

# Theory of laser-driven nonequilibrium superconductivity

*PRB 92, 224517 (2015)*

*PRB 93, 144506 (2016)*

Collaborators:

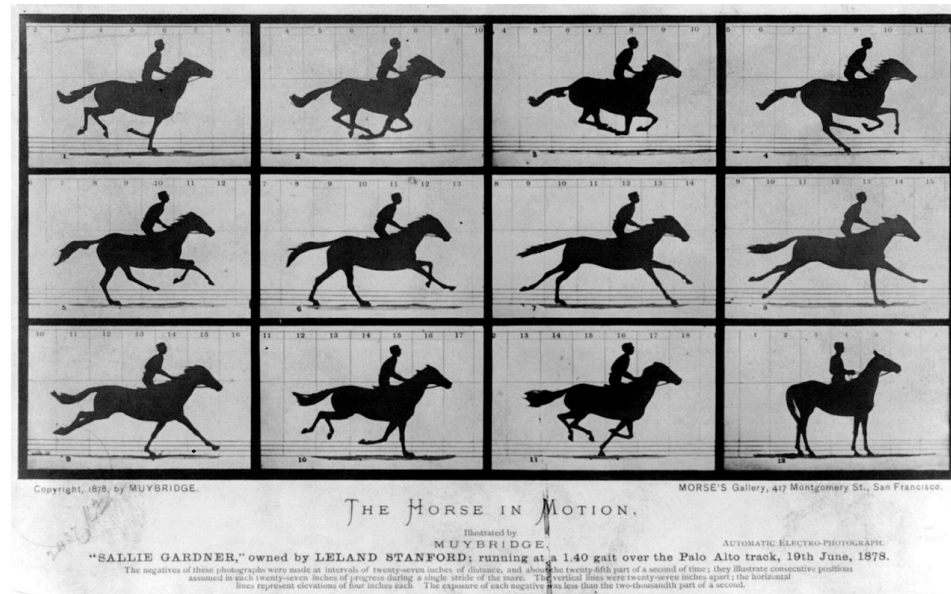
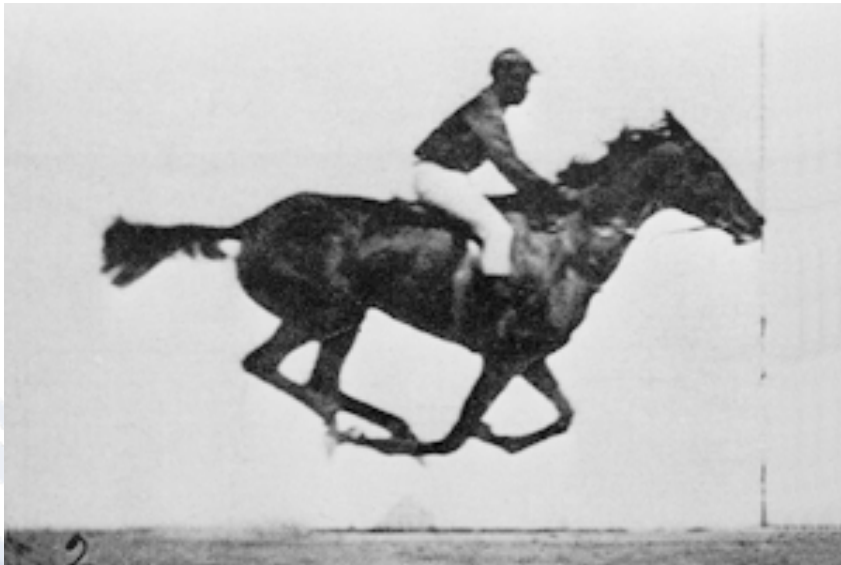
A. F. Kemper, B. Moritz, J. K. Freericks, T. P. Devereaux,  
A. Georges, C. Kollath, A. Tokuno

## Michael Sentef

### SIMES Seminar, February 12, 2016

# Pump-probe spectroscopy (1887)

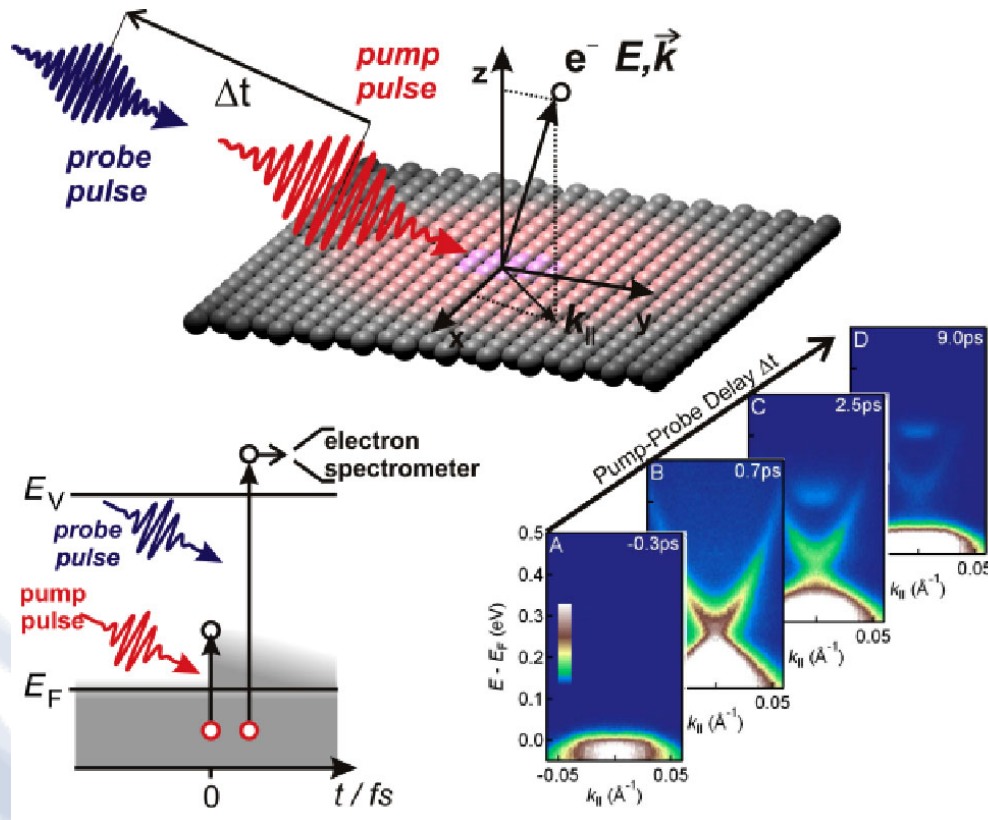
- stroboscopic investigations of dynamic phenomena



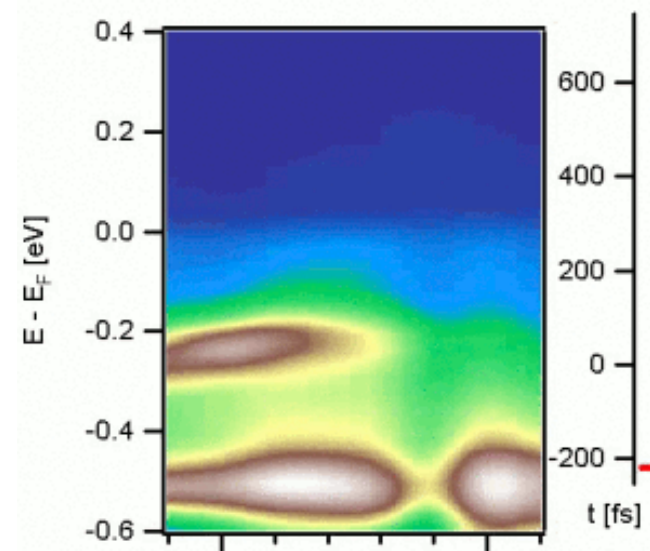
*Muybridge 1887*

# Pump-probe spectroscopy (today)

- stroboscopic investigations of dynamic phenomena



TbTe<sub>3</sub> CDW metal



*J. Sobota et al., PRL 108, 117403 (2012)*  
*F. Schmitt et al., Science 321, 1649 (2008)*  
 Image courtesy: J. Sobota / F. Schmitt

## *Understanding the nature of quasi-particles*

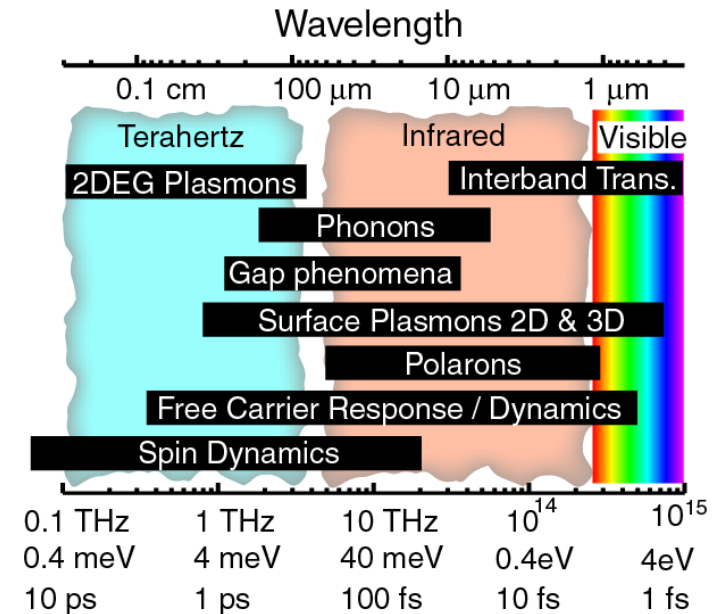
- Relaxation channels and dynamics

## *Understanding ordered phases*

- Collective oscillations
- Light-enhanced order
- Competing order parameters

## *Creating new states of matter*

- Photo-induced phase transitions
- Non-thermal phases



*Image courtesy:  
D. Basov*



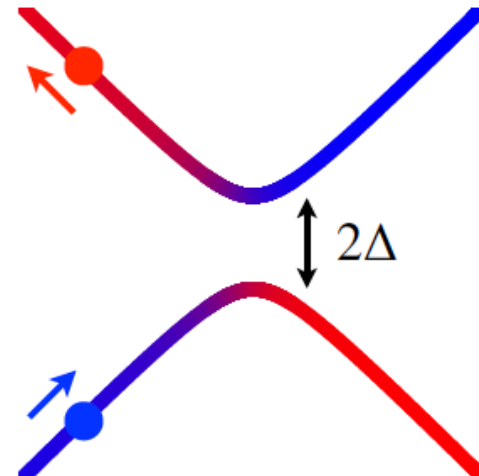
- Light-superconductor coupling
- Keldysh Green functions
- Ordered states: Driven superconductors
  - Higgs amplitude mode oscillations for optical pumping (1.5 eV laser)  
*PRB 92, 224517 (2015)*
  - light-enhanced superconductivity via coherent hopping control  
*arXiv:1505.07575*
- competing orders (preliminary results)

## Dynamics of superconductors

- Bogoliubov-de Gennes equation coupled to an electric field

$$i\partial_t \Psi_k = \begin{pmatrix} \overset{\text{electron}}{\downarrow} \epsilon_{k-eA(t)} & -\Delta^* \\ -\Delta & \overset{\text{hole}}{\uparrow} -\epsilon_{k+eA(t)} \end{pmatrix} \Psi_k$$

$$\Psi_k = \begin{pmatrix} c_{k\uparrow} \\ c_{-k\downarrow}^\dagger \end{pmatrix} : \text{Nambu spinor}$$

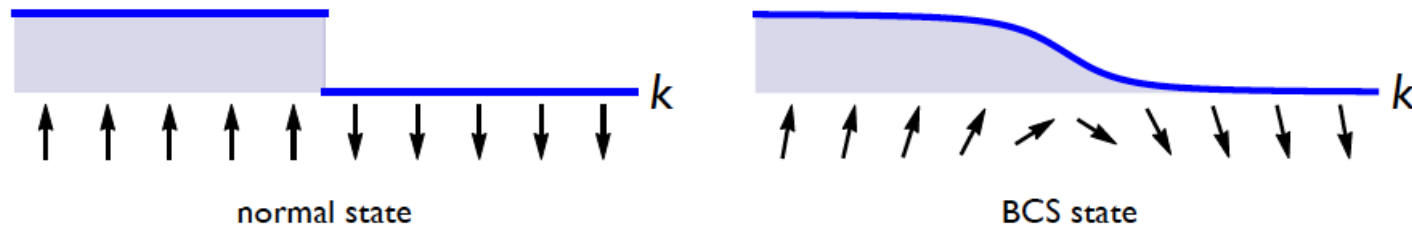


## Anderson pseudospin

$$\sigma_k = \frac{1}{2} \Psi_k^\dagger \cdot \tau \cdot \Psi_k \quad \text{Anderson, Phys. Rev. 112, 1900 (1958)}$$

$$\partial_t \sigma_k = 2 \mathbf{b}_k \times \sigma_k \quad \mathbf{b}_k = \left( -\Delta', -\Delta'', \frac{\epsilon_{k-eA(t)} + \epsilon_{k+eA(t)}}{2} \right)$$

Tsuji, Aoki, arXiv:1404.2711



- Particle-hole symmetric by construction.
- Linear response vanishes.

## Light-pseudospin coupling

$$\partial_t \boldsymbol{\sigma}_k = 2 \mathbf{b}_k \times \boldsymbol{\sigma}_k \quad \mathbf{b}_k = \left( -\Delta', -\Delta'', \frac{\epsilon_{k-\mathbf{e}A(t)} + \epsilon_{k+\mathbf{e}A(t)}}{2} \right)$$

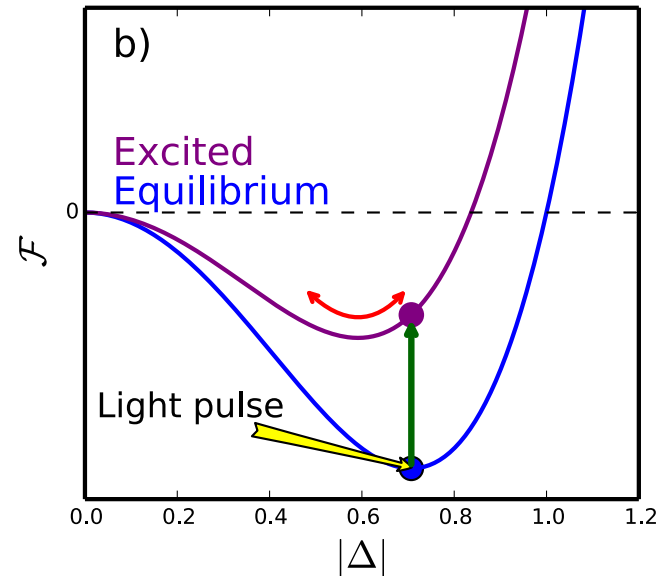
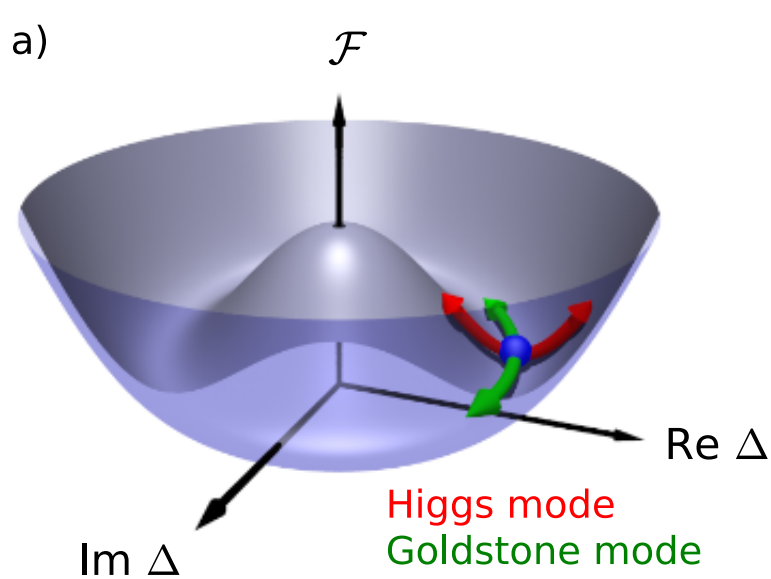
$$\epsilon(k - A) + \epsilon(k + A) = 2\epsilon(k) + \mathcal{O}(A^2),$$

$A^2$  coupling: „Anderson pseudospin (Higgs) resonance“ at  $2\omega = 2\Delta$

*Tsuji & Aoki, PRB 92, 064508 (2015)*

*R. Matsunaga et al., Science 345, 1145 (2014)*

# Higgs amplitude mode



Include the effects of driving field through Peierls substitution

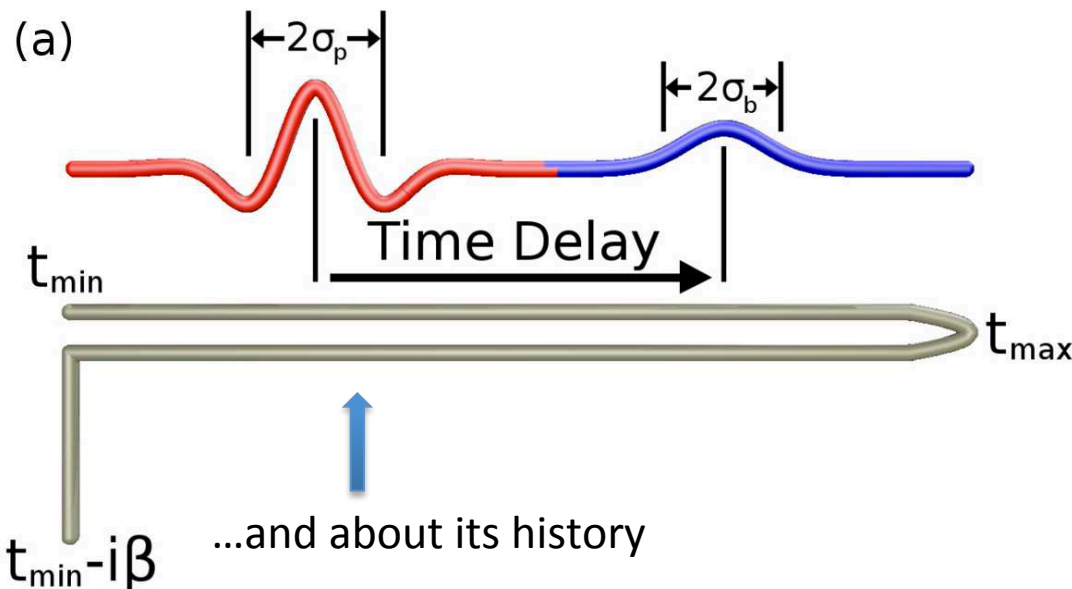
$$\mathbf{k} \rightarrow \mathbf{k} - e\mathbf{A}(t)$$

# Non-Equilibrium Keldysh Formalism

## Beyond BCS

$$G_k(t, t') = G_k^0(t, t') + \int dt_1 \int dt_2 G_k^0(t, t_1) \Sigma(t_1, t_2) G_k(t_2, t')$$

self-energy  $\Sigma$ :  
electron-electron scattering  
electron-phonon scattering  
...



System knows about its thermal initial state...

Include the effects of driving field through time-dependent electronic dispersion

$$\varepsilon(k) \rightarrow \varepsilon(k, t)$$



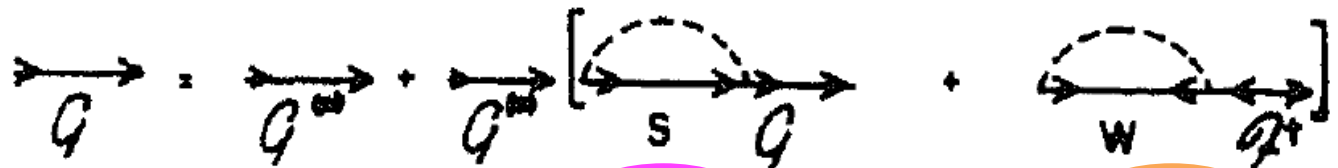
# Model and Method

$$\mathcal{H} = \sum_{\mathbf{k}\sigma} \epsilon(\mathbf{k}, t) c_{\mathbf{k}\sigma}^\dagger c_{\mathbf{k}\sigma} + \sum_{\mathbf{q}, \gamma} \Omega_\gamma b_{\mathbf{q}, \gamma}^\dagger b_{\mathbf{q}, \gamma} - \sum_{\mathbf{q}, \gamma, \sigma} g_\gamma c_{\mathbf{k}+\mathbf{q}\sigma}^\dagger c_{\mathbf{k}\sigma} \left( b_{\mathbf{q}, \gamma} + b_{-\mathbf{q}, \gamma}^\dagger \right)$$

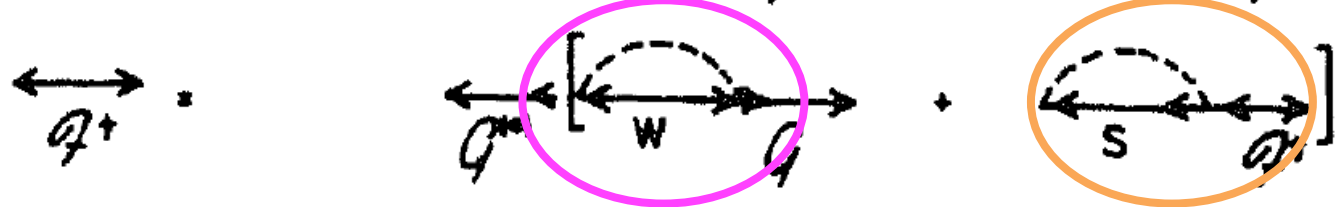
- electrons (2D square latt.) + spectrum of phonons + el-ph coupling (Holstein)
- Migdal-Eliashberg (1st Born) + phonon heat bath approximation

superconductor:

normal



anomalous

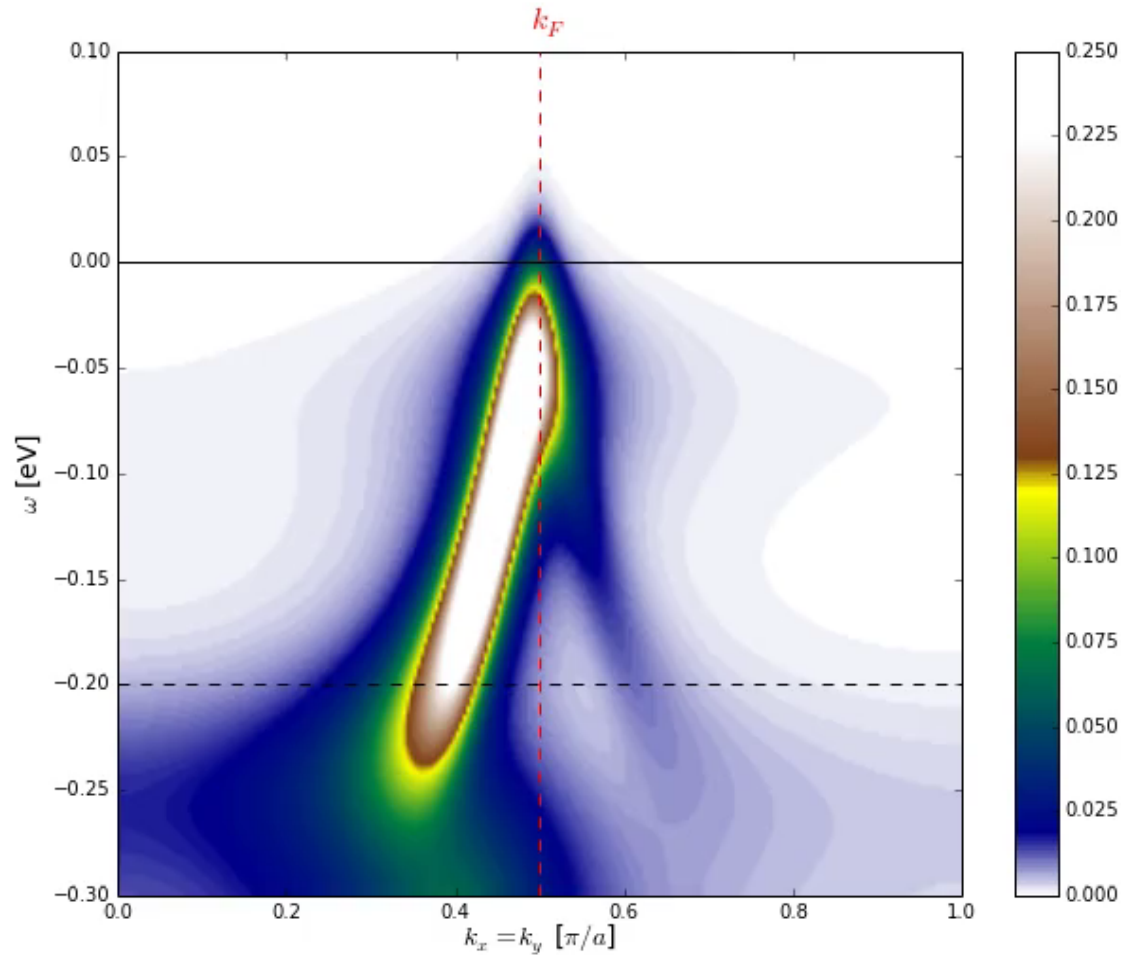


order parameter  $\Delta$ ,  
condensate dynamics

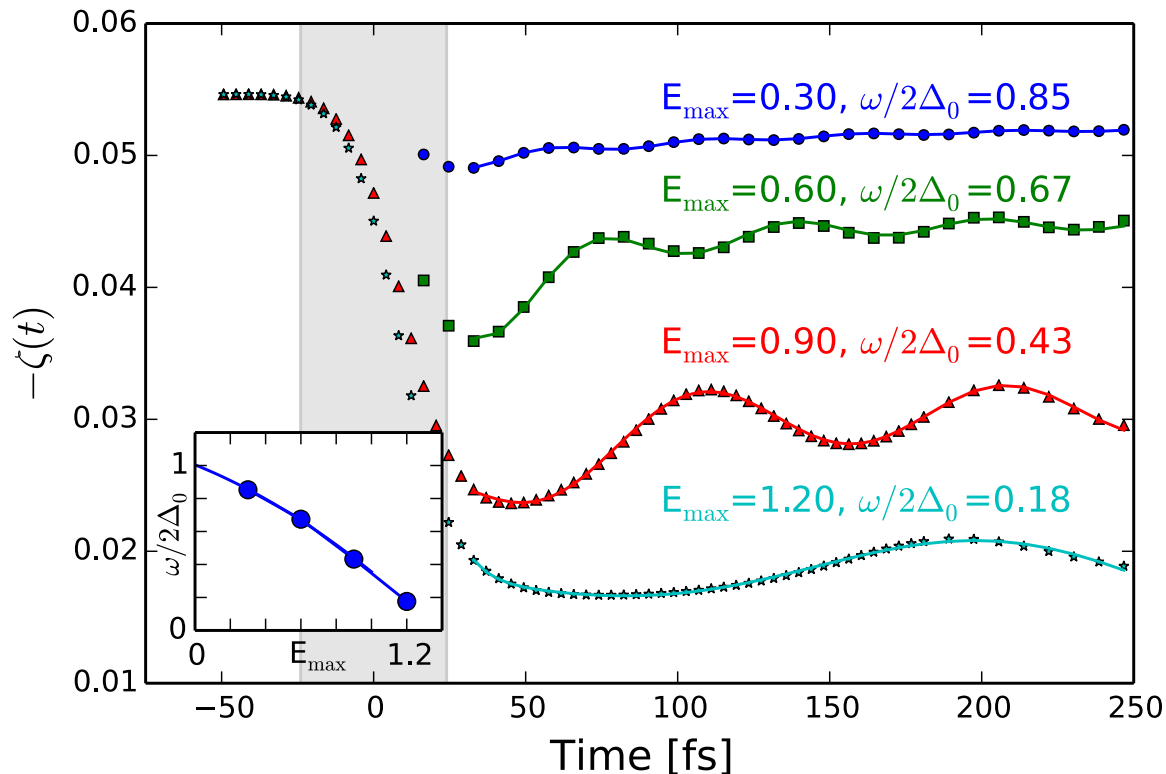
el-ph single-particle  
scattering

*cf. textbooks (Mahan, AGD, ...) for Migdal-Eliashberg approx.*

# Oscillations in photocurrent



# Amplitude mode oscillations



PRB 92, 224517  
(2015)

Amplitude (“Higgs”) mode oscillations predicted in time-resolved ARPES  
Reduced order parameter sets oscillation frequency  
Dissipation: Exciting Higgs even far away from gap resonance

Optics: Matsunaga et al., *Phys. Rev. Lett.* 111, 057002 (2013), *Science* 2014 [10.1126/science.1254697]

Theory: Volkov & Kogan 1974, Barankov PRL 2004, Yuzbashyan PRL 2006, Tsuji PRL 2013

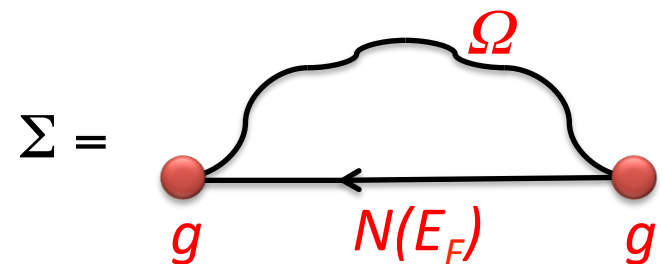
Max Planck Institute for the Structure and Dynamics of Matter

# How to enhance boson-mediated SC?

- BCS theory – plain vanilla SC (weak coupling)

$$\Delta \approx 2\hbar\Omega_c \exp(-1/V_0 N(E_F))$$

- effective attraction  $V_0 \sim g^2/(\hbar \Omega)$
- e-boson coupling  $g$
- boson frequency  $\Omega$
- electronic DOS  $N(E_F)$



Migdal-Eliashberg theory  
boson-mediated pairing

# How to enhance boson-mediated SC?

- **nonlinear phononics**  $Q^2Q$ : resonant excitation of vibrational modes – effects?

## 1. tune model parameters

- e-boson coupling  $g$
  - boson frequency  $\Omega$
- }  $\alpha^2F$  – Eliashberg function

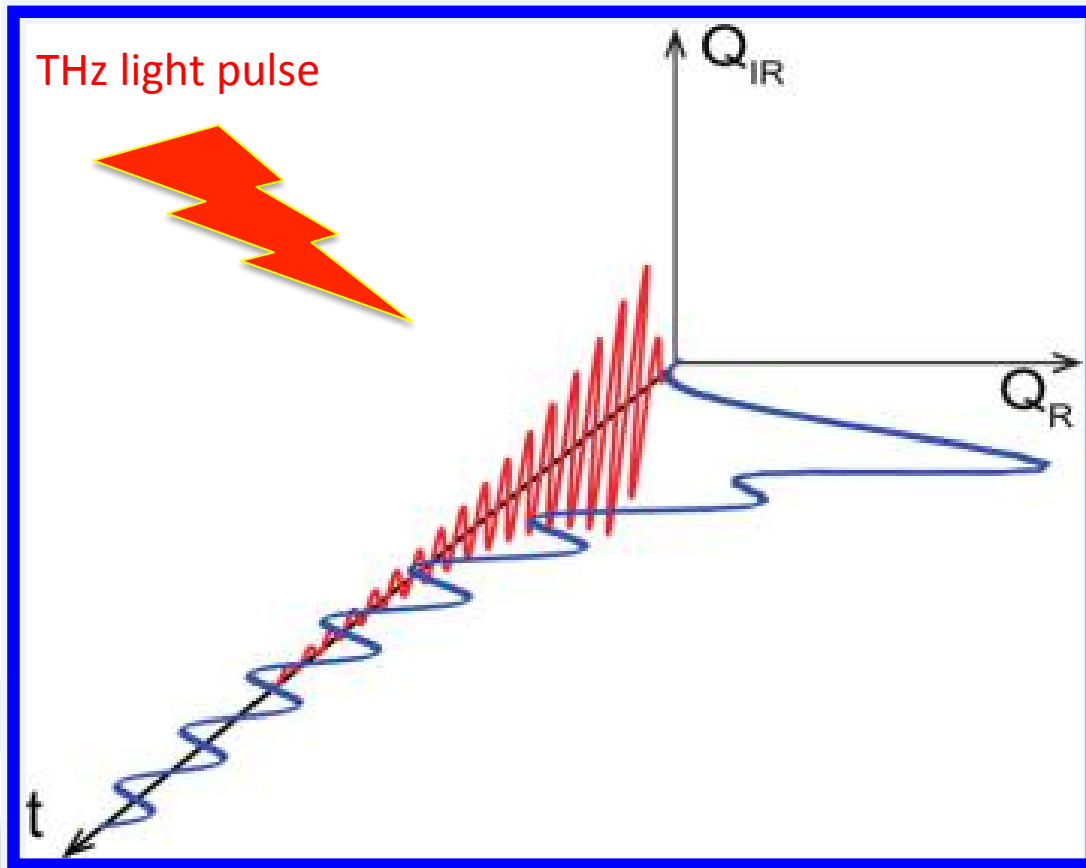
- **electronic DOS  $N(E_F)$**

**Gedankenexperiment (what if?)**

## 2. dynamical effect

- effective Hamiltonian (e.g., Floquet)

*also see: Knap et al., arXiv:1511.07874*



$$\ddot{Q}_{\text{IR}} + \Omega_{\text{IR}}^2 Q_{\text{IR}} = \frac{e^* E_0}{\sqrt{M_{\text{IR}}}} \sin(\Omega_{\text{IR}} t) F(t)$$

$$\ddot{Q}_{\text{RS}} + \Omega_{\text{RS}}^2 Q_{\text{RS}} = A Q_{\text{IR}}^2$$

Rectification of a second (Raman) phonon via coherent driving of a first (IR) phonon

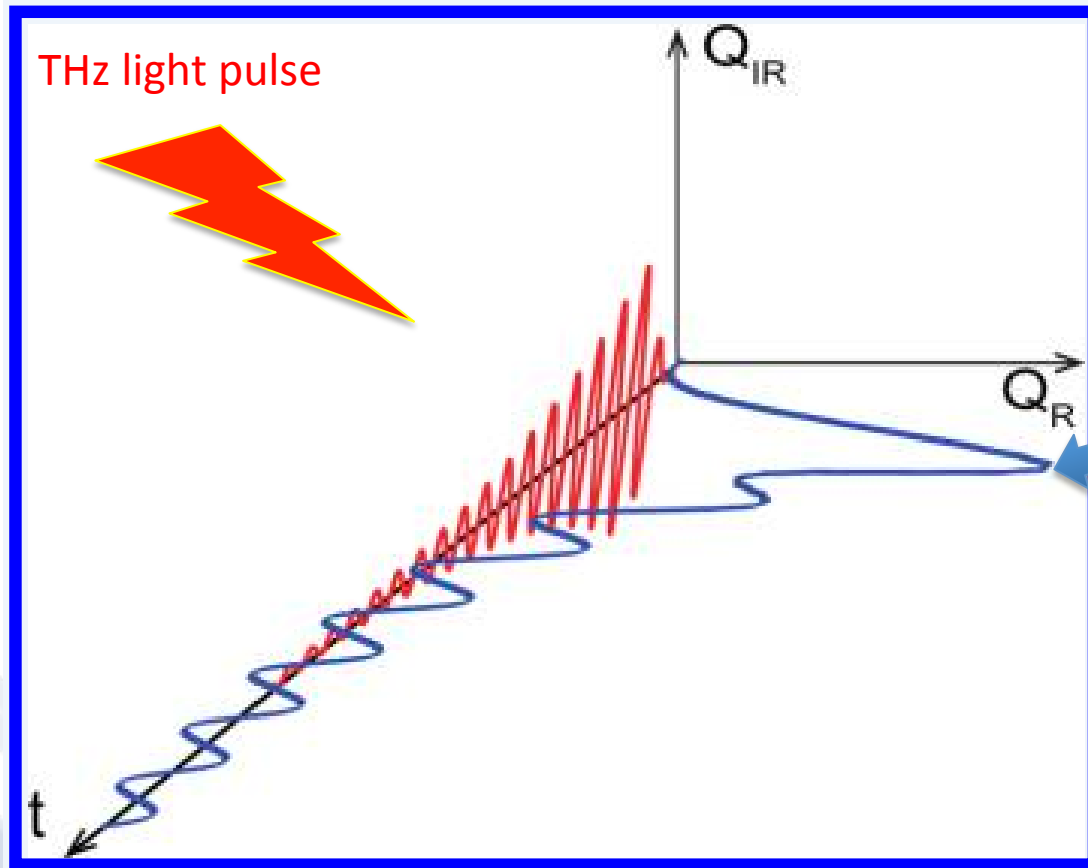
**„Nonlinear phononics“**

*M. Först et al., Nature Physics 7, 854 (2011)*

*A. Subedi, A. Cavalleri, A. Georges, PRB 89, 220301R (2014)*



# Classical lattice dynamics



Displaced Raman mode assumed to lead to temporal change of electronic hopping

**Gedankenexperiment (what if?)**

Include the effects of driving field through time-dependent electronic dispersion

$$\varepsilon(k) \rightarrow \varepsilon(k, t)$$

**„Nonlinear phononics“**

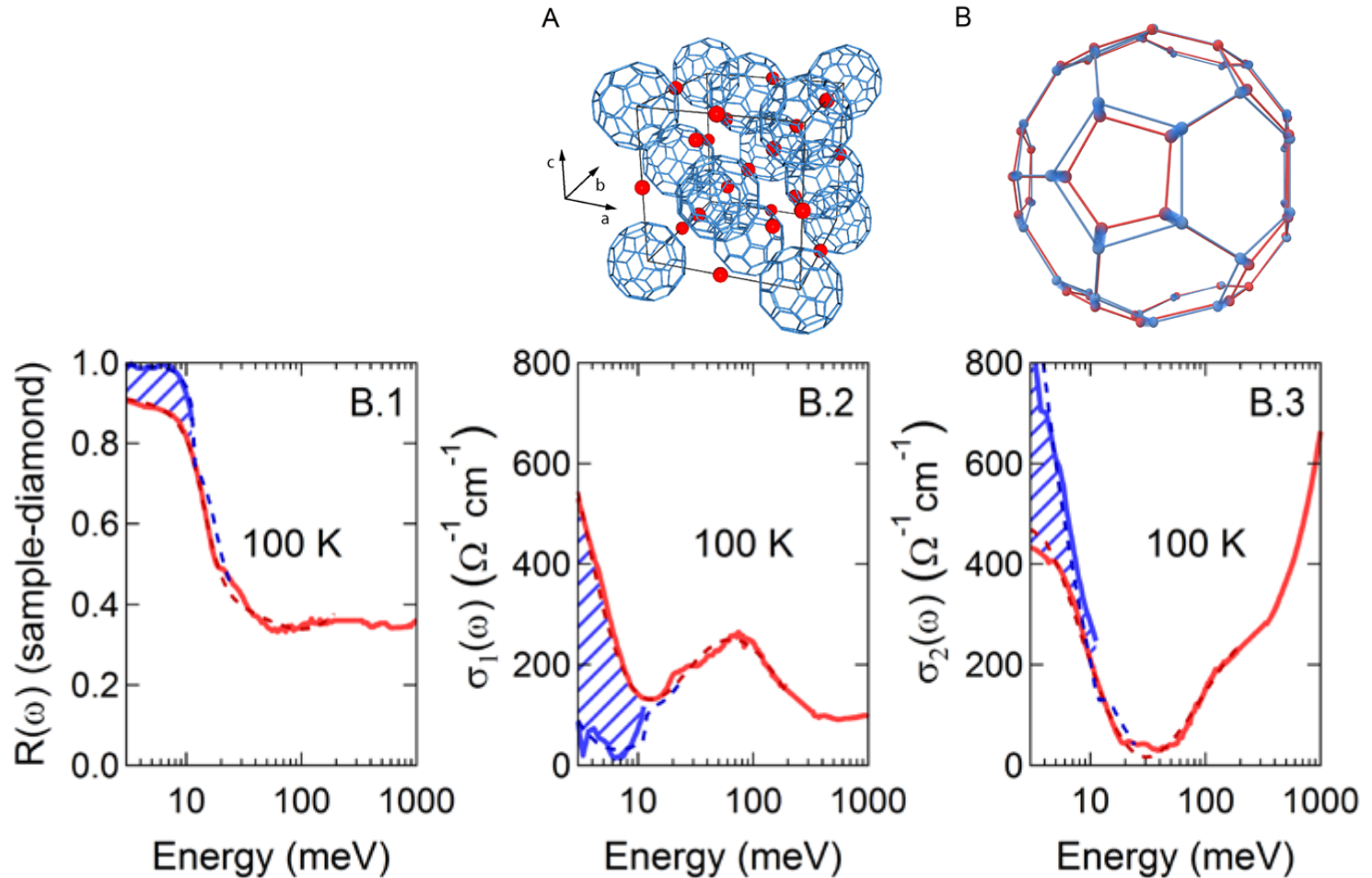
*M. Först et al., Nature Physics 7, 854 (2011)*

*A. Subedi, A. Cavalleri, A. Georges, PRB 89, 220301R (2014)*

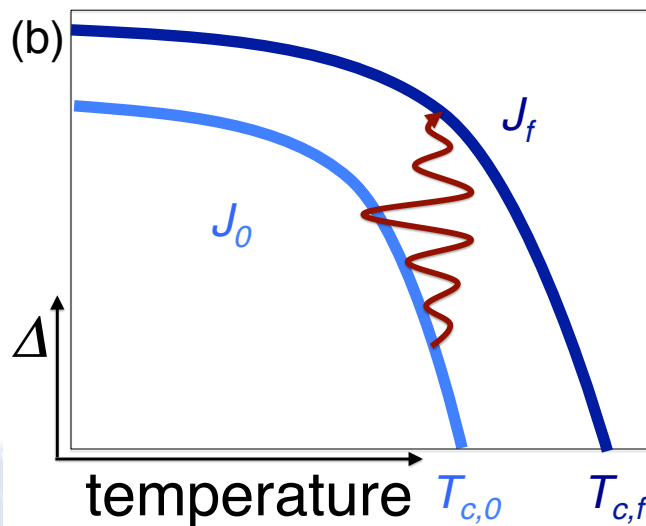
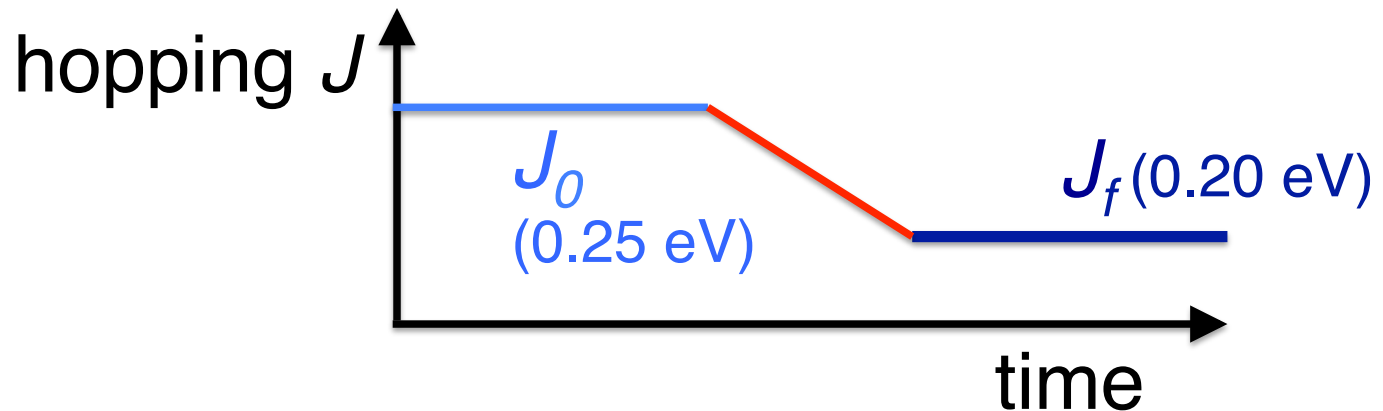
# Experimental motivation

**„An optically stimulated superconducting-like phase in K3C60 far above equilibrium  $T_c$ “**

*M. Mitrano et al., arXiv: 1505.04529 to appear in Nature*

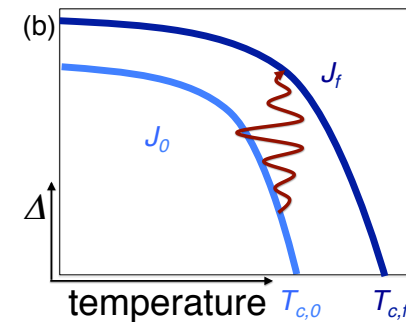
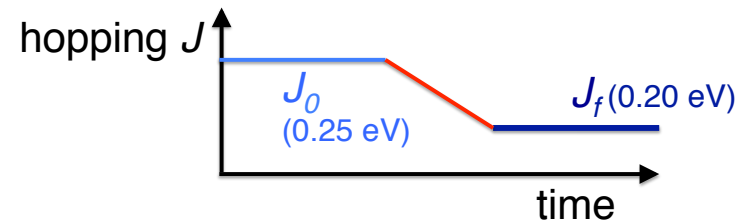
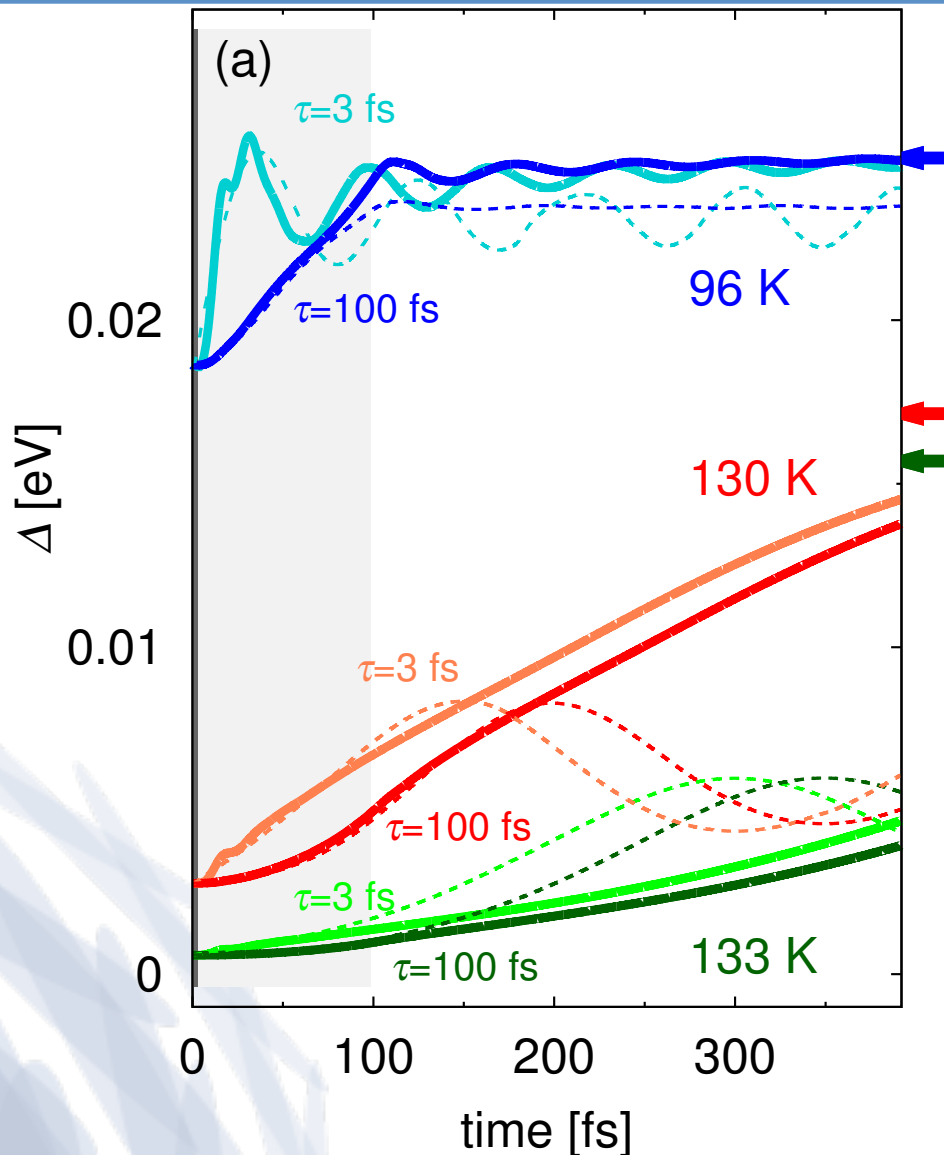


# Simplest model: hopping ramp



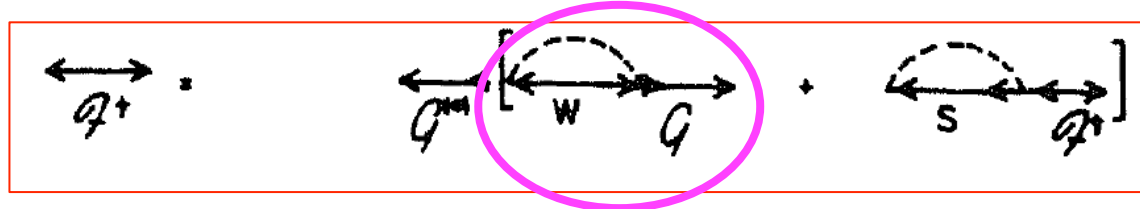
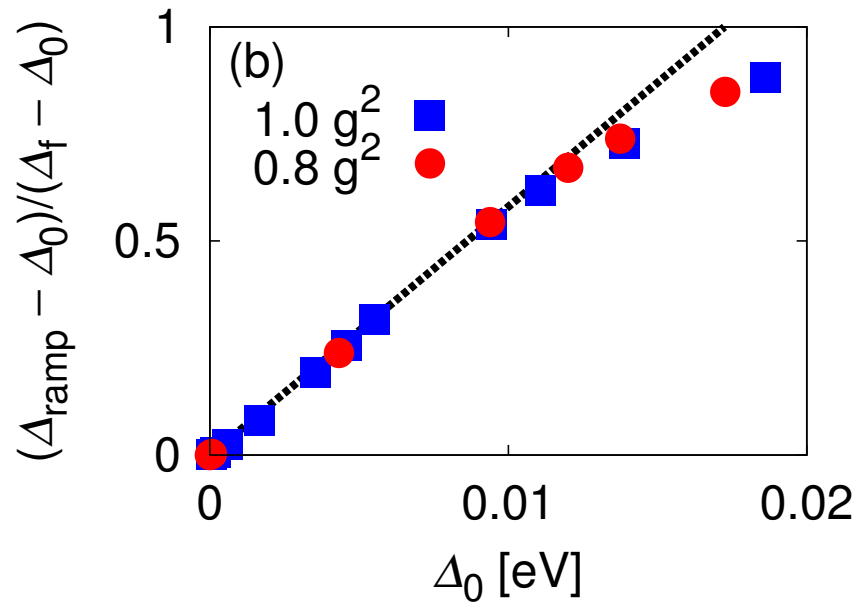
Equilibrium picture:  
Enhancement of SC via  
enhanced DOS at Fermi  
energy

# Superconductor evolution



Enhancement of SC  
strongly depends on  
initial thermal state

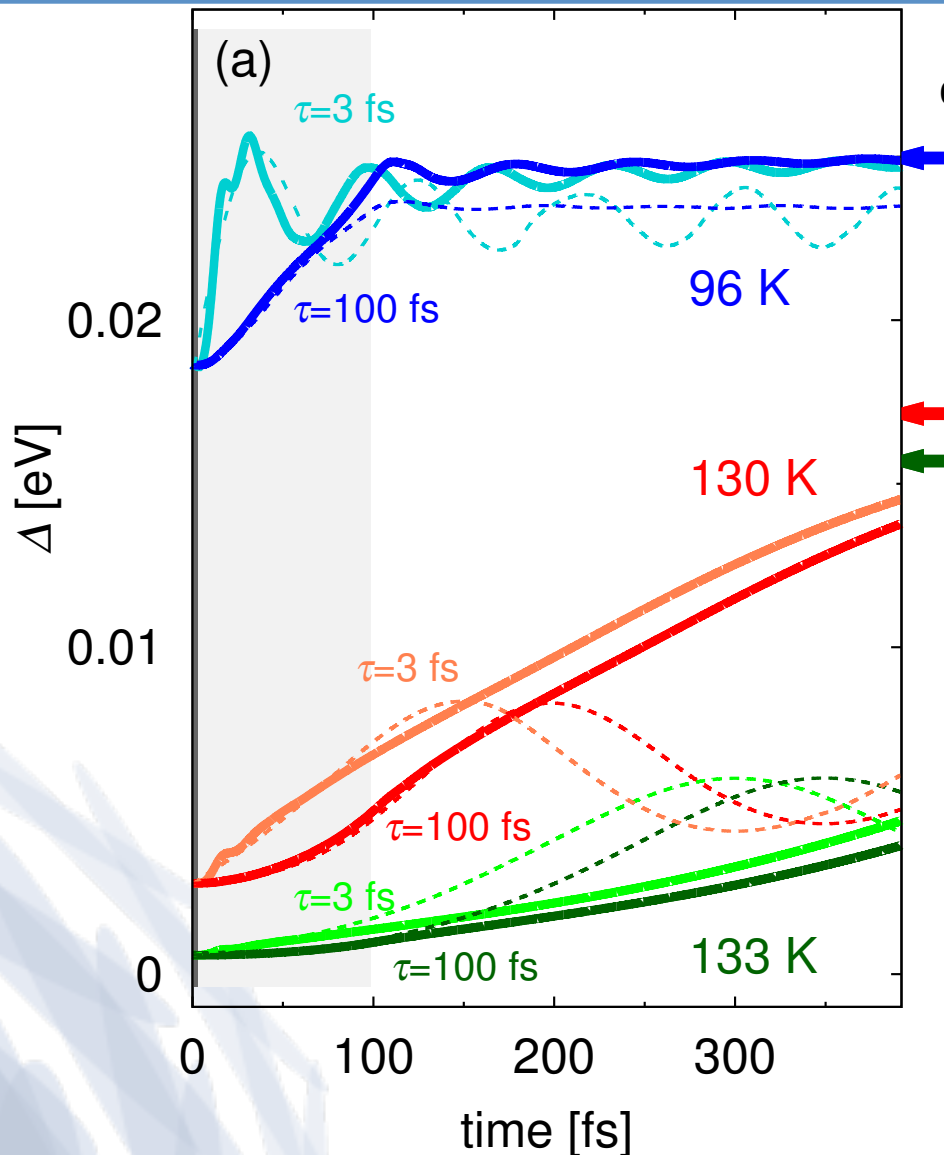
# Enhancement during ramp



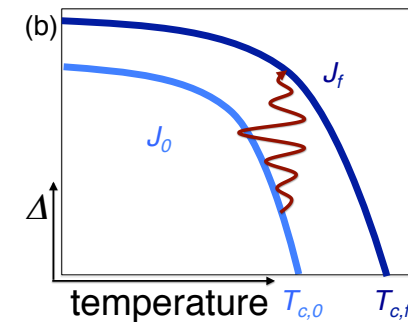
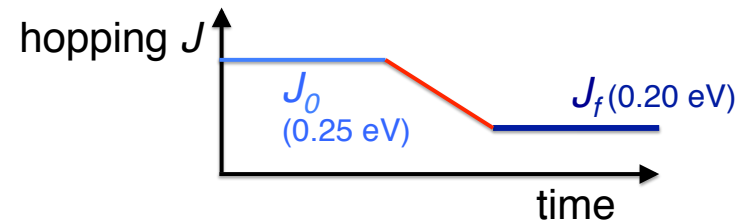
Order parameter enhancement  $\sim \Delta_0$

limit to time scale on which SC can be induced by quasistatic modification of effective pairing strength!

# Superconductor evolution



dashed: no dissipation (BCS only)



Dissipation helps  
enhancement for fast  
ramps



# Summary part 1

- Amplitude mode oscillations in pumped SC

*PRB 92, 224517 (2015)*

- Light-enhanced SC via nonlinear phononics

*arXiv:1505.07575*



A. F. Kemper



T. P. Devereaux



B. Moritz



J. K. Freericks



A. Georges



C. Kollath



GEORGETOWN UNIVERSITY



# Theory of laser-controlled competing orders



Akiyuki Tokuno, Antoine Georges, Corinna Kollath  
(Paris/Bonn)

## Why?

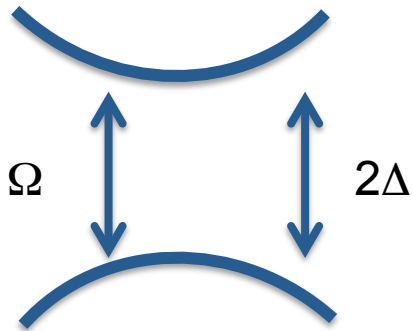
- **understand** ordering mechanisms
- **control** ordered states
- **induce** new states of matter

## How?

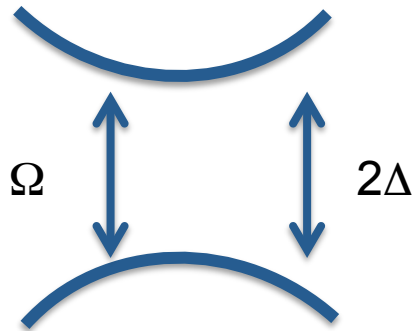
- **resonance** with something

Is there a **generic mechanism** to control ordered states?

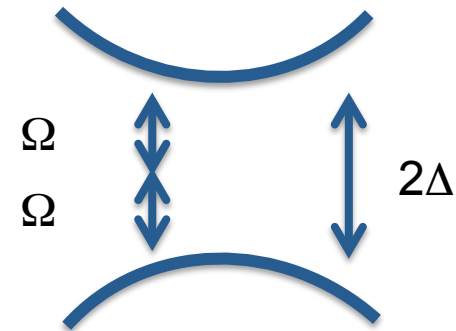
CDW  $\sim A$   
1-photon resonance



CDW  $\sim A$   
1-photon resonance

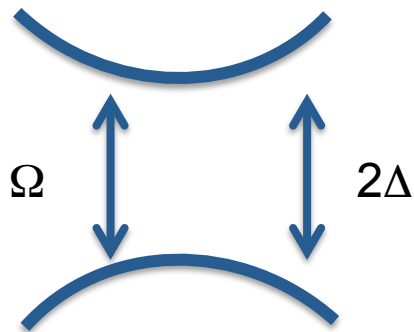


SC  $\sim A^2$   
2-photon resonance

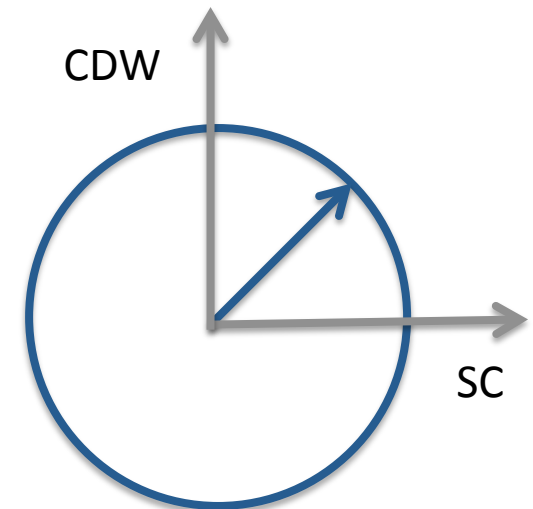
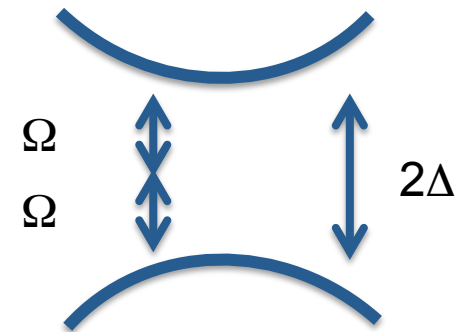


# Driven SC/CDW

CDW  $\sim A$   
1-photon resonance



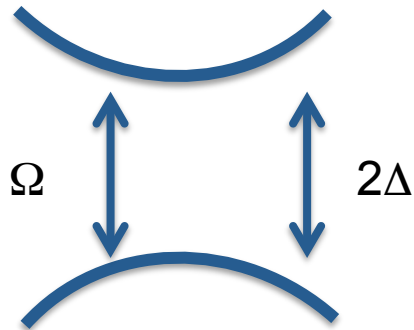
SC  $\sim A^2$   
2-photon resonance



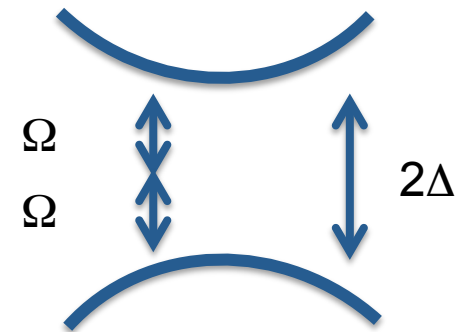


# Driven SC/CDW

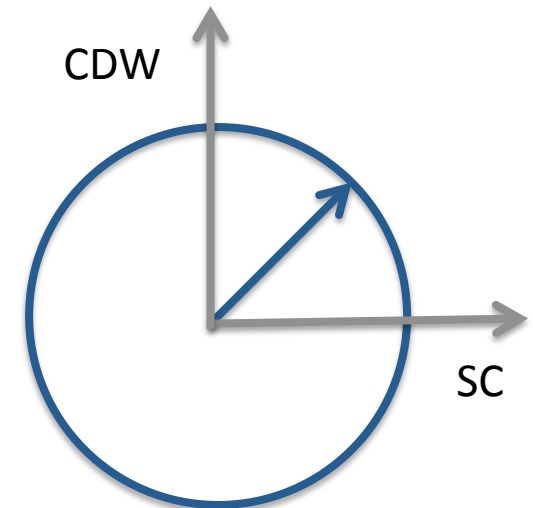
CDW  $\sim A$   
1-photon resonance



SC  $\sim A^2$   
2-photon resonance

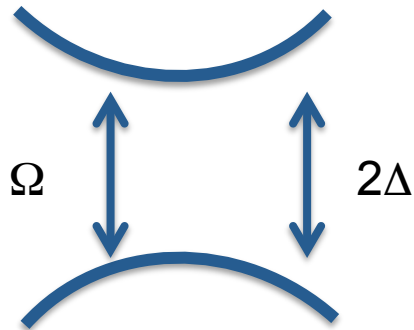


... laser lifts SC/CDW degeneracy

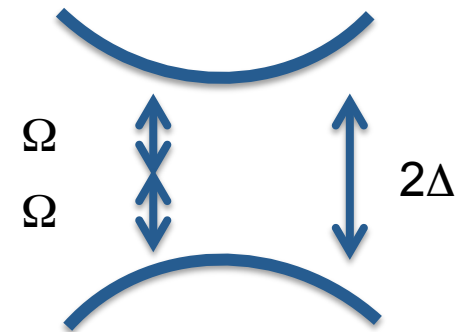


# Driven SC/CDW

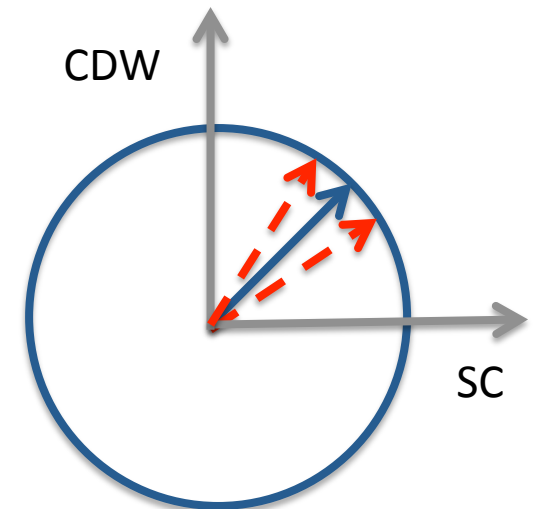
CDW  $\sim A$   
1-photon resonance



SC  $\sim A^2$   
2-photon resonance

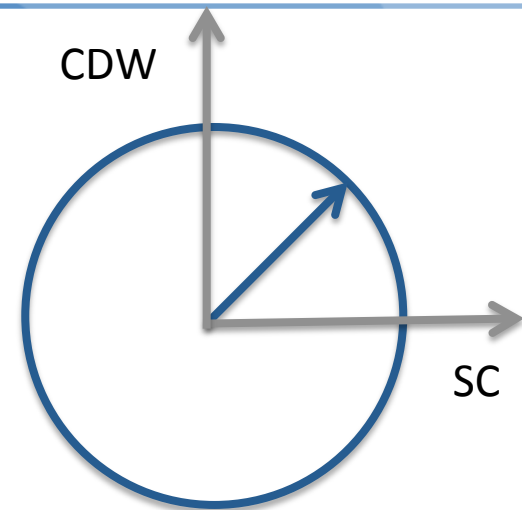


... laser lifts SC/CDW degeneracy  
... Goldstone-like collective mode?



# Competing orders

- attractive  $-U$  Hubbard model
- degeneracy of SC and CDW at perfect nesting
- $SO(4)$  symmetry (SC, CDW, eta pairing)



VOLUME 63, NUMBER 19

PHYSICAL REVIEW LETTERS

6 NOVEMBER 1989

## $\eta$ Pairing and Off-Diagonal Long-Range Order in a Hubbard Model

Chen Ning Yang



C. N. Yang  
(1957 Nobel for  
parity violation in  
weak interaction)



S.-C. Zhang  
(Topological  
Insulators)

Reprinted from Mod. Phys. Lett. B4 (1990) 759–766  
© World Scientific Publishing Company

## $SO_4$ SYMMETRY IN A HUBBARD MODEL

CHEN NING YANG

*Institute for Theoretical Physics, State University of New York,  
Stony Brook, NY 11794-3840, USA*

and

S. C. ZHANG

*IBM Research Division, Almaden Research Center,  
San Jose, CA 95120-6099, USA*

$$H = \sum_{k\sigma} \epsilon(k) n_{k\sigma} + U \sum_i n_{i\uparrow} n_{i\downarrow} = H_J + H_U,$$
$$\epsilon(k) = -2J(\cos(k_x) + \cos(k_y)),$$

attractive  $U$  + mean-field decoupling

$$\Delta_{SC} = U \sum_k f_k, \quad f_k \equiv \langle c_{-k\downarrow} c_{k\uparrow} \rangle \quad (\text{SC}),$$
$$\Delta_{CDW} = U \sum_k g_k, \quad g_k \equiv \frac{1}{2} \sum_{\sigma} \langle c_{k\sigma}^{\dagger} c_{k+Q\sigma} \rangle \quad (\text{CDW}),$$
$$\Delta_{\eta} = U \sum_k \eta_k. \quad \eta_k \equiv \langle c_{-(k+Q)\downarrow} c_{k\uparrow} \rangle \quad (\eta \text{ pairing}).$$

$$H_{MF} = \sum_k \begin{pmatrix} c_{k\uparrow}^\dagger \\ c_{k+Q\uparrow}^\dagger \\ c_{-k\downarrow} \\ c_{-(k+Q)\downarrow} \end{pmatrix}^T \begin{pmatrix} \epsilon(k-A) & \Delta_{CDW}^* & \Delta_{SC} & \Delta_\eta \\ \Delta_{CDW} & \epsilon(k+Q-A) & \Delta_\eta & \Delta_{SC} \\ \Delta_{SC}^* & \Delta_\eta^* & -\epsilon(k+A) & -\Delta_{CDW} \\ \Delta_\eta^* & \Delta_{SC}^* & -\Delta_{CDW}^* & -\epsilon(k+Q+A) \end{pmatrix} \begin{pmatrix} c_{k\uparrow} \\ c_{k+Q\uparrow} \\ c_{-k\downarrow}^\dagger \\ c_{-(k+Q)\downarrow}^\dagger \end{pmatrix}$$

4x4 matrix: SO(4) algebra

$$\begin{aligned} \Delta_{SC} &= U \sum_k f_k, & f_k &\equiv \langle c_{-k\downarrow} c_{k\uparrow} \rangle & (\text{SC}), \\ \Delta_{CDW} &= U \sum_k g_k, & g_k &\equiv \frac{1}{2} \sum_\sigma \langle c_{k\sigma}^\dagger c_{k+Q\sigma} \rangle & (\text{CDW}), \\ \Delta_\eta &= U \sum_k \eta_k. & \eta_k &\equiv \langle c_{-(k+Q)\downarrow} c_{k\uparrow} \rangle & (\eta \text{ pairing}). \end{aligned}$$

$$[G_k^<(t, t')]_{\alpha\beta} = +i\langle[\Psi_k^\dagger(t')]_\beta[\Psi_k(t)]_\alpha\rangle.$$

$$i\partial_t G_k^<(t, t) = [H_{MF}(k, t), G_k^<(t, t)].$$

$$\begin{aligned} i\partial_t n_k &= -\Delta_{SC}(f_k - f_k^*) + \Delta_{CDW}(g_k - g_k^*) - \Delta_\eta^* \eta_k + \Delta_\eta \eta_k^*, & \text{eta pairing provides coupling} \\ i\partial_t f_k &= \Delta_{SC}(1 - (n_k + n_{-k})) + (\epsilon(k - A) + \epsilon(k + A))f_k + \Delta_{CDW}(\eta_k + \eta_{k+Q}) - \Delta_\eta(g_k^* + g_{-k}^*), \\ i\partial_t g_k &= \Delta_{CDW}(n_k - n_{k+Q}) - 2\epsilon(k - A)g_k + \Delta_{SC}(\eta_k^* - \eta_{k+Q}) + \Delta_\eta f_k^* - \Delta_\eta^* f_{k+Q}, \\ i\partial_t \eta_k &= \eta_k(\epsilon(k - A) - \epsilon(k + A)) + \Delta_{CDW}(f_k + f_{k+Q}) - \Delta_{SC}(g_{-k} + g_k^*) - \Delta_\eta(n_k + n_{-(k+Q)} - 1). \end{aligned}$$

nonlinear equations + self-consistency:

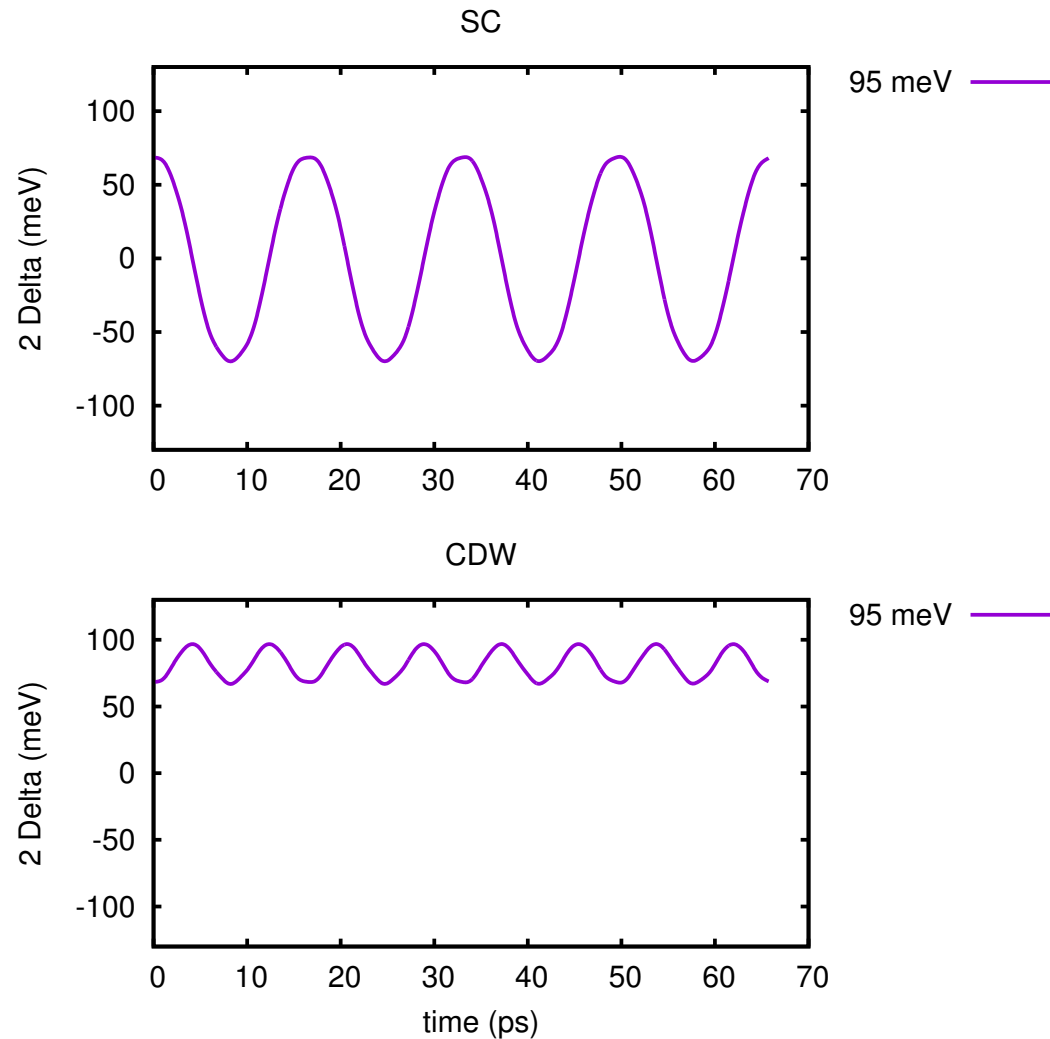
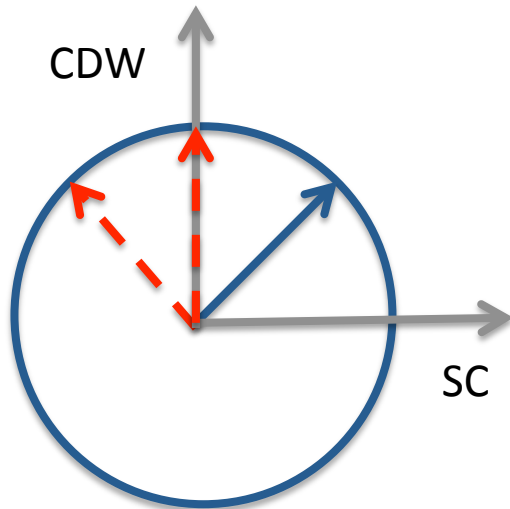
$$\begin{aligned} \Delta_{SC} &= U \sum_k f_k, \\ \Delta_{CDW} &= U \sum_k g_k, \\ \Delta_\eta &= U \sum_k \eta_k. \end{aligned}$$

# Laser hits degenerate orders

---

# Gap resonance – cw driving

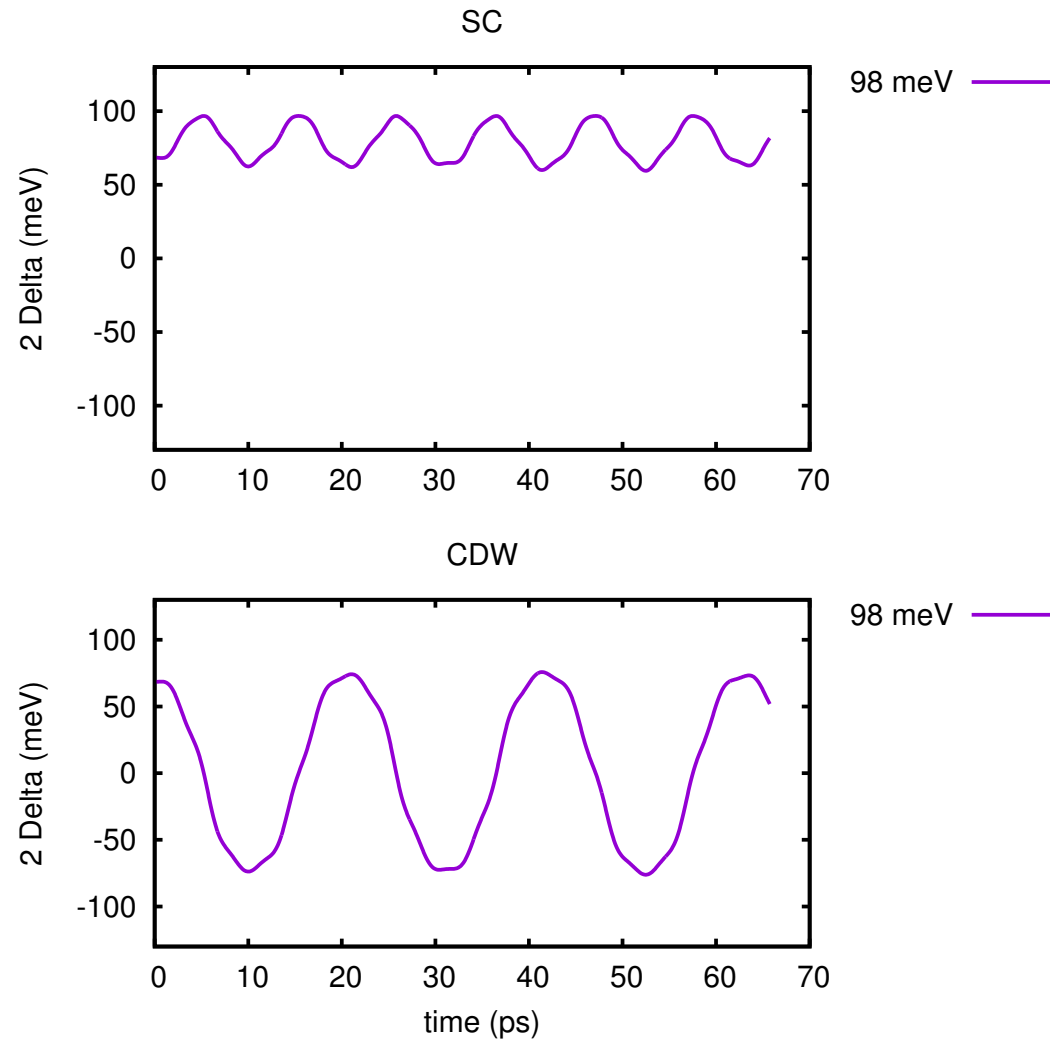
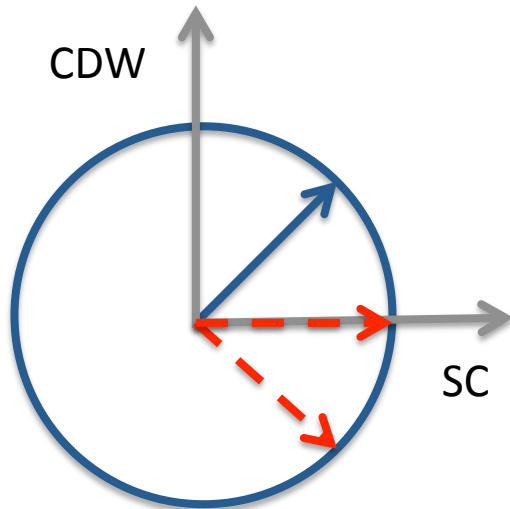
Below resonance:  
SC down, CDW up





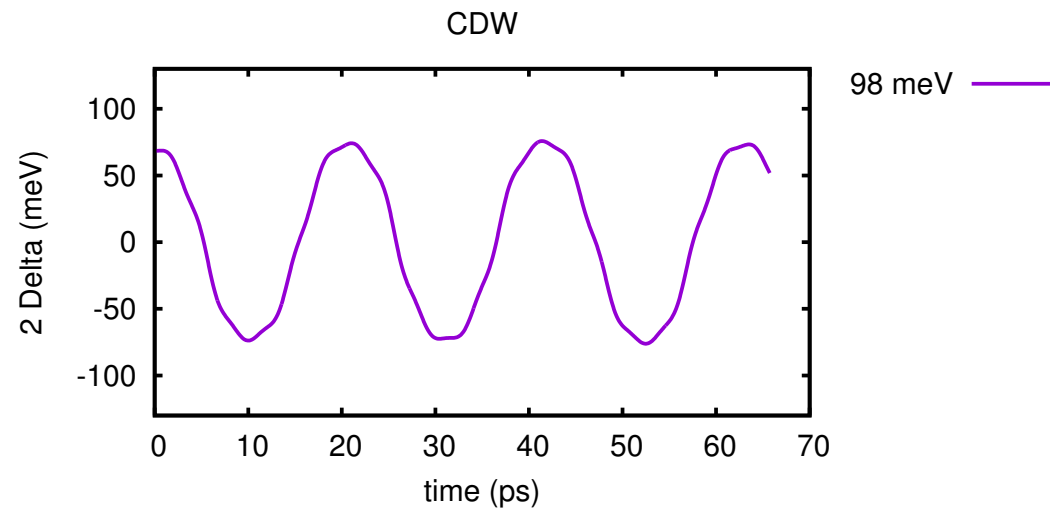
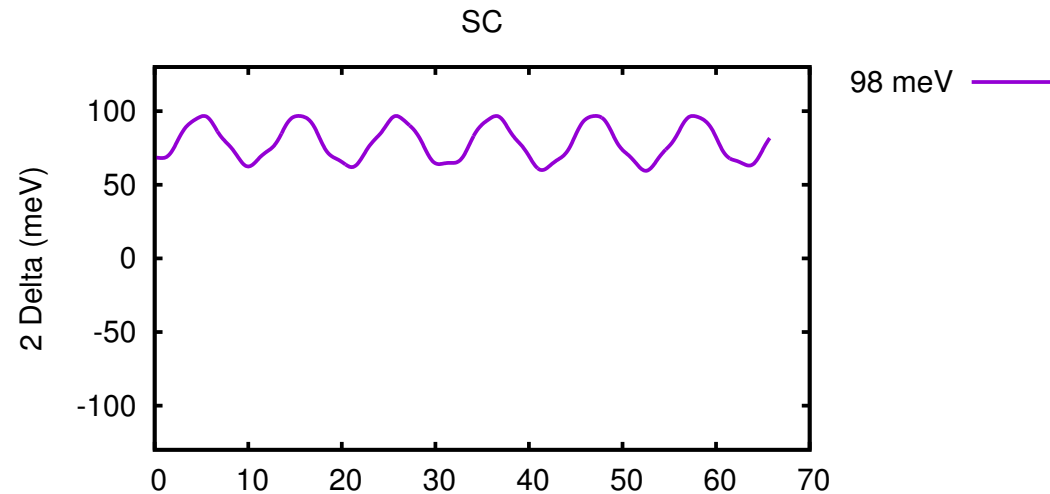
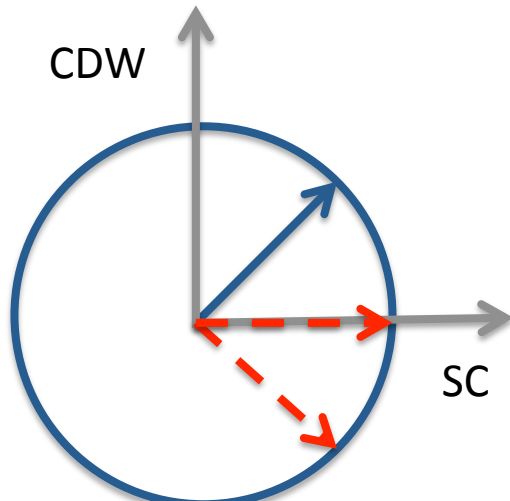
# Gap resonance – cw driving

Above resonance:  
SC up, CDW down

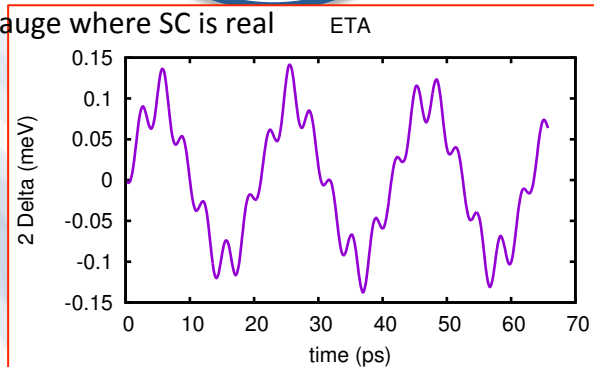


# Gap resonance – cw driving

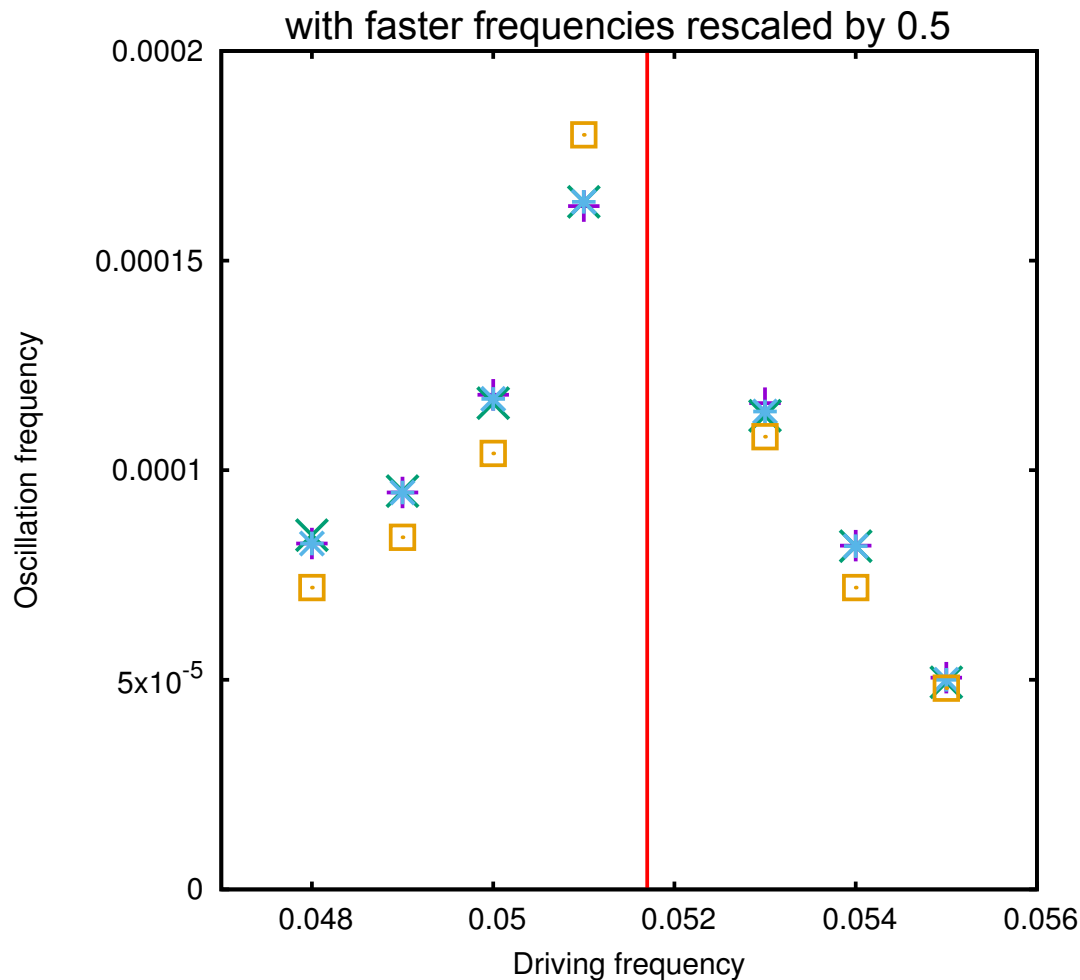
Above resonance:  
SC up, CDW down



imaginary in gauge where SC is real



# Gap resonance – cw driving



SC freq  
CDW freq  
ETA freq  
ETA ampl (x4)

oscillation frequency set  
by light-induced eta  
pairing amplitude,  
which gives „mass“ to  
collective mode

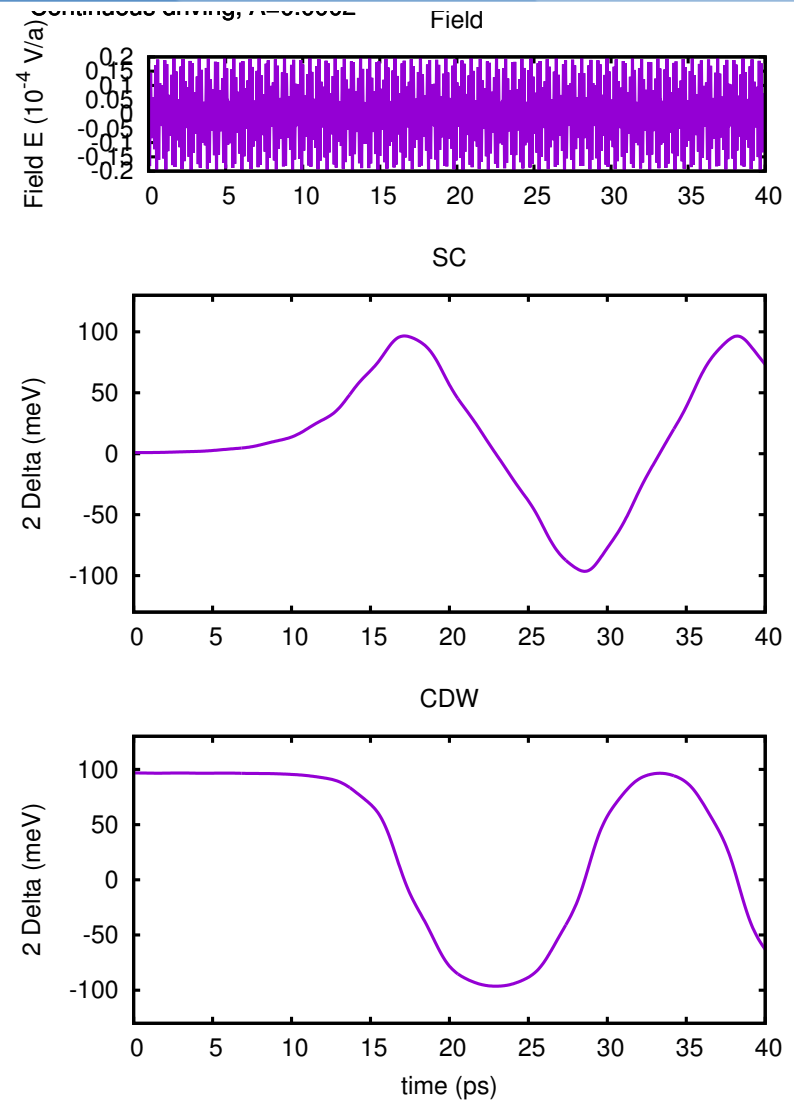
resonant behavior at  
 $\Omega=2\Delta$

# Can we bring SC alive?

# Can we bring SC alive?

CDW initial state

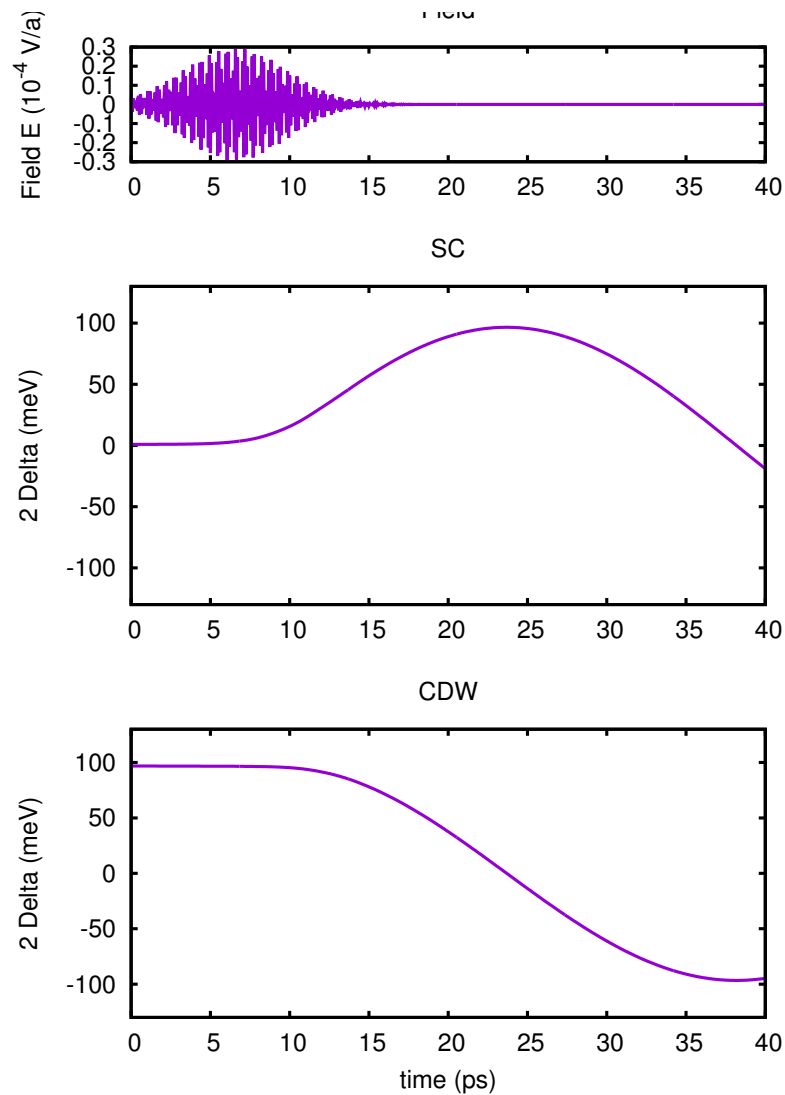
SC comes alive!



# Can we bring SC alive? – pulsed field

CDW initial state

SC comes alive!



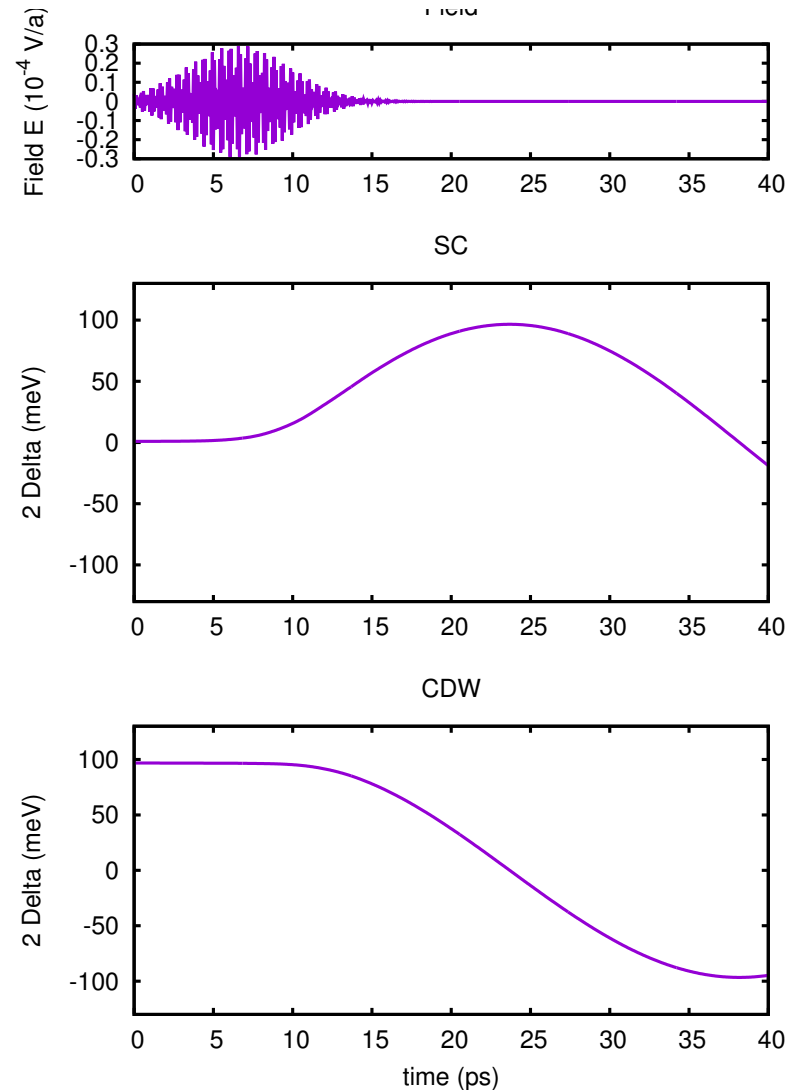
# What controls the dynamics?

Short times: laser control

Long times: dissipation?

## Questions:

1. Can we use dissipation to stabilize SC?
2. Nondegenerate case?



- laser-controlled switching between SC/CDW
- light-induced eta pairing and a collective mode
- analytical theory?
- light-induced long-lived superconductivity possible?

