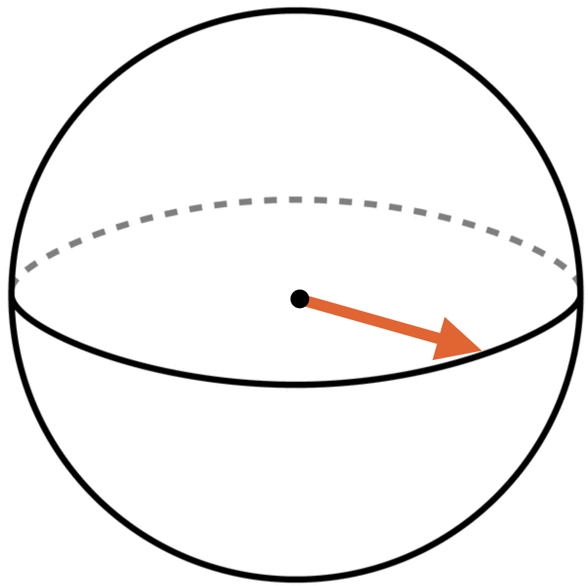
Bloch sphere

Visualizes states in a two-level system



Honeycomb lattice wavefunctions

$$\left|\psi_k\right\rangle =$$



$$\cdot\; e^{i\,\boldsymbol{k}\cdot\boldsymbol{r}}$$

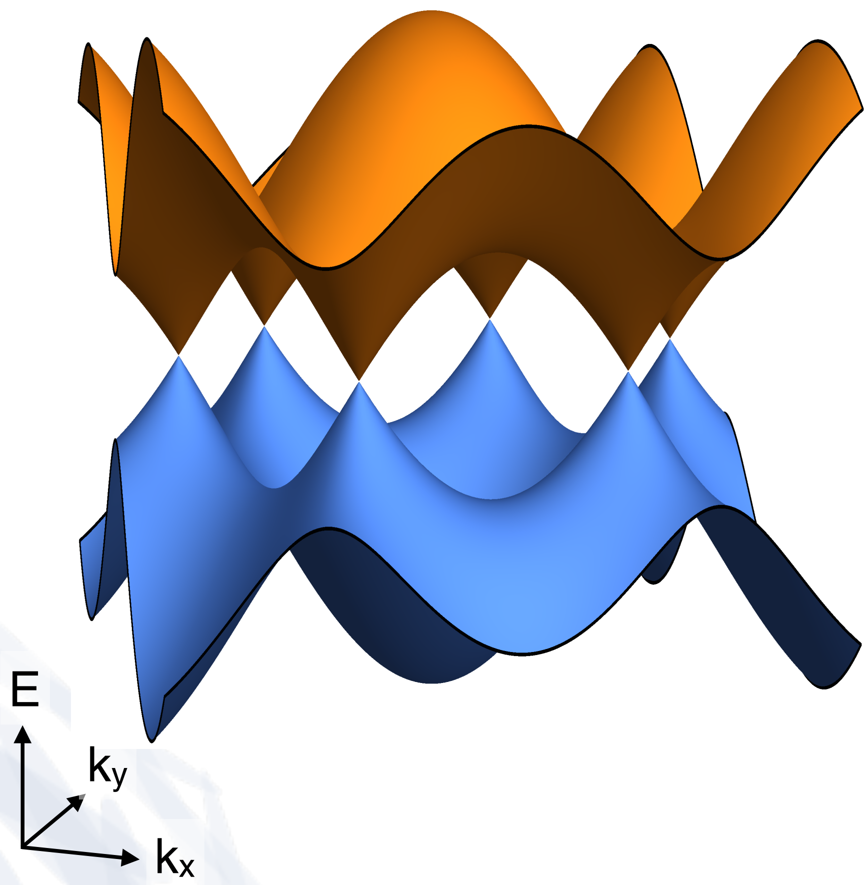
Pseudospin

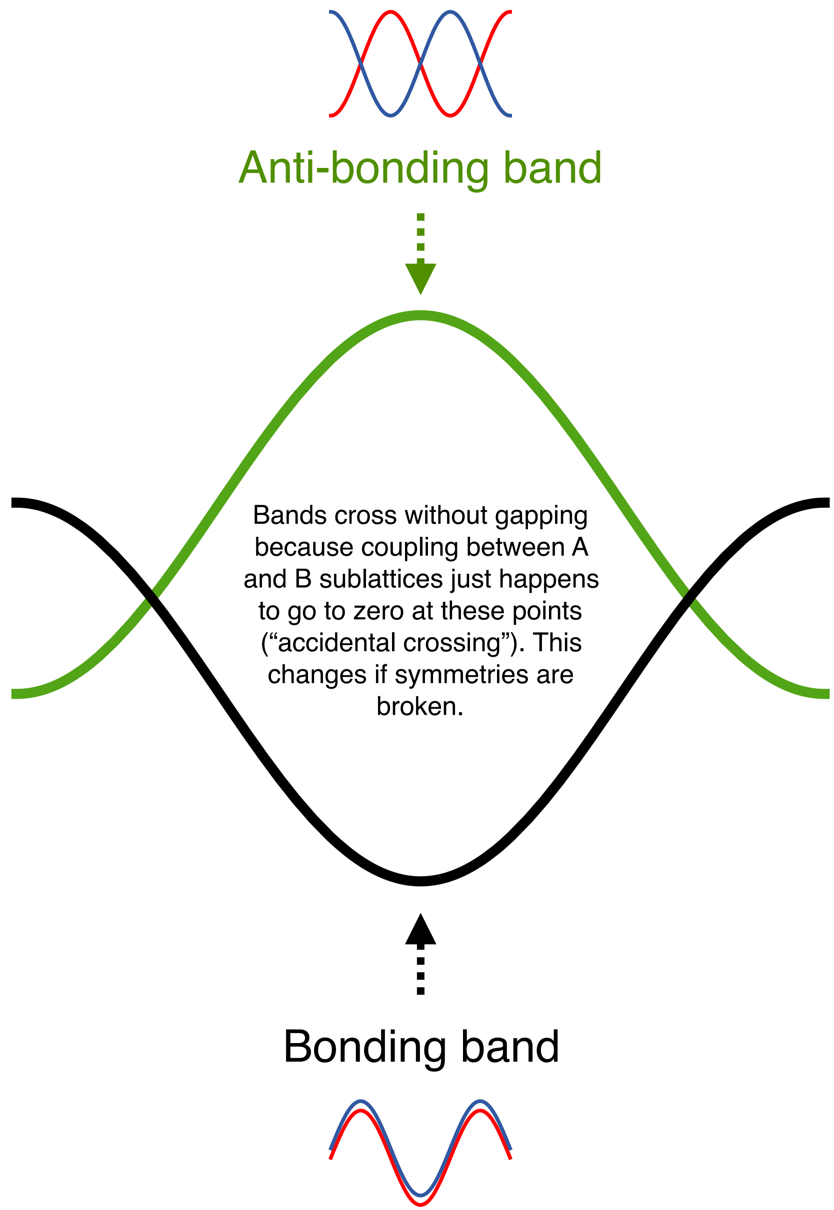
Shows how the states on the A- and B-sublattices superpose

In Graphene, electrons are always equally distributed between the identical A and B sublattices. This means that the pseudospin always lies on the equator of the Bloch sphere, indicating that the A and B sublattice states have equal amplitude, but there can be a phase difference between them.

Graphene: electronic structure

Identical A- and B-sublattices made of carbon

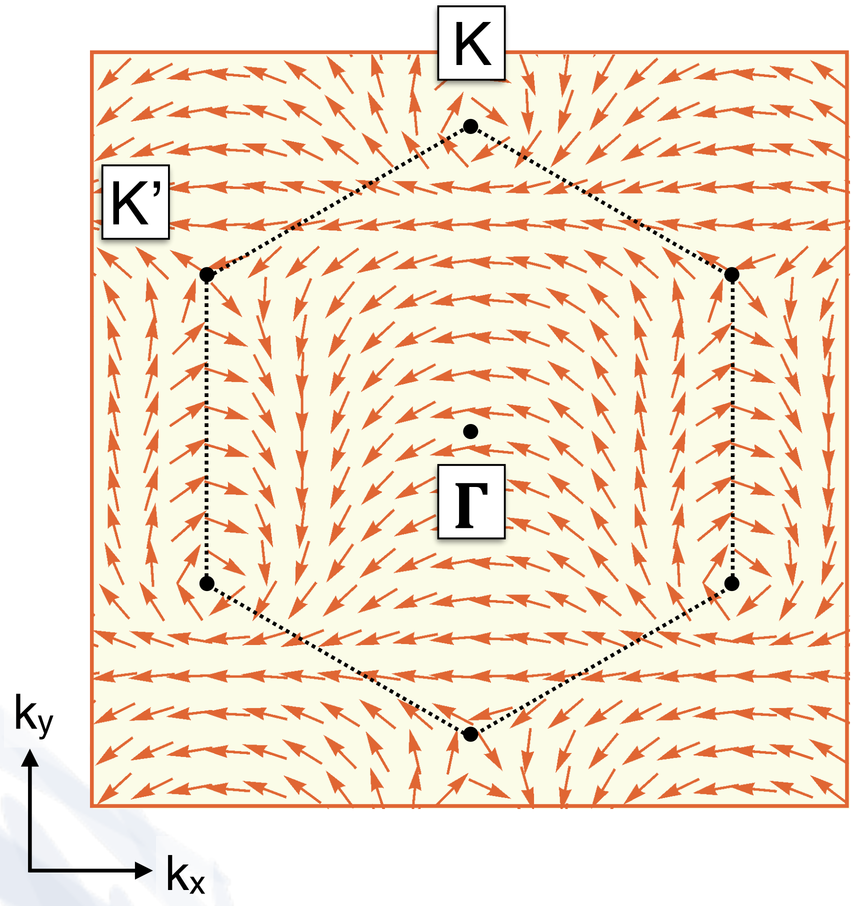


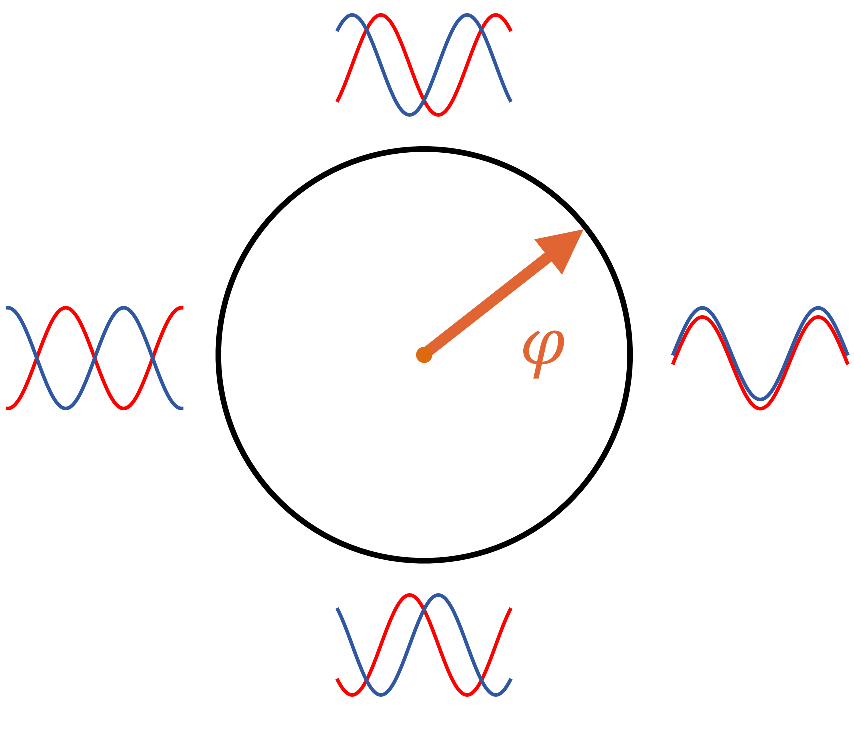


P.R. Wallace (1947)

Graphene: pseudospin texture

Graphene conduction band

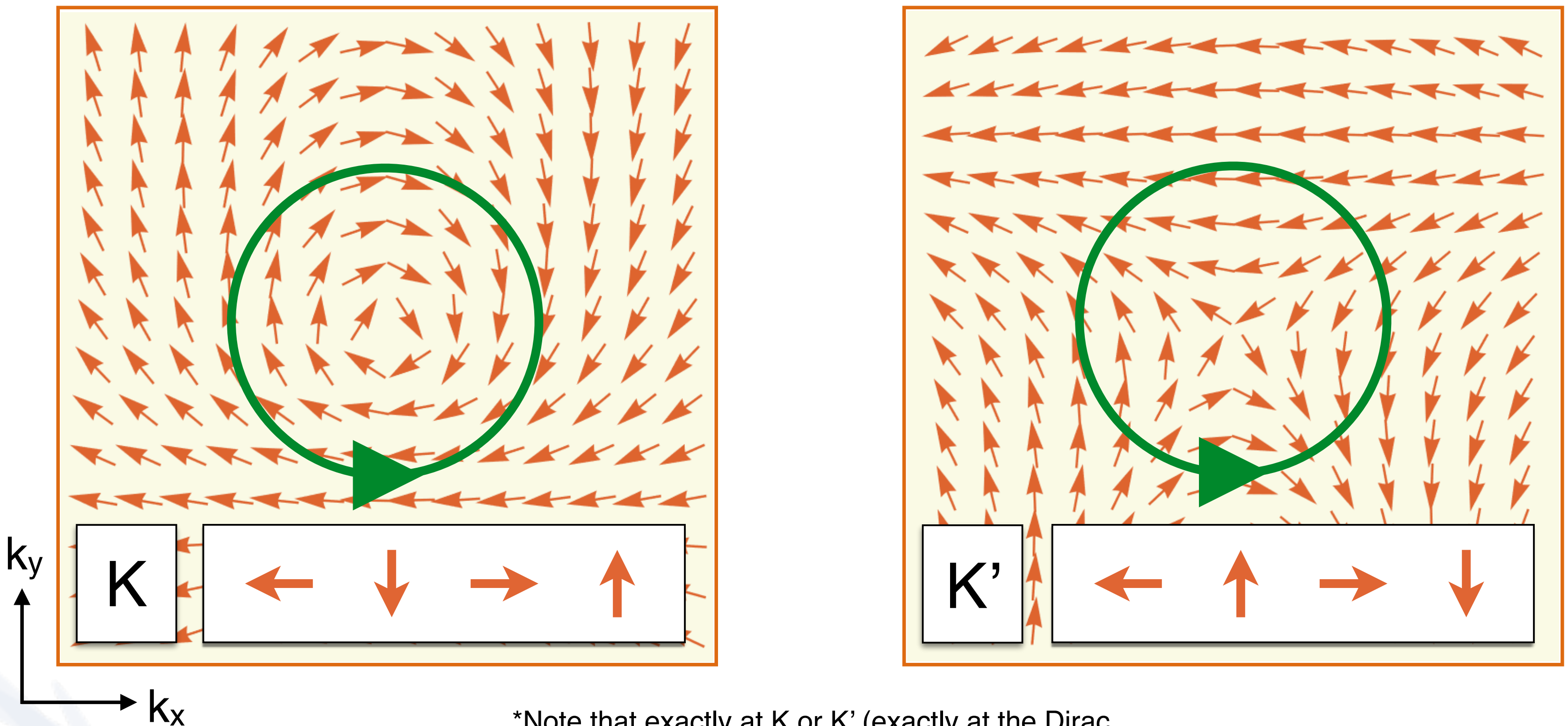




Bloch sphere equator

Chirality

The difference between K and K’



*Note that <u>exactly</u> at K or K’ (exactly at the Dirac point), the pseudospin vector is undefined

Eigenstates’ phase winds in opposite directions at K and K’

Topological states of matter

mpsd

Dirac fermions in pseudospin representation: Decompose into Pauli matrices

$$H(K+q) = \begin{pmatrix} m_K & q_x + iq_y \\ q_x - iq_y & -m_K \end{pmatrix}$$

$$= p_x\sigma_x + p_y\sigma_y + p_z\sigma_z$$

$p_x = q_x$
$p_y = q_y$
$p_z = m_K$

Pseudospin winding <-> Berry phase
Berry phase on a closed loop around Dirac point is quantized = +/- π
+/- sign depends on sign of mass term m_K

+/- ½ Dirac monopole

Chern number C = sum of Dirac monopoles in the Brillouin zone
Distinguishes trivial from nontrivial (topological) insulators

C=0 C≠0

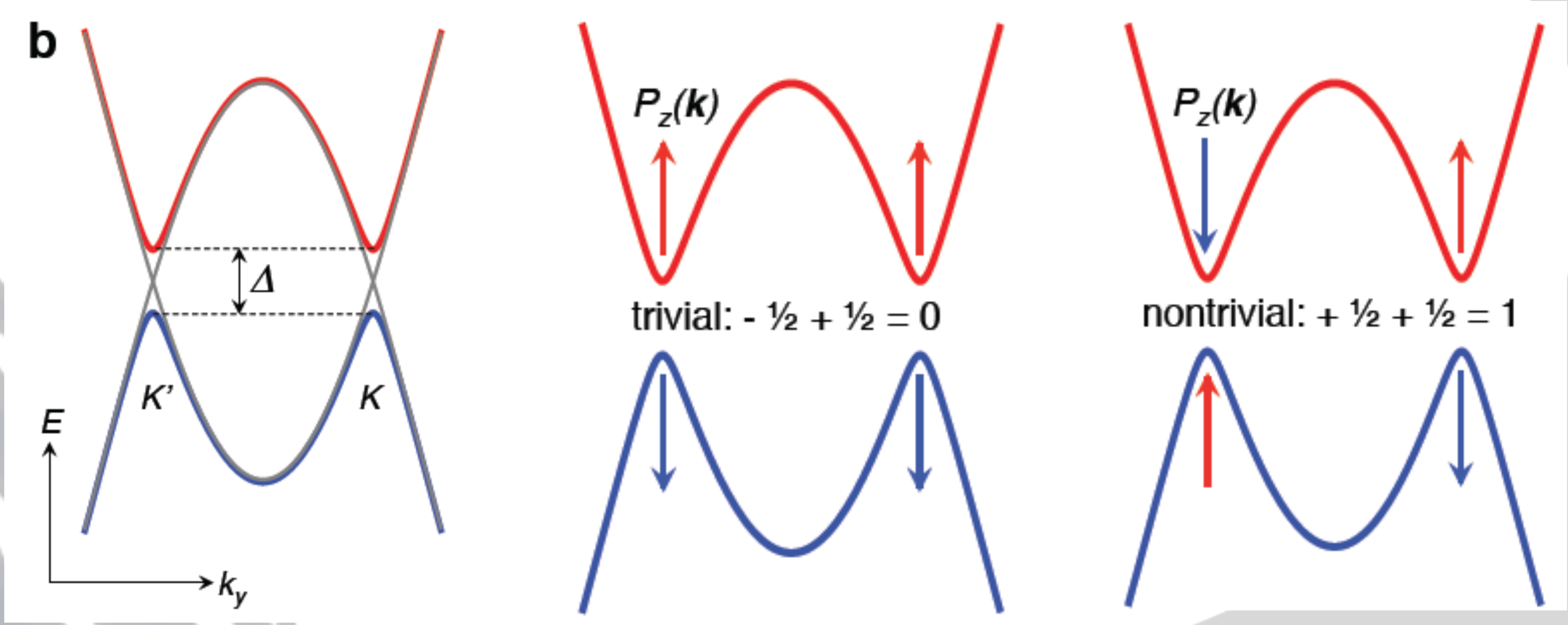
Topological states of matter

$$H(K'+q) = \begin{pmatrix} \boxed{m_{K'}} & q_x \boxed{-} iq_y \\ q_x + iq_y & -m_{K'} \end{pmatrix} \qquad H(K+q) = \begin{pmatrix} \boxed{m_K} & q_x \boxed{+} iq_y \\ q_x - iq_y & -m_K \end{pmatrix}$$

***K* vs. *K'*: opposite winding of in-plane pseudospin**

***m*$_K$ = *m*$_{K'}$**
trivial insulator

***m*$_K$ = − *m*$_{K'}$**
nontrivial insulator



M. Sentef et al., Nat. Comm. 6, 7047 (2015)

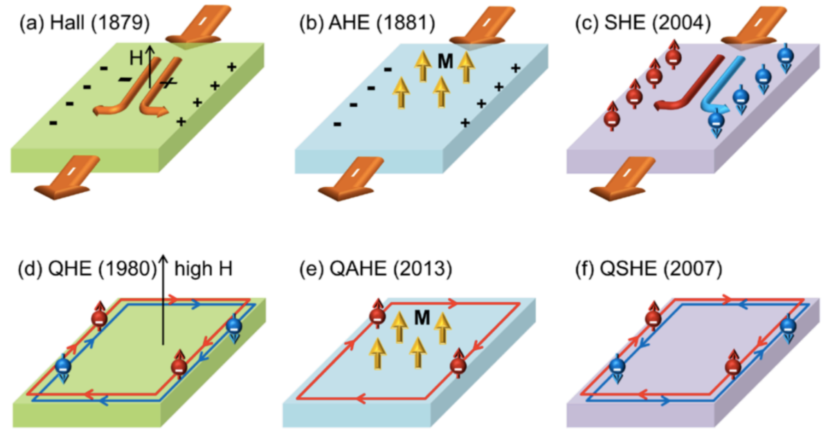
Chern number and quantum Hall effect

$J_{Hall} = \sigma_{Hall} E_{DC}$

Hall conductance $\sigma_{Hall} = C\ e^2/h$

C = Chern number

= Berry curvature integrated over occupied states



resonancescience.org

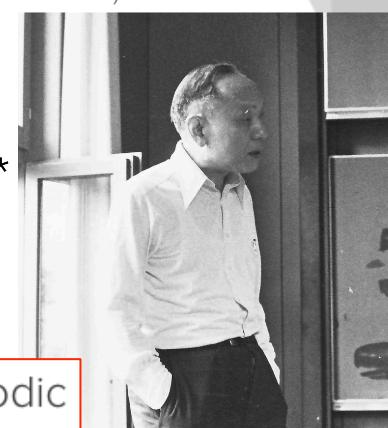
(bulk-boundary correspondence: C=#edge channels)

Fair Use Images / Wikipedia

„Kubo = Chern“

Japanese physicist = Chinese mathematician*

*quote by Shou-Cheng Zhang



Oberwolfach Photo Collection / Wikipedia

(c) Nobel Media AB, Photo: A. Mahmoud

Quantized Hall Conductance in a Two-Dimensional Periodic Potential

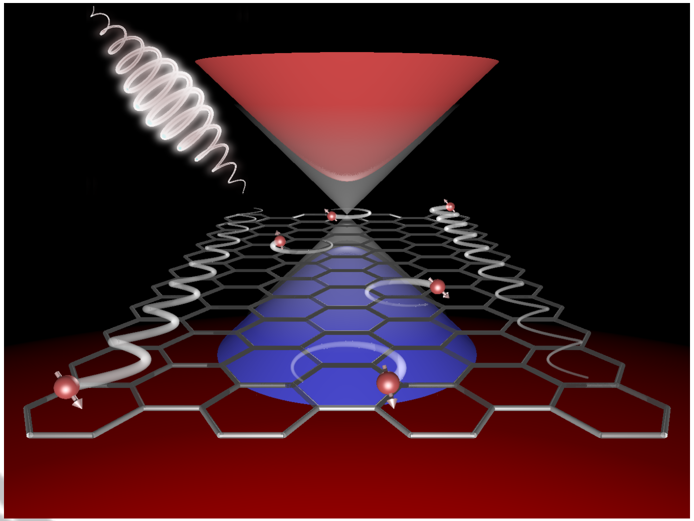
D. J. Thouless, M. Kohmoto, M. P. Nightingale, and M. den Nijs

Phys. Rev. Lett. **49**, 405 – Published 9 August 1982

Physics See Focus story: Nobel Prize—Topological Phases of Matter

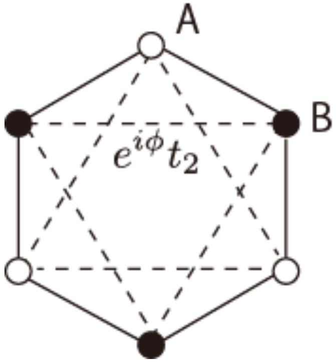
② Floquet topological states

Graphene + circularly polarized light (breaks trs)

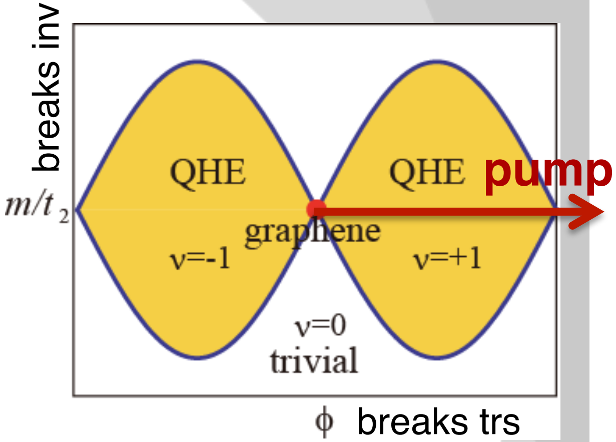


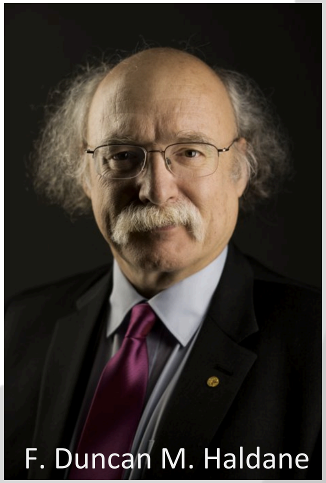
?

Haldane model (PRL 61, 2015 (1988))



Local flux ϕ
Staggered field *m*
Fictitious fields!

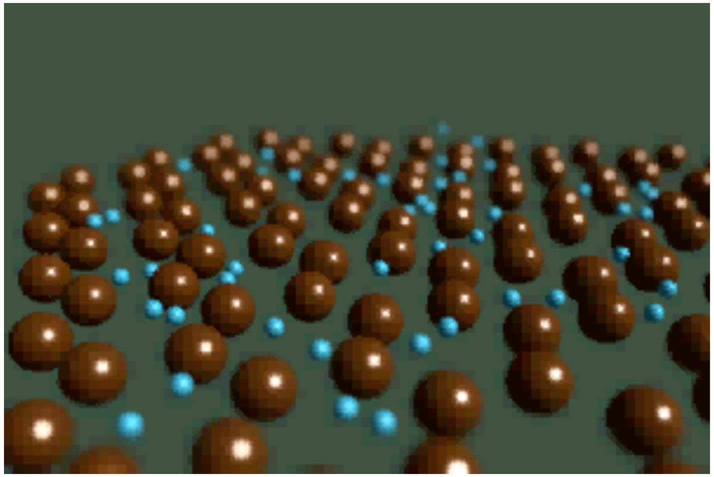




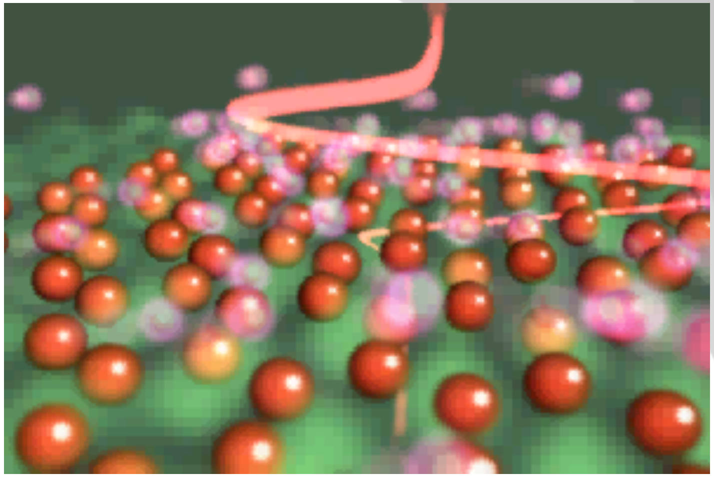
Artistic view of Floquet states

mpsd

by Koichiro Tanaka (Kyoto university)



electrons in solids



Floquet state (photo-dressed state)

$$H$$

$$H_{\text{eff}}$$

$$H_{\text{eff}} = H_0 + \frac{[H_{-1}, H_1]}{\Omega} + \mathcal{O}(\Omega^{-2})$$

Floquet states of matter

mpsd

time periodic system

$$i\partial_t \psi = H(t)\psi \qquad H(t) = H(t+T) \qquad \Omega = 2\pi/T$$

"Floquet mapping"

=discrete Fourier trans.

$$\Psi(t) = e^{-i\varepsilon t} \sum_m \phi^m e^{-im\Omega t}$$

Floquet Hamiltonian (static eigenvalue problem)

$$\sum_{m=-\infty}^{\infty} \mathcal{H}^{mn} \phi_\alpha^m = \varepsilon_\alpha \phi_\alpha^n$$

ε: Floquet quasi-energy

$$(\mathcal{H})^{mn} = \frac{1}{T} \int_0^T dt H(t) e^{i(m-n)\Omega t} \boxed{+ m\delta_{mn}\Omega I}$$

comes from the $i\partial_t$ term

$$H_m = \mathcal{H}^{m0}$$

~ absorption of m "photons"

Floquet states of matter

mpsd

Time-periodic quantum system = Floquet theory (exact) ~ effective theory

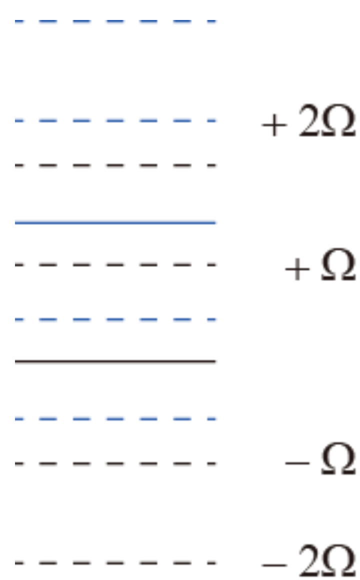
$$i\partial_t\psi = H(t)\psi$$

$$H(t) = H(t+T)$$

two states + periodic driving

$$\mathcal{H}\phi = \varepsilon\phi$$

Floquet theory



n-photon dressed state
Floquet side bands

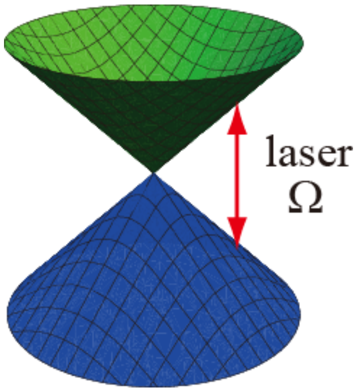
$$H_{\text{eff}} = H_0 + \frac{[H_{-1}, H_1]}{\Omega} + \mathcal{O}(\Omega^{-2})$$

Fictitious fields!

projection to the original Hilbert space

Hilbert space size = original system

Dirac fermion + circularly polarized laser



coupling to AC field

$$\boldsymbol{k} \longrightarrow \boldsymbol{k} + \boldsymbol{A}(t)$$

$$k = k_x + ik_y$$
$$\boldsymbol{A}(t) = (F/\Omega \cos \Omega t,\, F/\Omega \sin \Omega t)$$
$$A = F/\Omega$$

time dependent Schrödinger equation

$$i\partial_t \psi_k = \begin{pmatrix} 0 & k + Ae^{i\Omega t} \\ \bar{k} + Ae^{-i\Omega t} & 0 \end{pmatrix} \psi_k$$

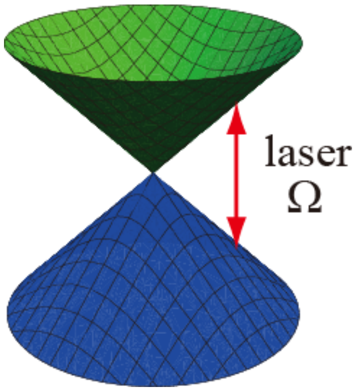
Floquet theory

$$(\mathcal{H})^{mn} = \frac{1}{T}\int_0^T dt H(t) e^{i(m-n)\Omega t} + m\delta_{mn}\Omega I$$

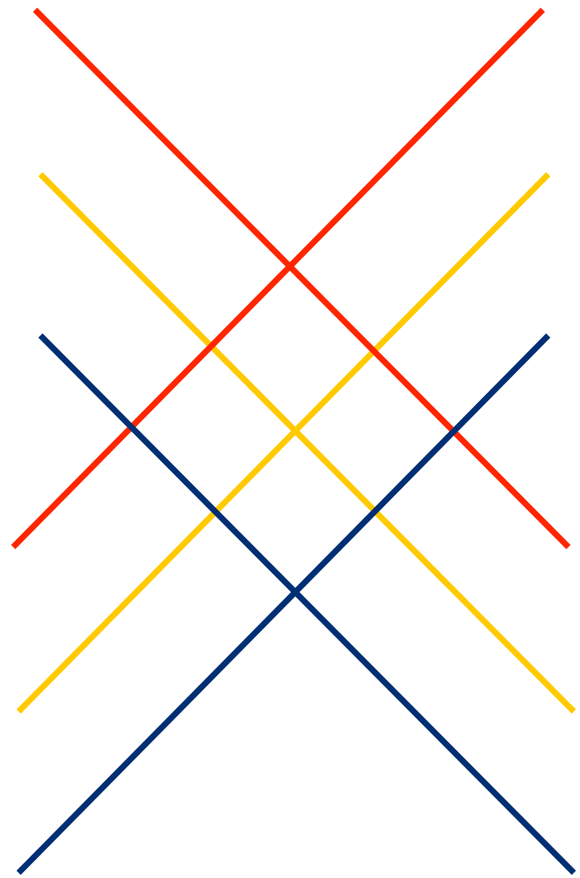
$$H^{\text{Floquet}} = \begin{pmatrix} \Omega & k & 0 & A & 0 & 0 \\ \bar{k} & \Omega & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & k & 0 & A \\ A & 0 & \bar{k} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\Omega & k \\ 0 & 0 & A & 0 & \bar{k} & -\Omega \end{pmatrix}$$

truncated at m=0,+1, -1 for display

Dirac fermion + circularly polarized laser



$$H^{\text{Floquet}} = \begin{pmatrix} \Omega & k & 0 & A & 0 & 0 \\ \bar{k} & \Omega & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & k & 0 & A \\ A & 0 & \bar{k} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\Omega & k \\ 0 & 0 & A & 0 & \bar{k} & -\Omega \end{pmatrix}$$

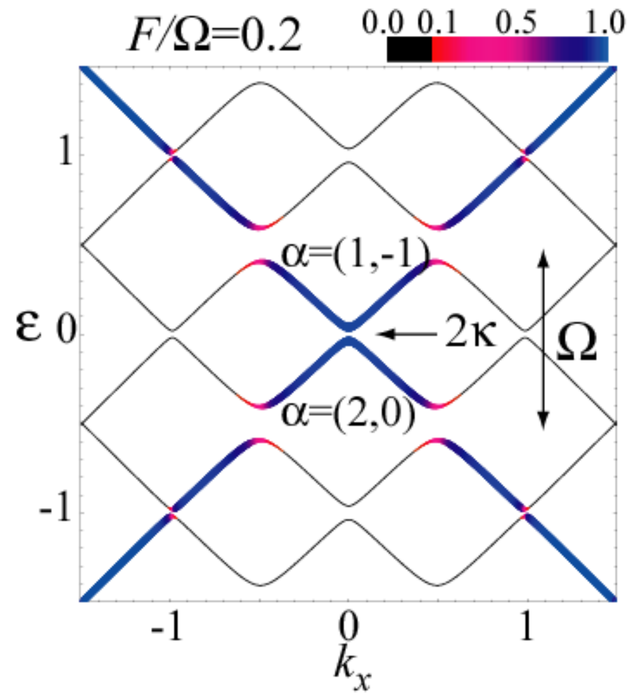


Dirac fermion + circularly polarized laser

Mass term =
synthetic field stemming from a real time-dependent field A(t)

$$\kappa = \frac{\sqrt{4A^2+\Omega^2}-\Omega}{2} \sim A^2/\Omega$$

$$H^{\mathrm{Floquet}} = \begin{pmatrix} \Omega & k & 0 & A & 0 & 0 \\ \bar{k} & \Omega & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & k & 0 & A \\ A & 0 & \bar{k} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\Omega & k \\ 0 & 0 & A & 0 & \bar{k} & -\Omega \end{pmatrix}$$



1-photon absorbed state

0-photon absorbed state

-1-photon absorbed state

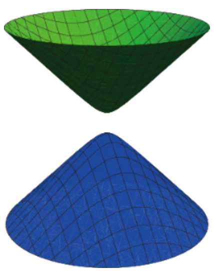
Oka and Aoki,
PRB 79, 081406 (2009)

Dirac fermion + circularly polarized laser

Projection to the original Hilbert space

$$H^{\text{Floquet}} = \begin{pmatrix} \Omega & k & 0 & A & 0 & 0 \\ \bar{k} & \Omega & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & k & 0 & A \\ A & 0 & \bar{k} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\Omega & k \\ 0 & 0 & A & 0 & \bar{k} & -\Omega \end{pmatrix}$$

near Dirac point



2κ

2nd order perturbation

$$H_{\text{eff}} = H_0 + \boxed{\frac{[\overset{\sim A\sigma_-}{\boxed{H_{-1}}}, \overset{\sim A\sigma_+}{\boxed{H_1}}]}{\Omega}} + \mathcal{O}(A^4)$$

Dynamical gap

$$\kappa = \frac{\sqrt{4A^2 + \Omega^2} - \Omega}{2} \sim A^2/\Omega$$

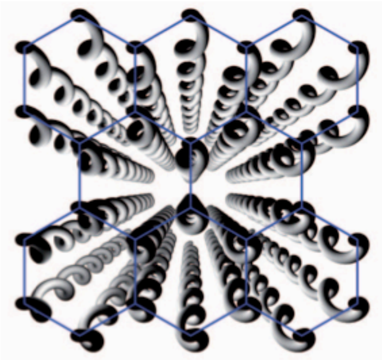
Mass term =
synthetic field stemming from a
real time-dependent field A(t)

$$\sim v(k_x\sigma_y - \tau_z k_y \sigma_x) \boxed{\pm \tau_z \frac{v^2 A^2}{\Omega} \sigma_z}$$

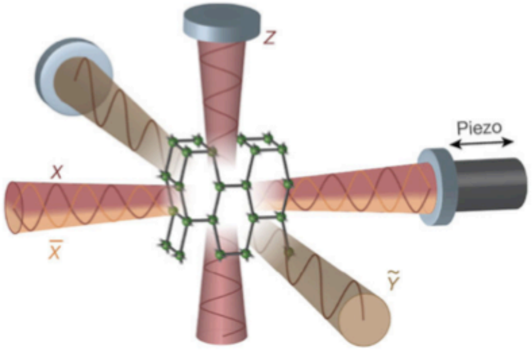
$$A = F/\Omega$$

Related experiments

Observed in quantum simulation experiments

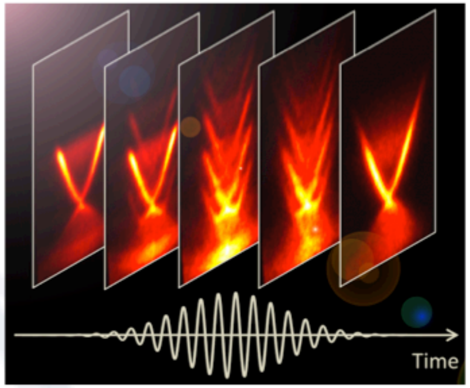


Photonic waveguides
Rechtsman *et. al*, Nature (2013)

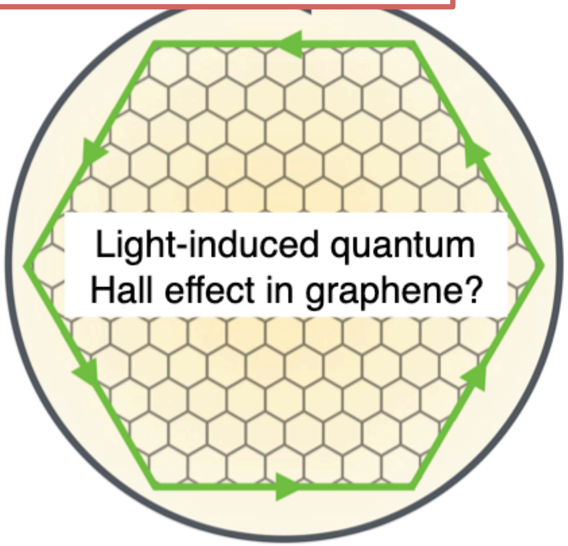


Optical lattices
Jotzu *et. al*, Nature (2014)

„Floquet engineering of artificial gauge fields"

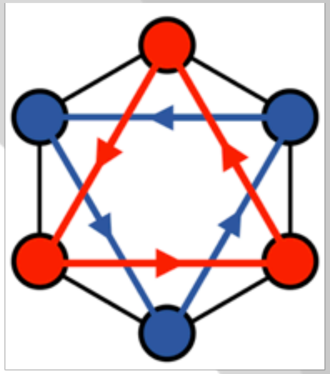


ARPES Bi_2Se_3
Wang *et. al*, Science (2013)

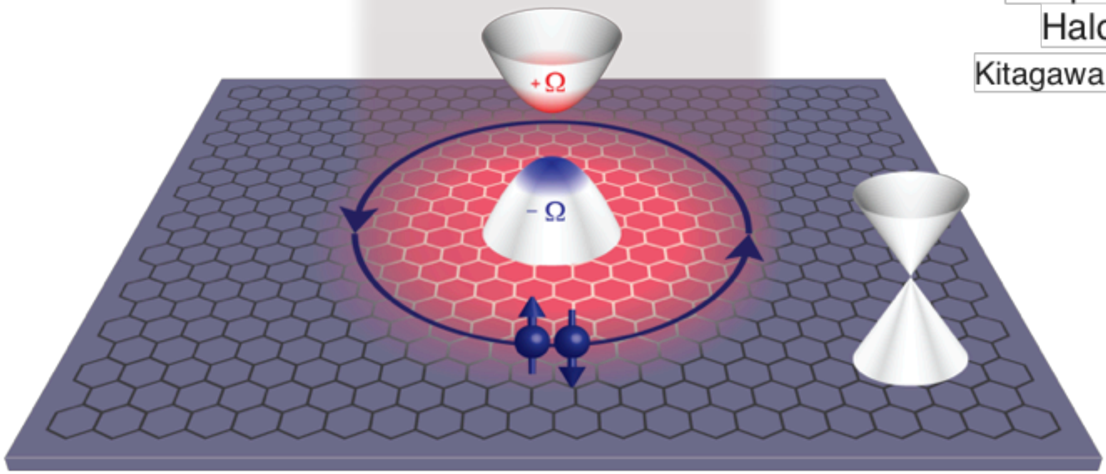


③ Light-induced Hall effect in graphene

*bulk-edge correspondence:
topological gap in bulk implies
topologically protected edge states
along interface to trivial material/vacuum



Floquet-engineered
Haldane Model
Kitagawa *et al.* PRB (2011)



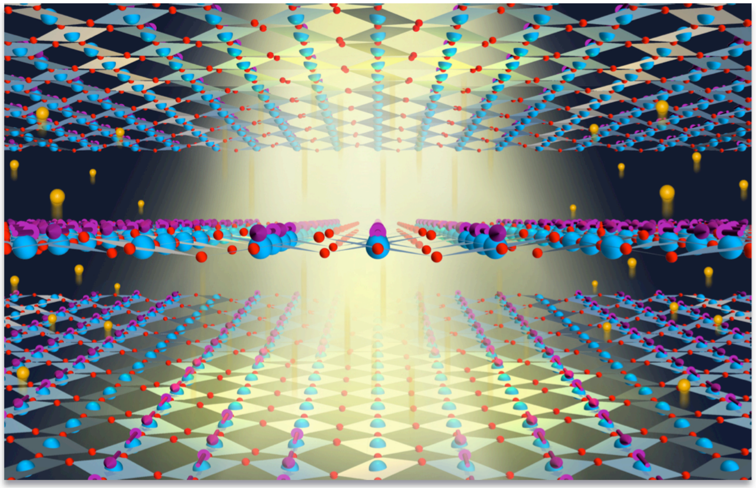
Graphene

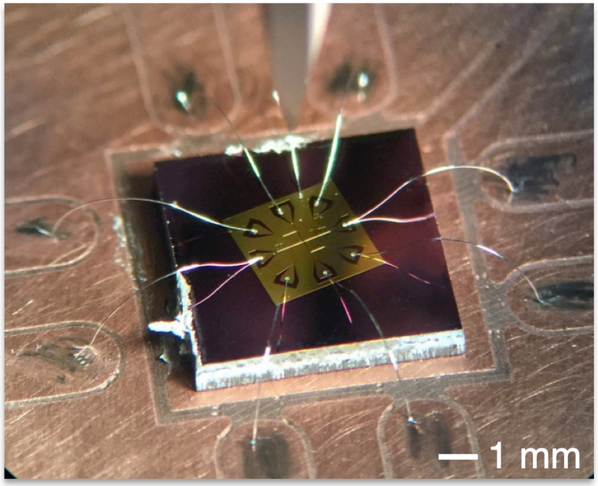
T. Oka & H. Aoki, PRB (2009)

J. McIver et al., Light-induced anomalous Hall effect in graphene, arXiv:1811.03522, Nat. Phys. 2019

Femtosecond science on-chip

Probing ultrafast electrical transport in solids

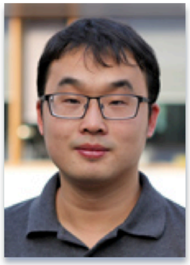




Coherent electromagnetic control of quantum materials

Probe ultrafast electrical transport on-chip

Benedikt Schulte



Eryin Wang

James McIver

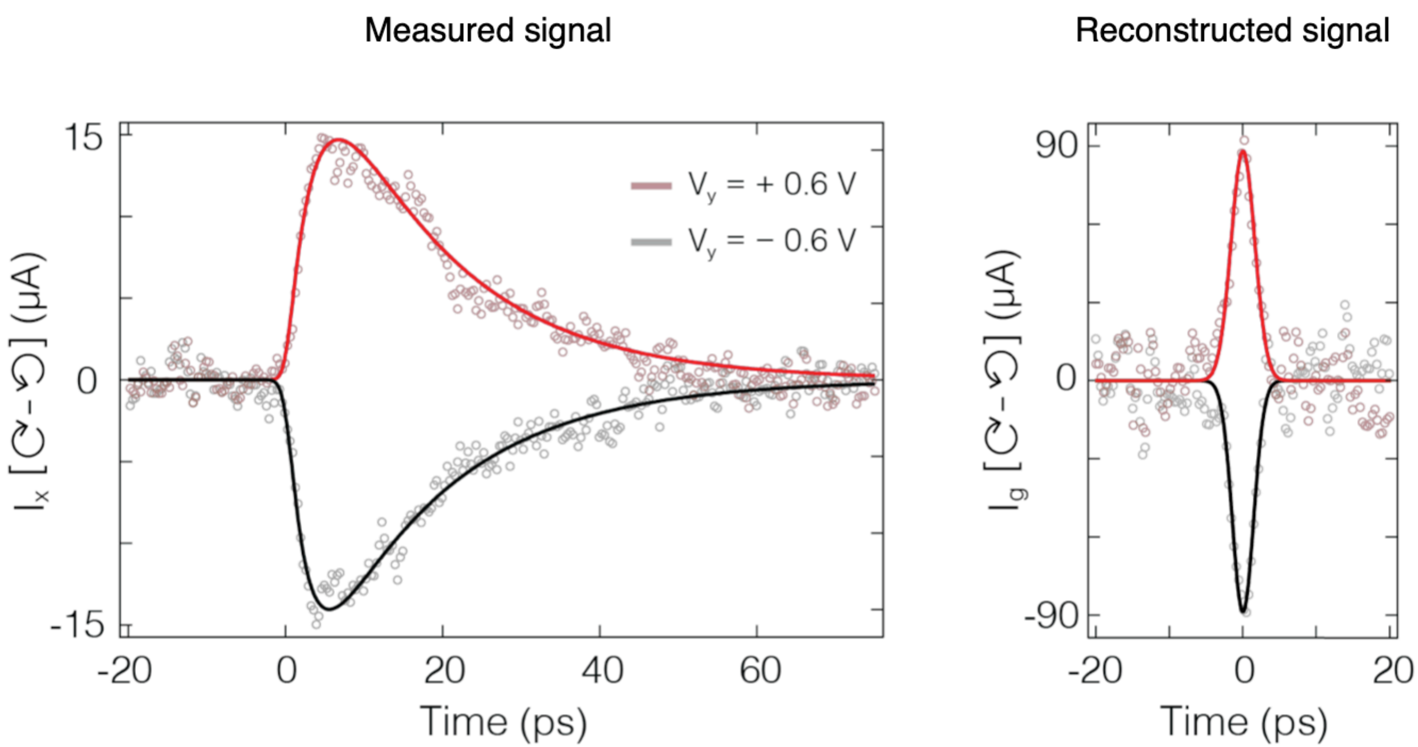
Toru Matsuyama

Guido Meier

Andrea Cavalleri

Light-induced anomalous Hall effect

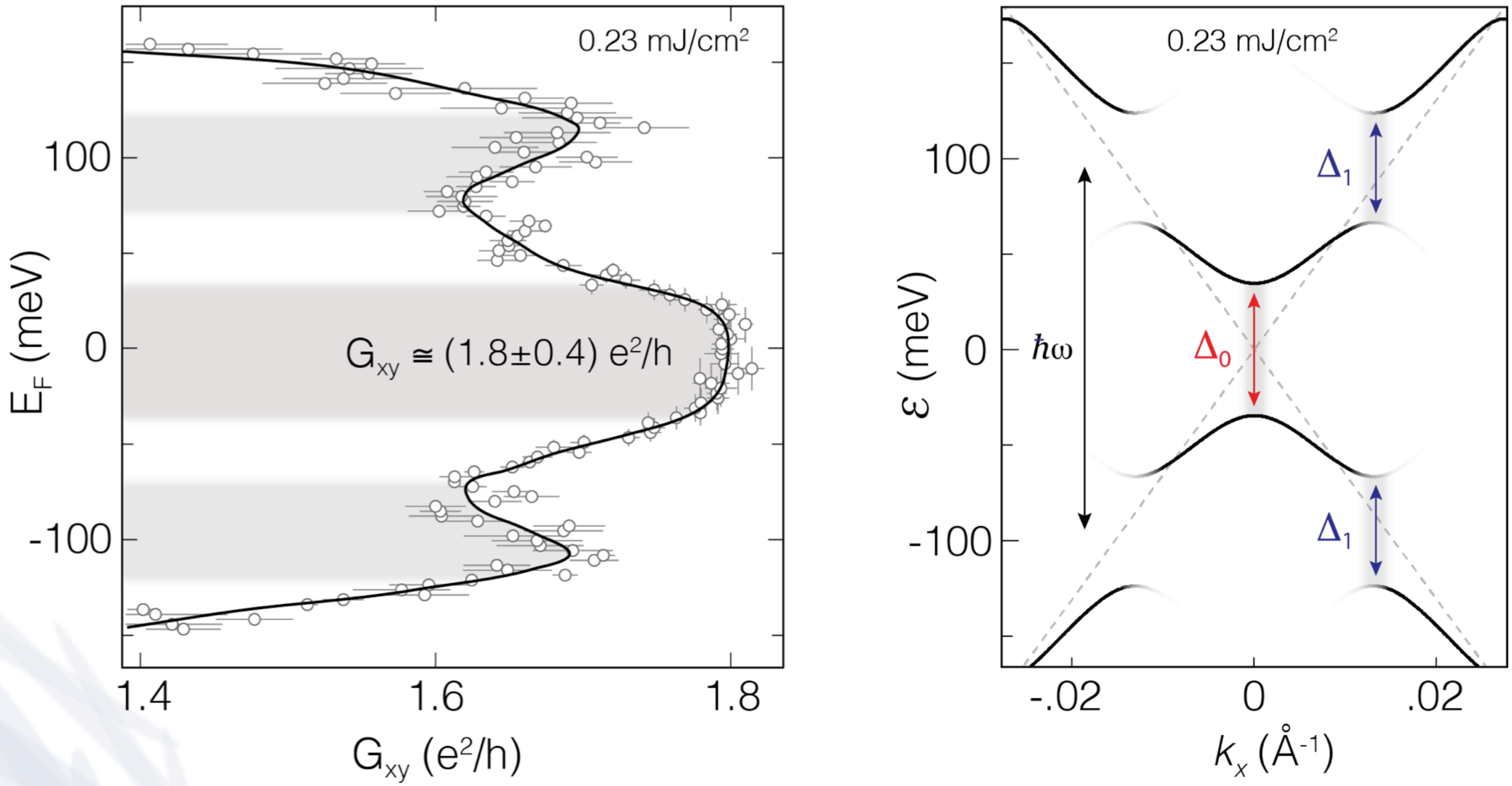
Key signature of emergent topological properties in graphene



McIver *et. al.* arXiv:1811.03522

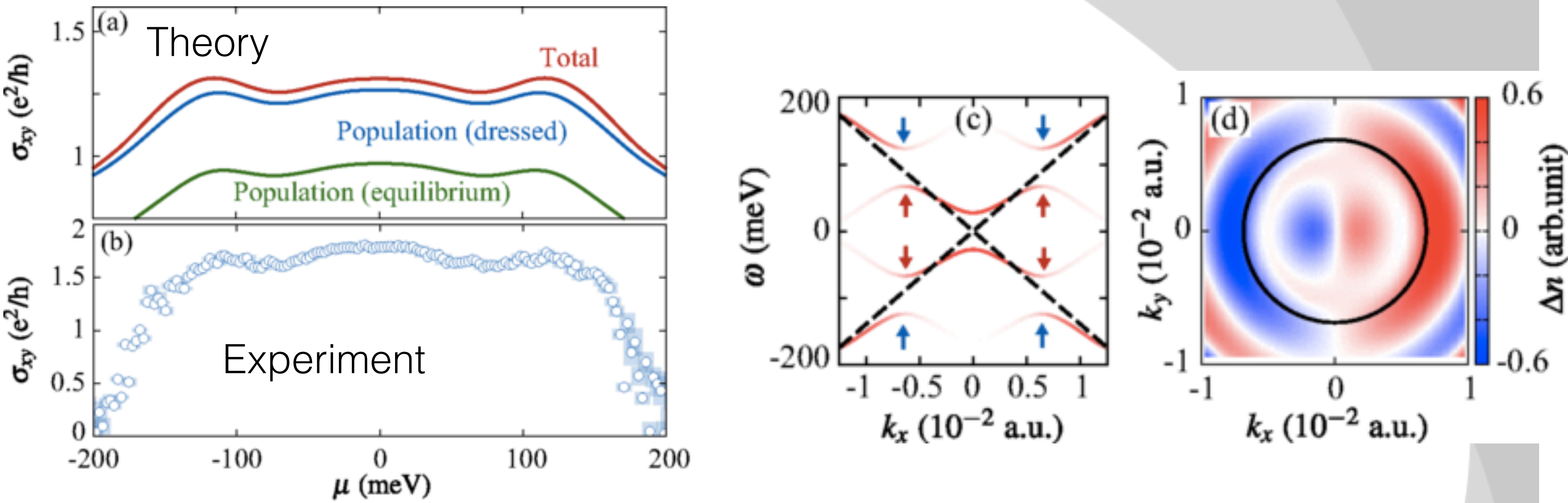
Non-equilibrium topological state

Transport from photon-dressed topological bands



McIver *et. al.* arXiv:1811.03522

Theory of light-induced Hall effect



Floquet topology and light-induced population effects both important

S. A. Sato et al., Microscopic theory for the light-induced anomalous Hall effect in graphene, Phys. Rev. B 99, 214302 (2019)

③ Acknowledgments graphene work


Andrea Cavalleri


Benedikt Schulte

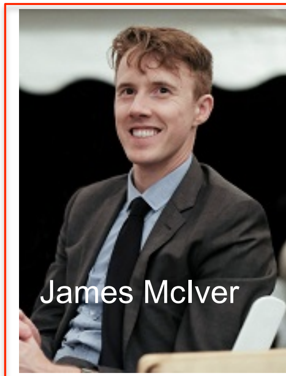

Falk Stein


Gregor Jotzu


Toru Matsuyama

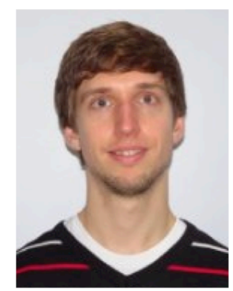

Guido Meier


Angel Rubio



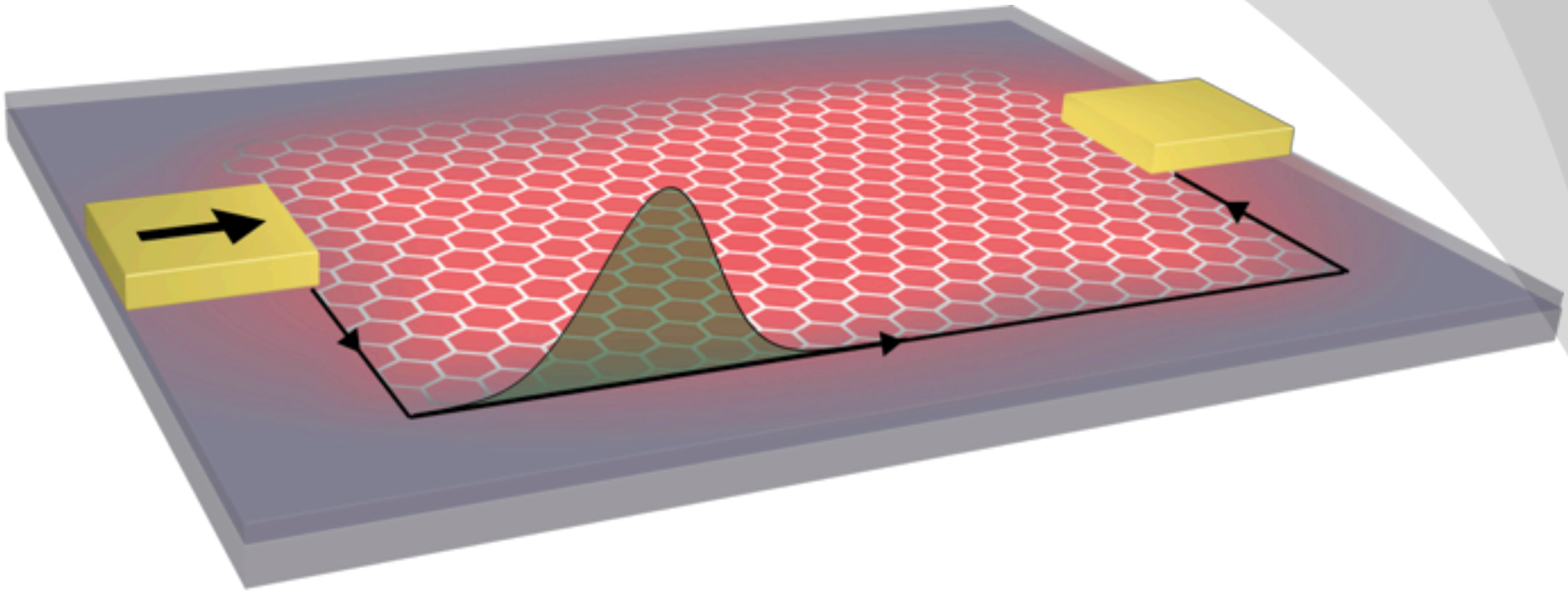

Shunsuke Sato


Ludwig Mathey


Marlon Nuske

Light-induced edge states

Topological transport on demand



Big picture: light-induced edge states

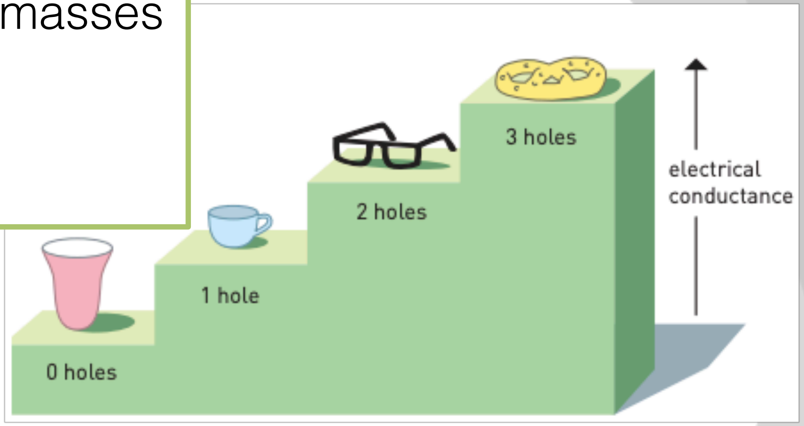
Unifying themes in physics?

Physics Nobel Prize 2019

Physics Nobel Prize 2016

Dirac fermions
Majorana fermions
Weyl fermions
gaps / particle masses
gauge fields

topology?



universe = coffee mug

material = coffee mug

Thank you for your attention!

④ Optical control of Majoranas

mpsd

PHYSICAL REVIEW B 82, 184516 (2010)

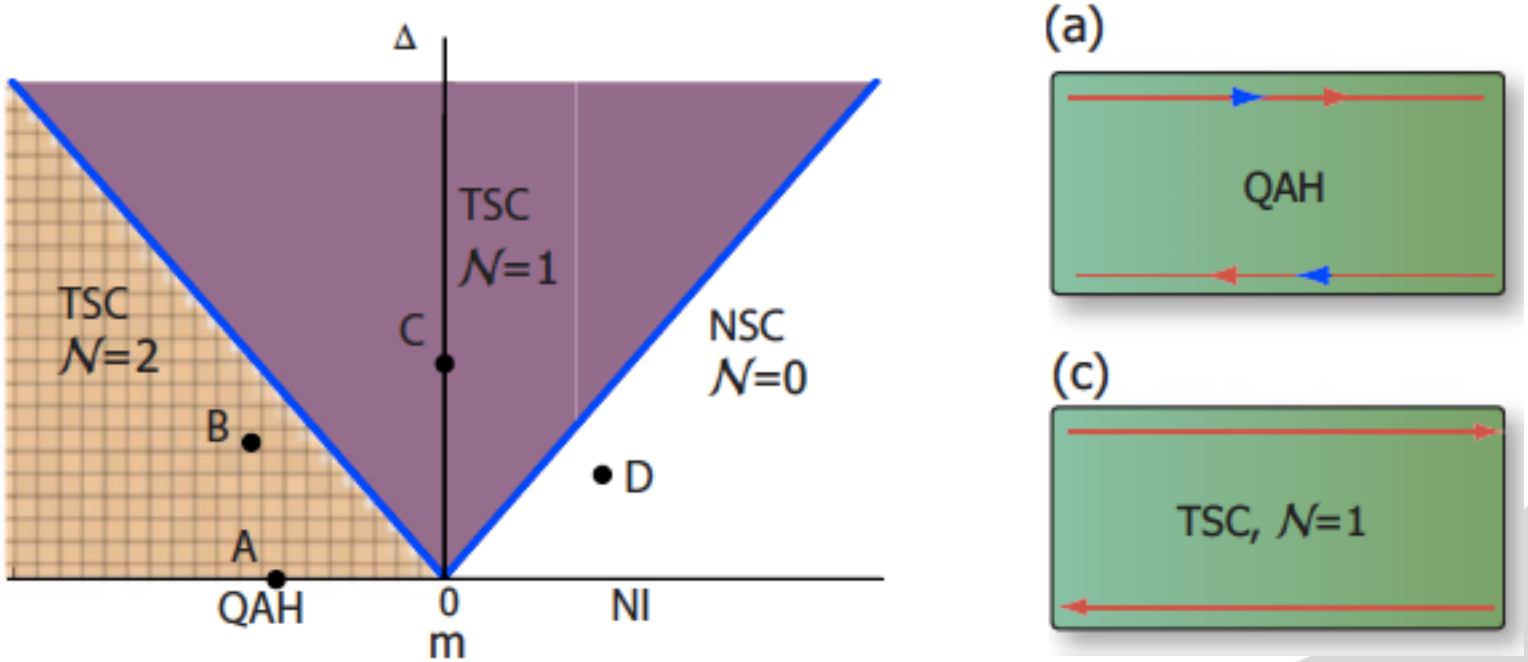
Chiral topological superconductor from the quantum Hall state

Xiao-Liang Qi,[1,2] Taylor L. Hughes,[1,3] and Shou-Cheng Zhang[1]

Chiral topological superconductor

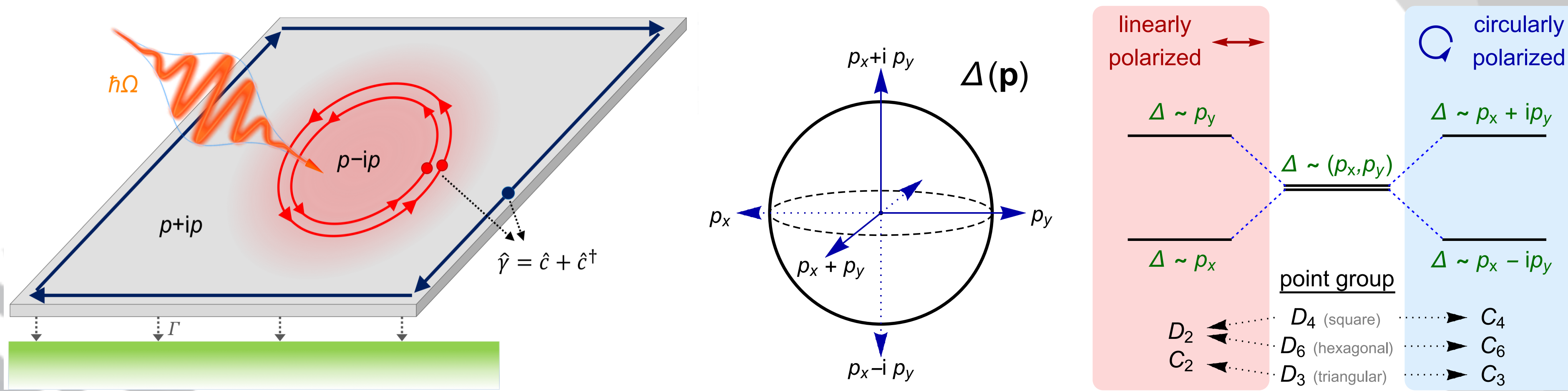
=

2 x quantum anomalous Hall insulator + superconductivity



④ Optical control of Majoranas

Can one switch the chirality of a 2D topological superconductor with light pulses?

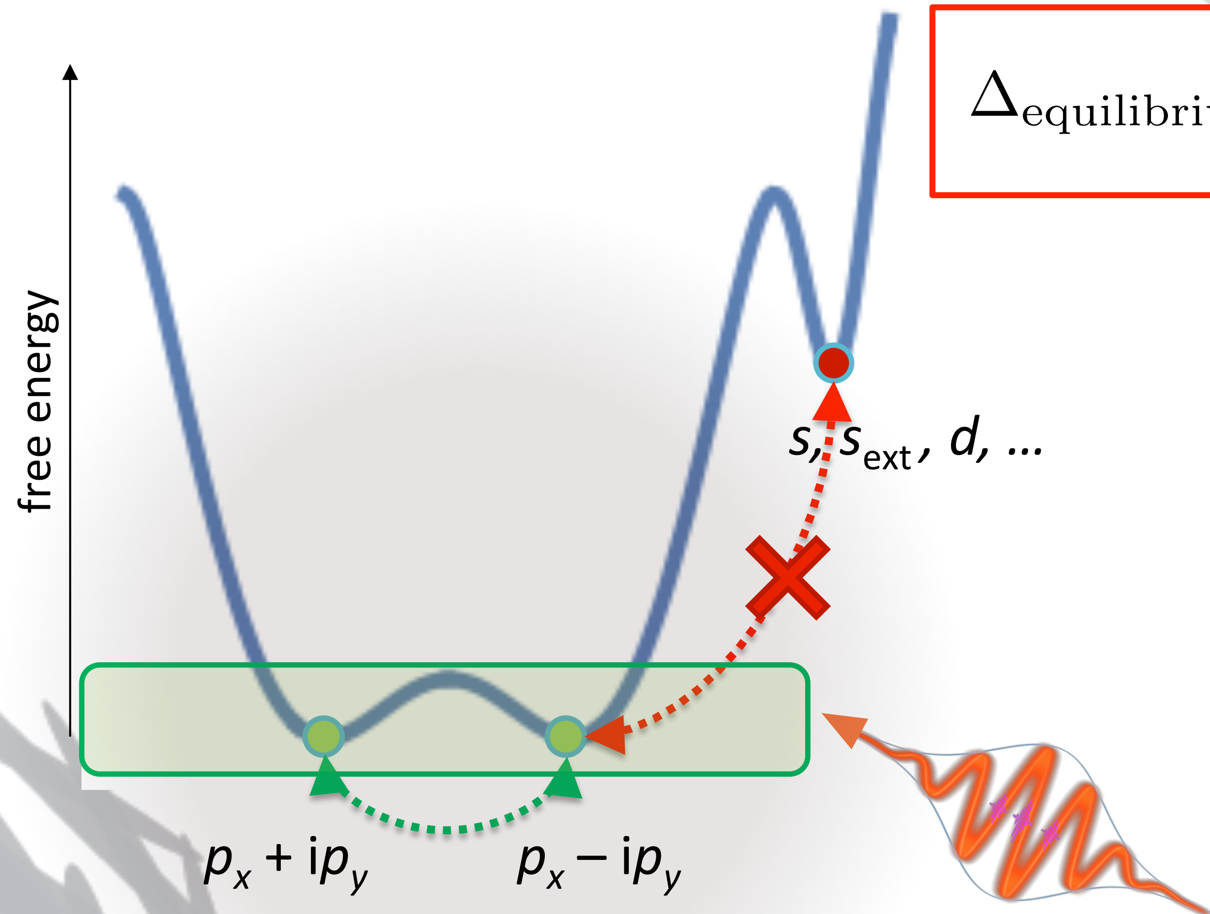


key idea: use two-pulse sequence with linearly and circularly polarized light

Nonequilibrium pathway to switching

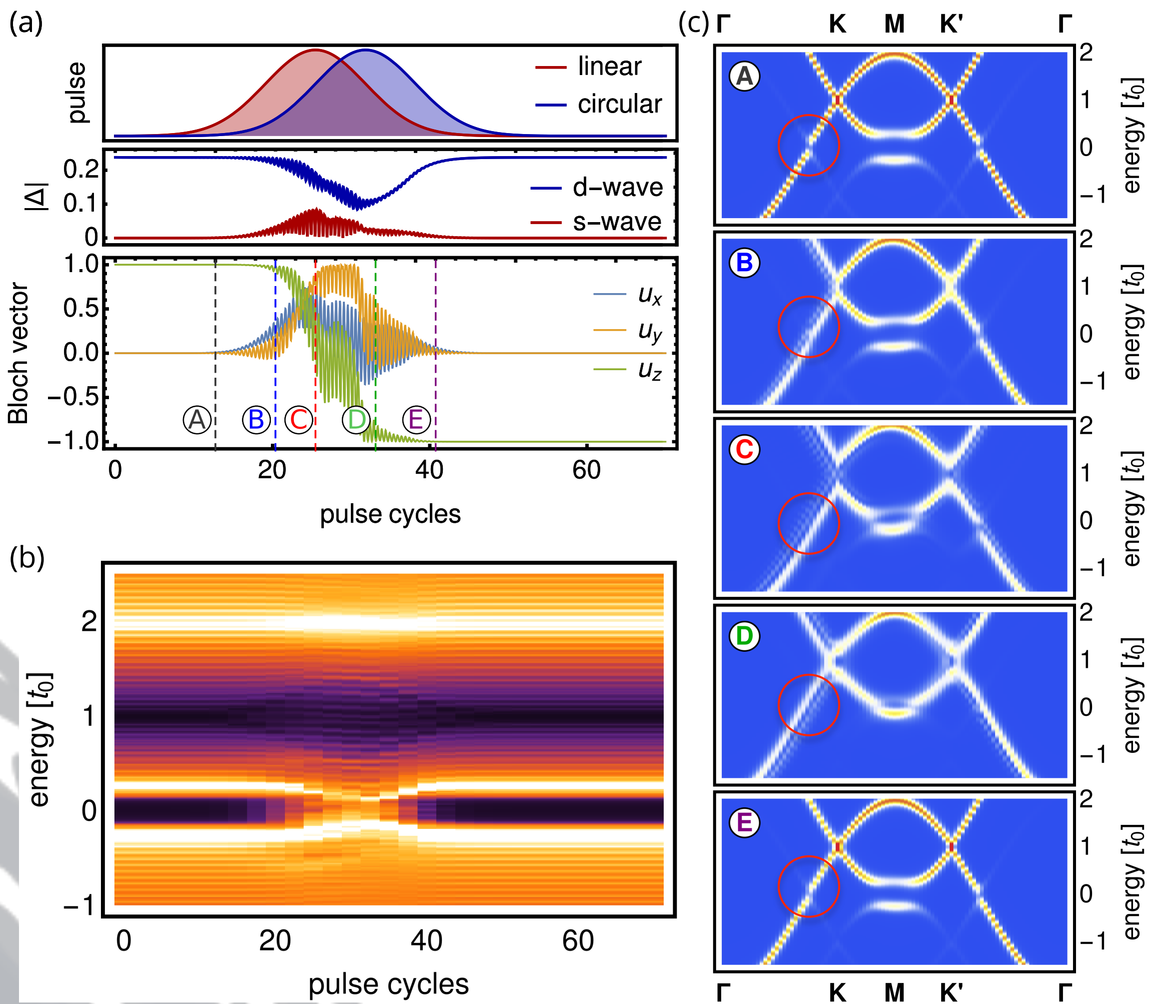
mpsd

$$\Delta_{\text{equilibrium}} \sim \begin{cases} p_x + ip_y \\ p_x - ip_y \end{cases}$$



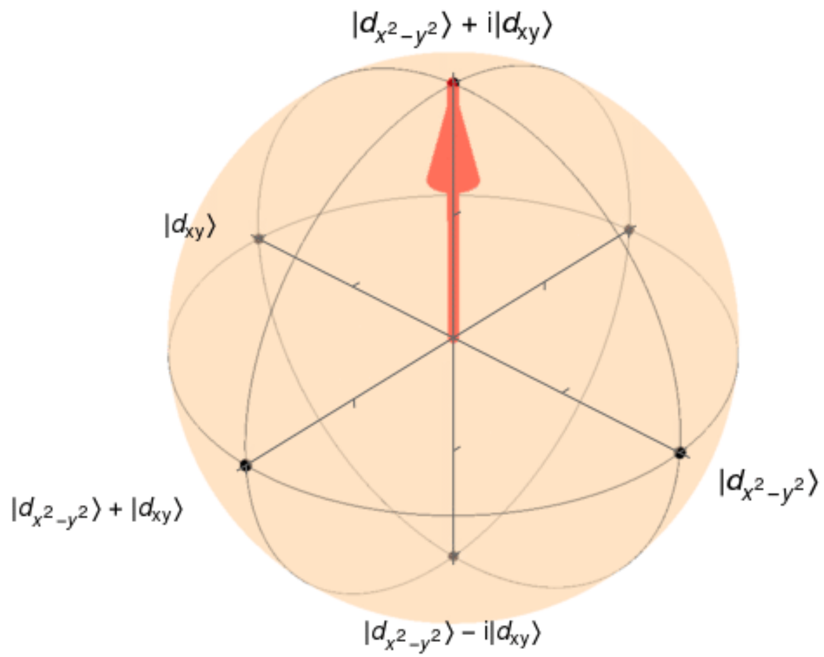
$$\Delta_{\text{non-eq}}(t) \sim \cos(\theta)\ \text{“}p_x + ip_y\text{”}\ +\ \sin(\theta)e^{i\phi}\ \text{“}p_x - ip_y\text{”}$$

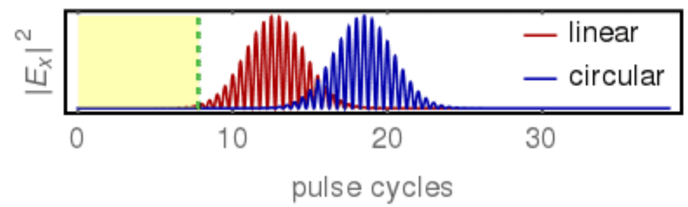
Optical control of Majoranas



two-pulse sequence reverses d+id state in graphene

Bloch vector rotation



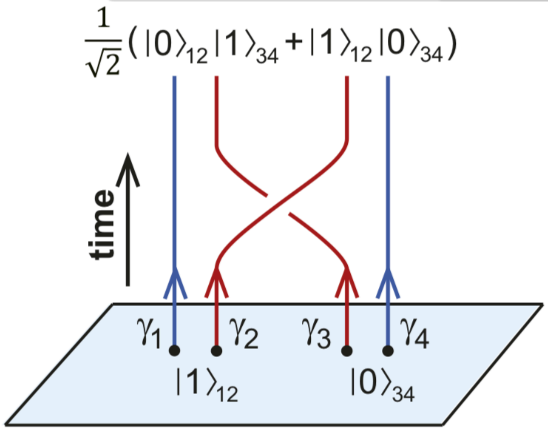


A „programmable“ topological quantum computer?

non-Abelian statistics of Majorana fermions:

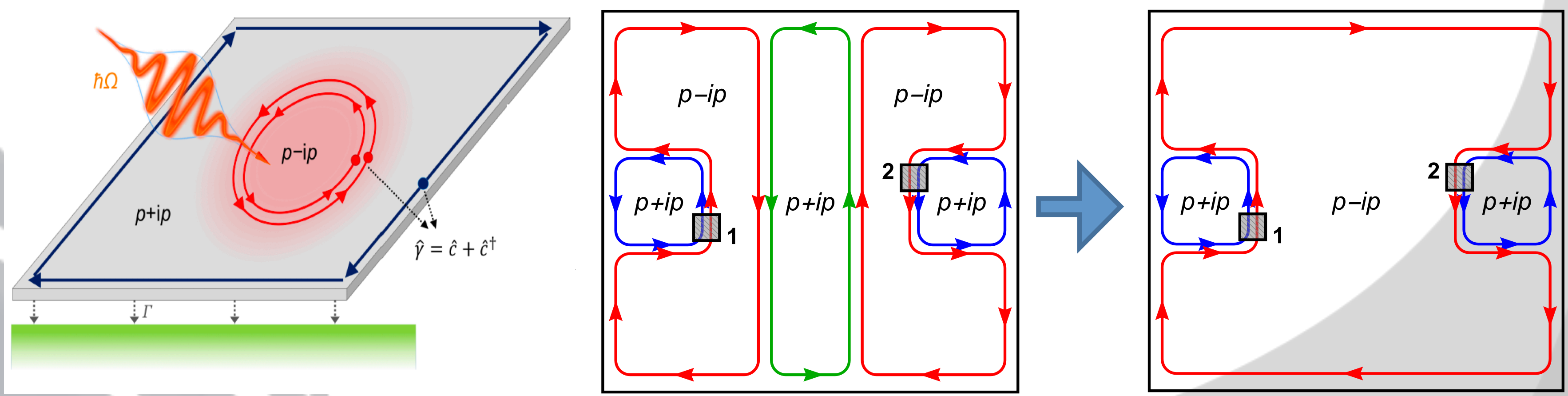
- half-quantum vortices of chiral superconductors host single Majorana fermions
- Two Majoranas represent one electron: **½ + ½ = 1**

  → Braiding between Majoranas is a non-Abelian operation in electron (charge) basis!



Ivanov, PRL 86, 268 (2001)
B. Lian et al., PNAS 115, 10938 (2018)

simplest operation: a **switchable Hadamard gate**



④ Acknowledgments Majorana work

- All-optical control of chiral Majorana modes
- towards arbitrarily programmable quantum computer?

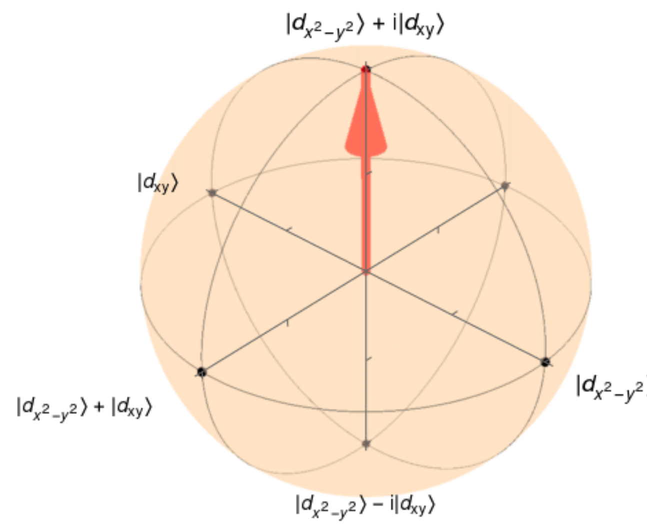
„program the gate optically, read it out electrically“

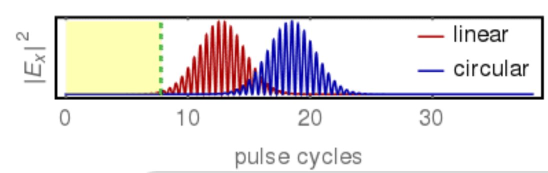
M. Claassen et al.,
Nat. Phys. 15, 766 (2019)

M. Claassen

D. Kennes

M. Zingl





Unifying themes in physics?

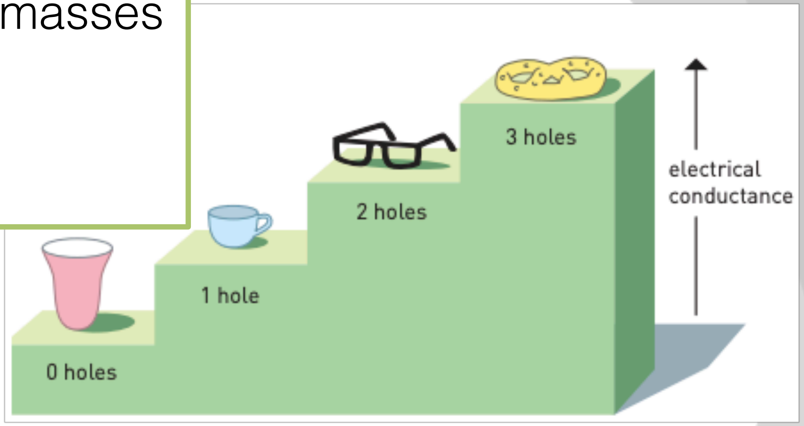
mpsd

Physics Nobel Prize 2019

Physics Nobel Prize 2016

Dirac fermions
Majorana fermions
Weyl fermions
gaps / particle masses
gauge fields

topology?



THE NOBEL PRIZE IN PHYSICS 2016, THE ROYAL SWEDISH ACADEMY OF SCIENCES, HTTP://KVA.SE

universe = coffee mug

material = coffee mug

Thank you for your attention!