

Theoretical simulations of pump-probe spectroscopies in solids

Michael A. Sentef

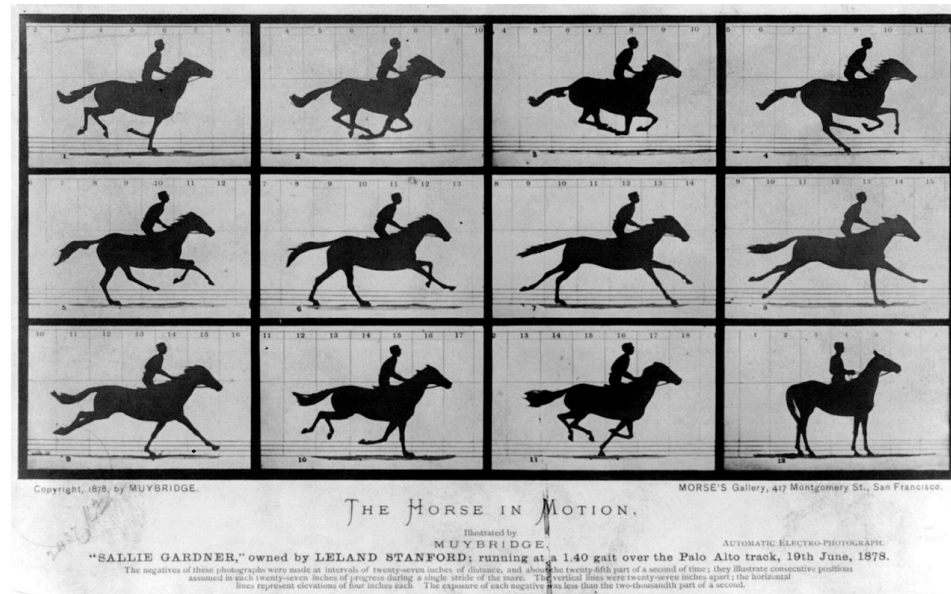
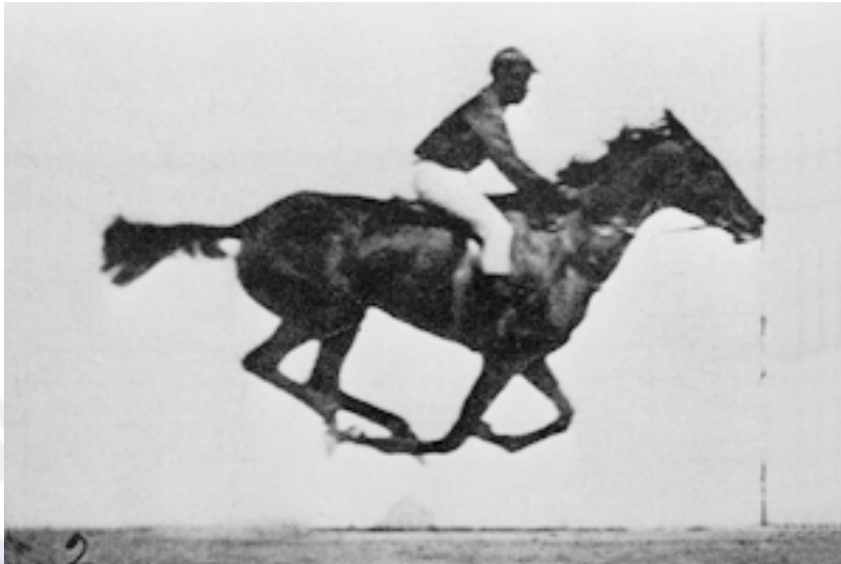
lab.sentef.org

Max-Planck Institute for the Structure and Dynamics of Matter, Hamburg

DPG Spring Meeting, Dresden, March 2017

Pump-probe spectroscopy (1887)

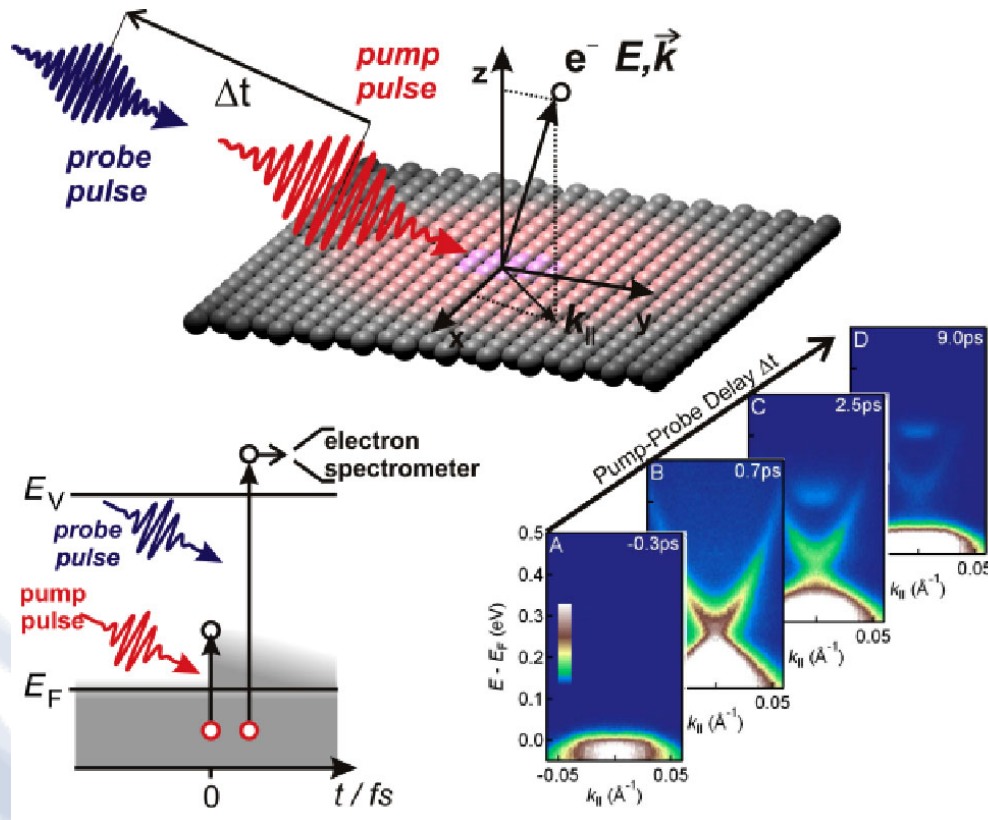
- stroboscopic investigations of dynamic phenomena



Muybridge 1887

Pump-probe spectroscopy (today)

- stroboscopic investigations of dynamic phenomena



TbTe3 CDW metal

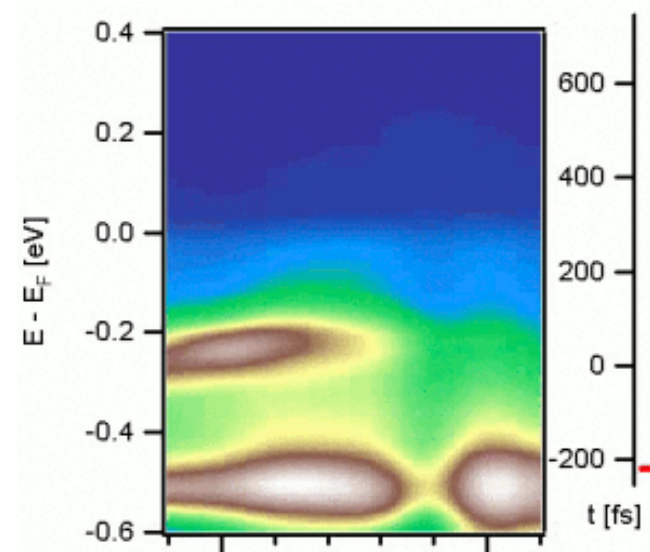


Image courtesy:
J. Sobota / F. Schmitt

Non-Equilibrium Keldysh Formalism

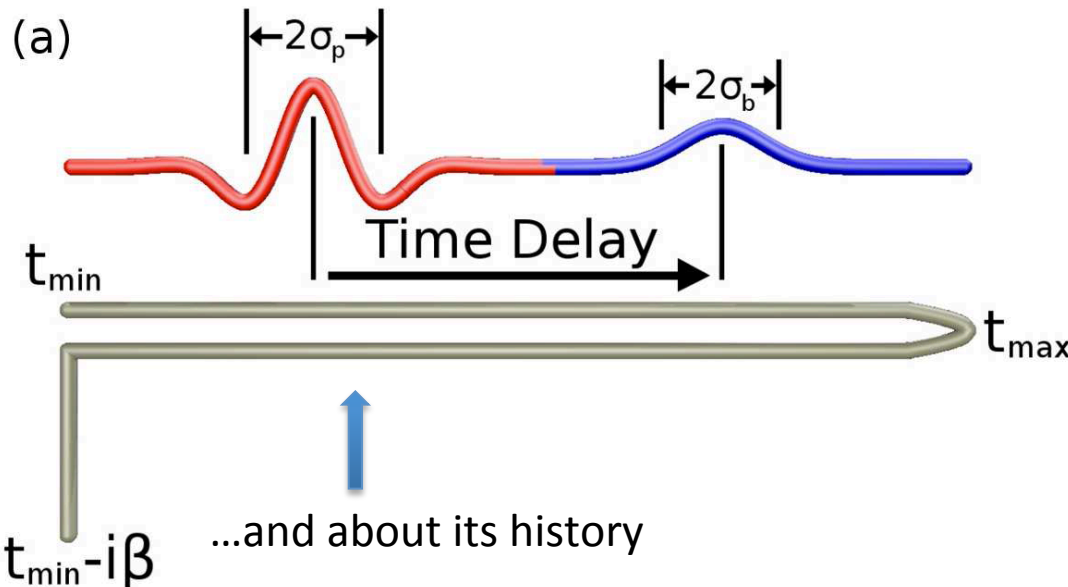
$$G_k(t, t') = G_k^0(t, t') + \int dt_1 \int dt_2 G_k^0(t, t_1) \Sigma(t_1, t_2) G_k(t_2, t')$$

self-energy Σ :
electron-electron scattering
electron-phonon scattering

same problem as in equilibrium
(but worse):
use your favorite self-energy
approximation, e.g. perturbation
theory, nonequilibrium DMFT, ...

Include the effects of driving
field through time-
dependent electronic
dispersion

$$\varepsilon(k) \rightarrow \varepsilon(k, t)$$



System knows about its thermal initial
state...

Electron-boson coupling

PRX 3, 041033 (2013)

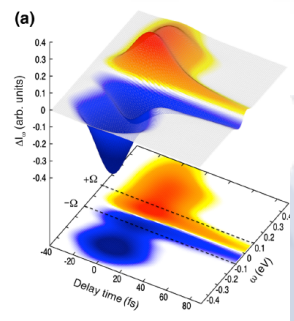
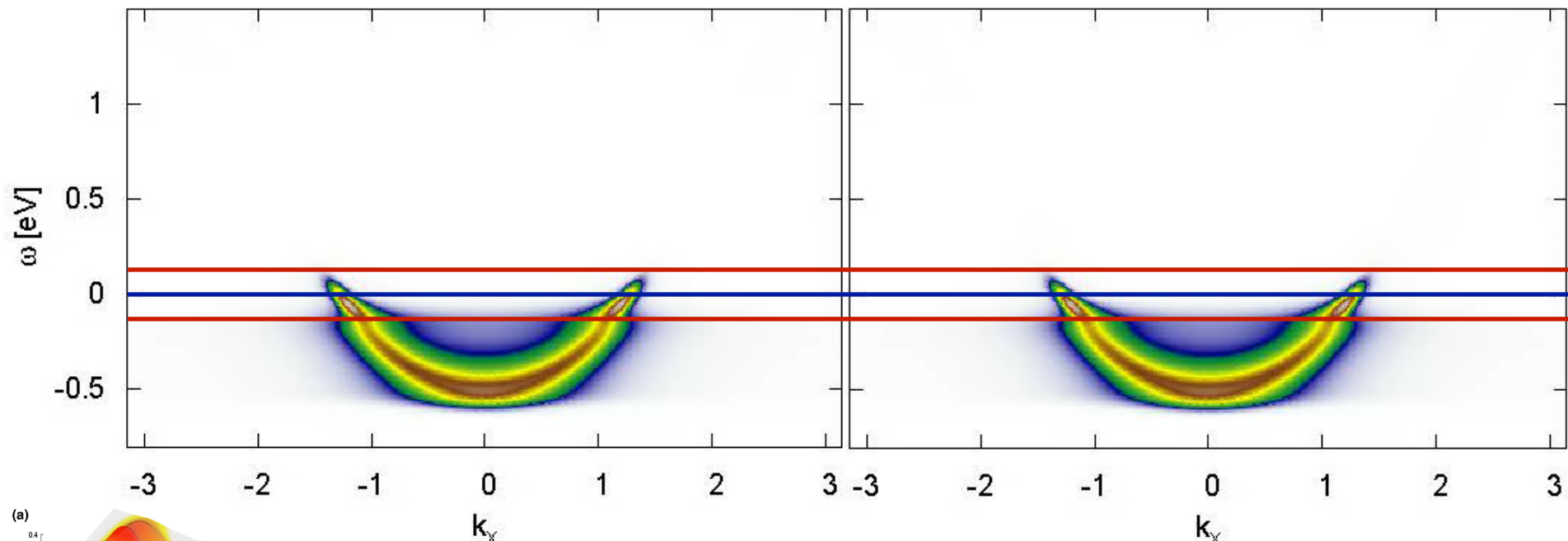
Weak pump

Strong pump

$t = -65.00$

time unit = 0.66 fs

$t = -65.00$



boson window effect for fast versus slow relaxation

nonlinear response for strong pump

Ultrafast Materials Science today

Understanding the nature of quasiparticles

- Relaxation dynamics

- Control of couplings**

PRL 111, 077401 (2013)

PRX 3, 041033 (2013)

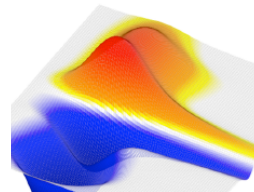
PRB 87, 235139 (2013)

PRB 90, 075126 (2014)

Nature Commun. 7, 13761 (2016)

PRB 95, 024304 (2017)

arXiv:1702.00952



Understanding ordered phases

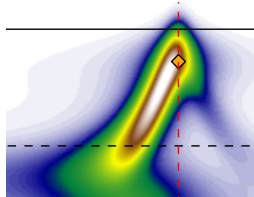
- Collective oscillations

- Competing order parameters**

PRB 92, 224517 (2015)

PRB 93, 144506 (2016)

PRL 118, 087002 (2017)



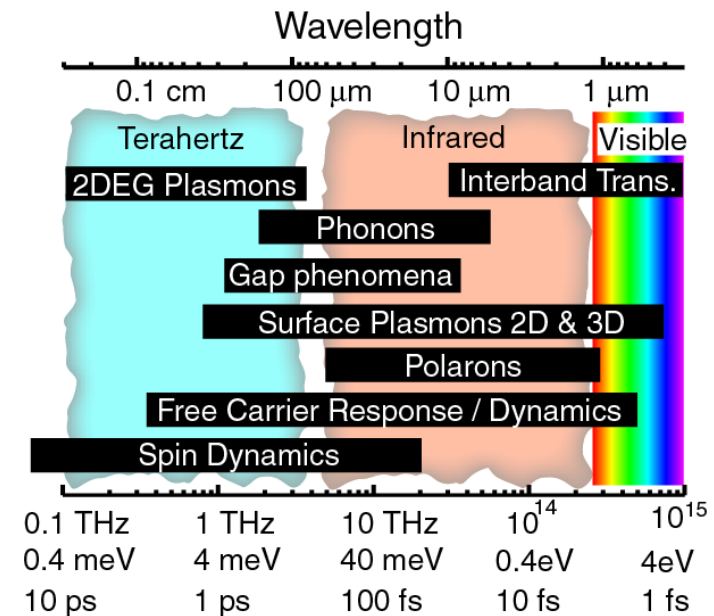
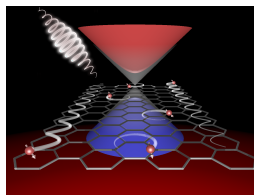
Creating new states of matter

- Photo-induced phase transitions

- Floquet topological states**

Nature Commun. 6, 7047 (2015)

Nature Commun. 8, 13940 (2017)



*Image courtesy:
D. Basov*

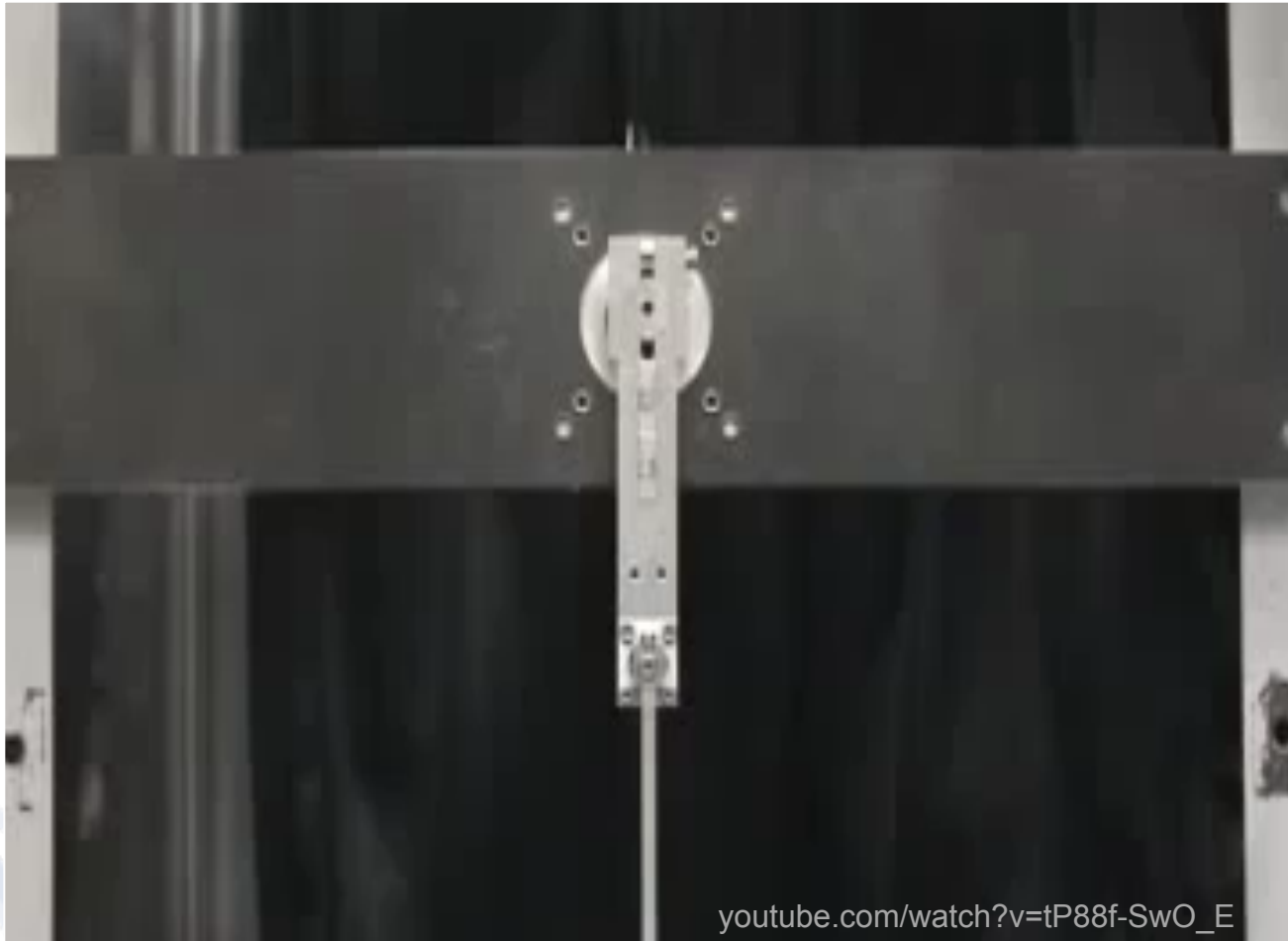
- Part I: Floquet engineering of topological solids
 - Floquet Chern insulator in graphene
Nature Commun. 6, 7047 (2015)
 - Floquet-Weyl semimetal in Na_3Bi
Nature Commun. 8, 13940 (2017)
- Part II: Light-enhanced electron-phonon coupling
PRB 95, 024304 (2017)
arXiv:1702.00952
- Part III: Laser-controlled competing orders
PRL 118, 087002 (2017)

„as time permits“

I. Floquet engineering in solids

Driven is different

Kapitza pendulum



dynamical stabilization of a metastable state

$\omega \rightarrow \text{infinity}$

Kapitza class, dynamical stabilization

Bukov, d'Alessio, Polkovnikov, Adv. Phys. 64, 139-226 (2015)

$\omega \rightarrow \text{finite but largest scale}$

Floquet engineering

$\omega \rightarrow \text{resonances}$

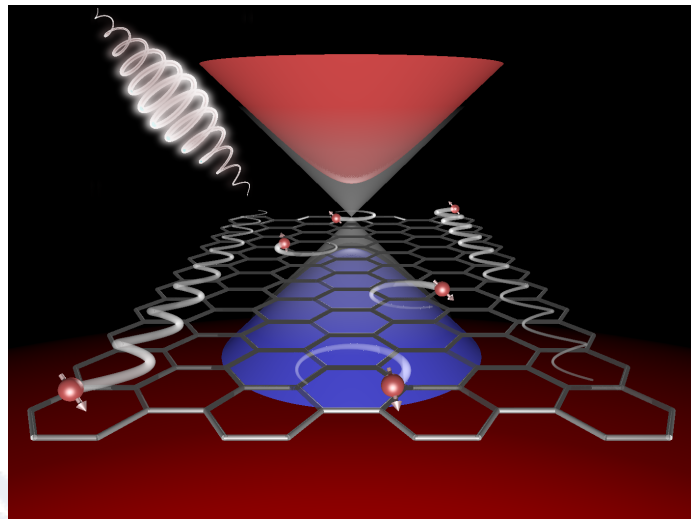
sidebands, huge effects, detuning

$\omega \rightarrow \text{smallest}$

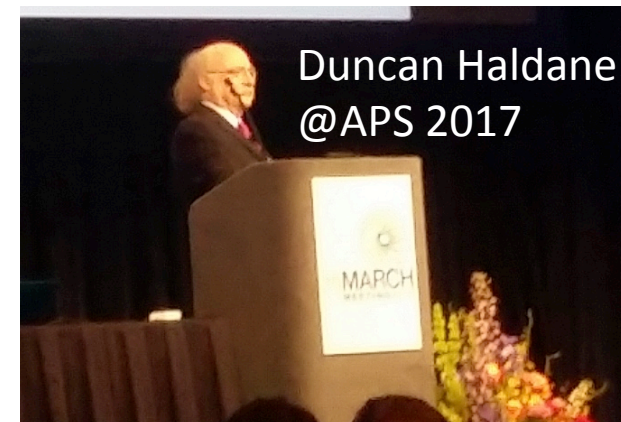
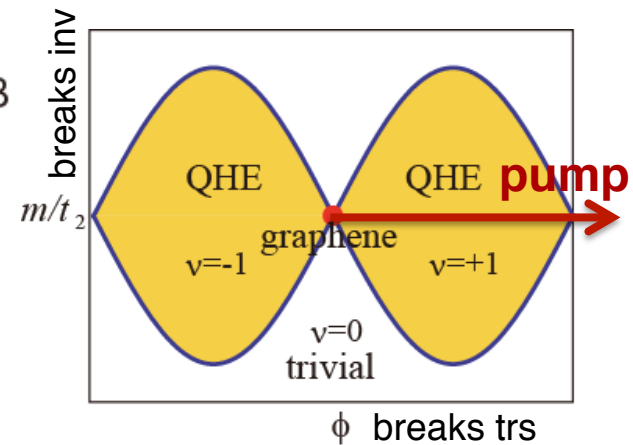
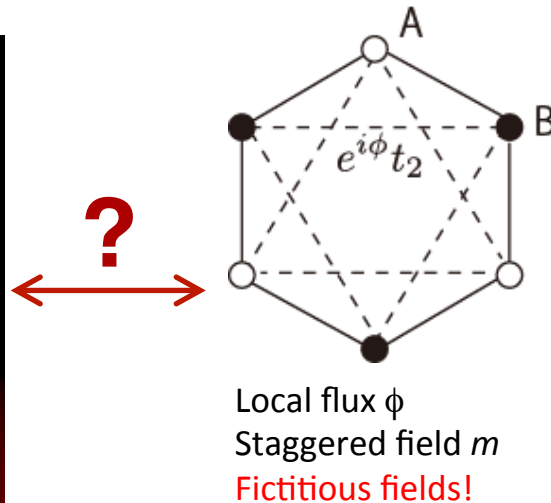
dc physics, adiabatic evolution

Floquet topological states

Graphene + circularly polarized light (breaks trs)



Haldane model (PRL 61, 2015 (1988))



Floquet engineering in a nutshell

time periodic system

$$i\partial_t\psi = H(t)\psi \quad H(t) = H(t+T) \quad \Omega = 2\pi/T$$

“Floquet mapping”
= Bloch state in time



$$\Psi(t) = e^{-i\varepsilon t} \sum_m \phi^m e^{-im\Omega t}$$

Floquet Hamiltonian (static eigenvalue problem)

$$\sum_{m=-\infty}^{\infty} \mathcal{H}^{mn} \phi_{\alpha}^m = \varepsilon_{\alpha} \phi_{\alpha}^n \quad \varepsilon: \text{Floquet quasi-energy}$$

$$(\mathcal{H})^{mn} = \frac{1}{T} \int_0^T dt H(t) e^{i(m-n)\Omega t} + m\delta_{mn}\Omega I$$

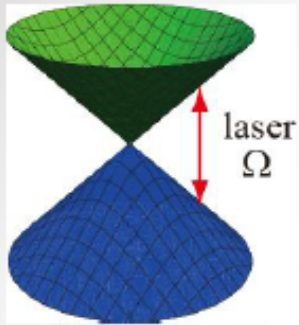
~ absorption of m “photons”



slides courtesy of Takashi Oka

Floquet spectrum: Dirac model + circularly polarized laser

TO, Aoki 2009



$$H^{\text{Floquet}} = \begin{pmatrix} \Omega & k & 0 & A & 0 & 0 \\ \bar{k} & \Omega & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & k & 0 & A \\ A & 0 & \bar{k} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\Omega & k \\ 0 & 0 & A & 0 & \bar{k} & -\Omega \end{pmatrix}$$

0-photon absorbed state

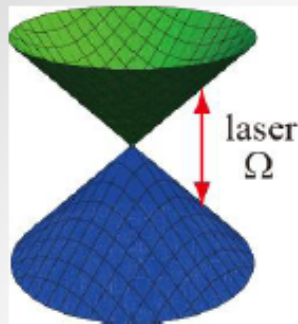


0-photon absorbed state

k_x

Floquet spectrum: Dirac model + circularly polarized laser

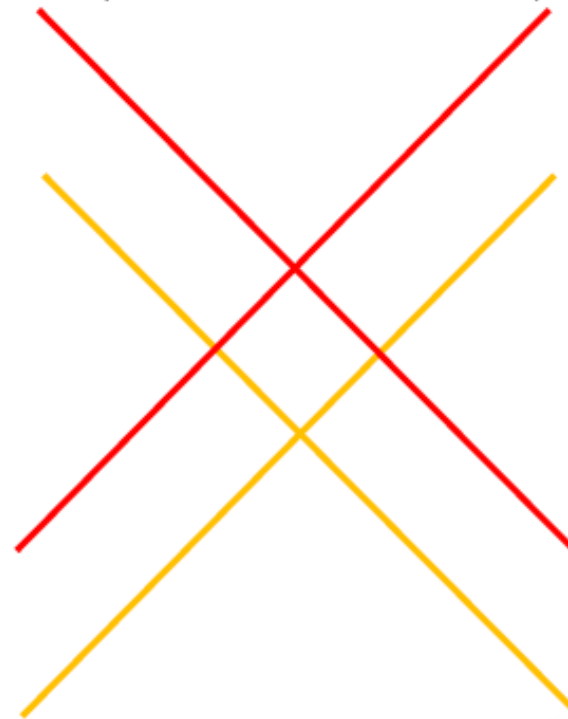
TO, Aoki 2009



$$H^{\text{Floquet}} = \begin{pmatrix} \boxed{\Omega} & \boxed{k} & 0 & A & 0 & 0 \\ \boxed{\bar{k}} & \boxed{\Omega} & 0 & 0 & 0 & 0 \\ 0 & 0 & \boxed{0} & \boxed{k} & 0 & A \\ A & 0 & \boxed{\bar{k}} & \boxed{0} & 0 & 0 \\ 0 & 0 & 0 & 0 & -\Omega & k \\ 0 & 0 & A & 0 & \bar{k} & -\Omega \end{pmatrix}$$

1-photon absorbed state

0-photon absorbed state



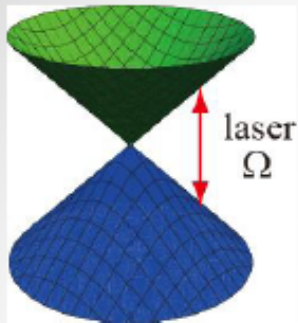
1-photon absorbed state

0-photon absorbed state

$\longrightarrow k_x$

Floquet spectrum: Dirac model + circularly polarized laser

TO, Aoki 2009



$$H^{\text{Floquet}} = \begin{pmatrix} \boxed{\Omega} & \boxed{k} & 0 & A & 0 & 0 \\ \boxed{\bar{k}} & \boxed{\Omega} & 0 & 0 & 0 & 0 \\ 0 & 0 & \boxed{0} & \boxed{k} & 0 & A \\ A & 0 & \boxed{\bar{k}} & \boxed{0} & 0 & 0 \\ 0 & 0 & 0 & 0 & \boxed{-\Omega} & \boxed{k} \\ 0 & 0 & A & 0 & \boxed{\bar{k}} & \boxed{-\Omega} \end{pmatrix}$$

1-photon absorbed state

0-photon absorbed state

-1-photon absorbed state



1-photon absorbed state

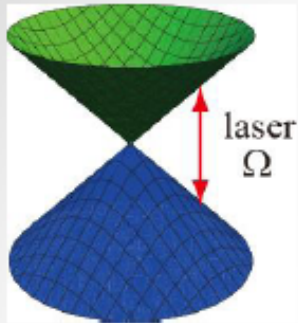
0-photon absorbed state

-1-photon absorbed state

k_x

Floquet spectrum: Dirac model + circularly polarized laser

TO, Aoki 2009

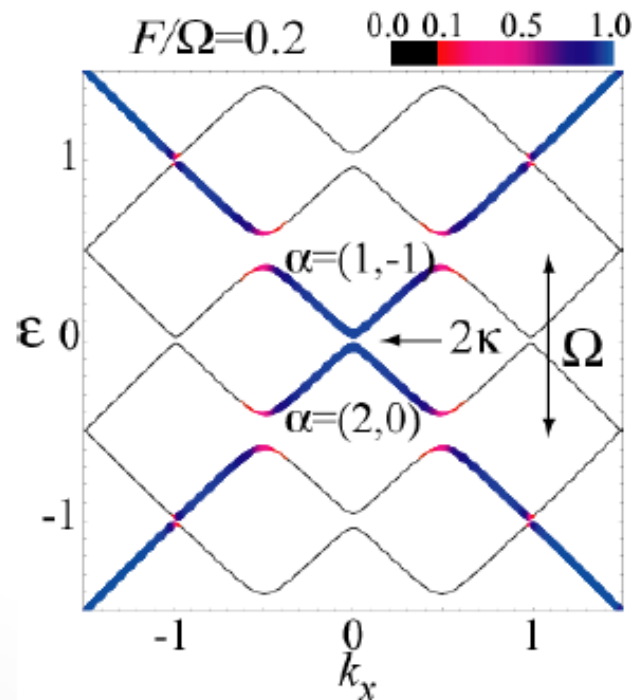


$$H^{\text{Floquet}} = \begin{pmatrix} \boxed{\Omega} & \boxed{k} & \boxed{0} & \boxed{A} & 0 & 0 \\ \boxed{\bar{k}} & \boxed{\Omega} & \boxed{0} & \boxed{0} & 0 & 0 \\ 0 & 0 & \boxed{0} & \boxed{k} & \boxed{0} & \boxed{A} \\ \boxed{A} & \boxed{0} & \boxed{\bar{k}} & \boxed{0} & \boxed{0} & \boxed{0} \\ 0 & 0 & \boxed{0} & \boxed{0} & \boxed{-\Omega} & \boxed{k} \\ 0 & 0 & \boxed{A} & \boxed{0} & \boxed{\bar{k}} & \boxed{-\Omega} \end{pmatrix}$$

1-photon absorbed state

0-photon absorbed state

-1-photon absorbed state



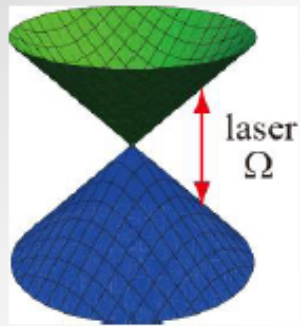
1-photon absorbed state

0-photon absorbed state

-1-photon absorbed state

Floquet spectrum: Dirac model + circularly polarized laser

TO, Aoki 2009

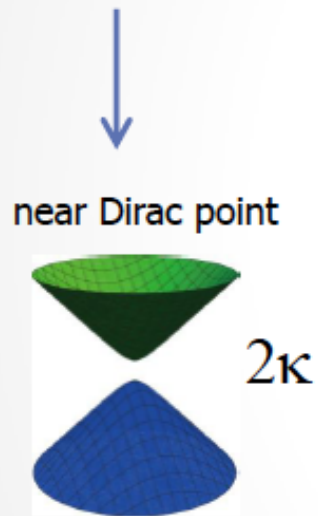


$$H^{\text{Floquet}} = \begin{pmatrix} \boxed{\Omega} & \boxed{k} & \boxed{0} & \boxed{A} & 0 & 0 \\ \boxed{\bar{k}} & \boxed{\Omega} & \boxed{0} & \boxed{0} & 0 & 0 \\ \boxed{0} & \boxed{0} & \boxed{0} & \boxed{k} & \boxed{0} & \boxed{A} \\ \boxed{A} & \boxed{0} & \boxed{\bar{k}} & \boxed{0} & \boxed{0} & \boxed{0} \\ 0 & 0 & 0 & 0 & -\Omega & k \\ 0 & 0 & A & 0 & \bar{k} & -\Omega \end{pmatrix}$$

1-photon absorbed state

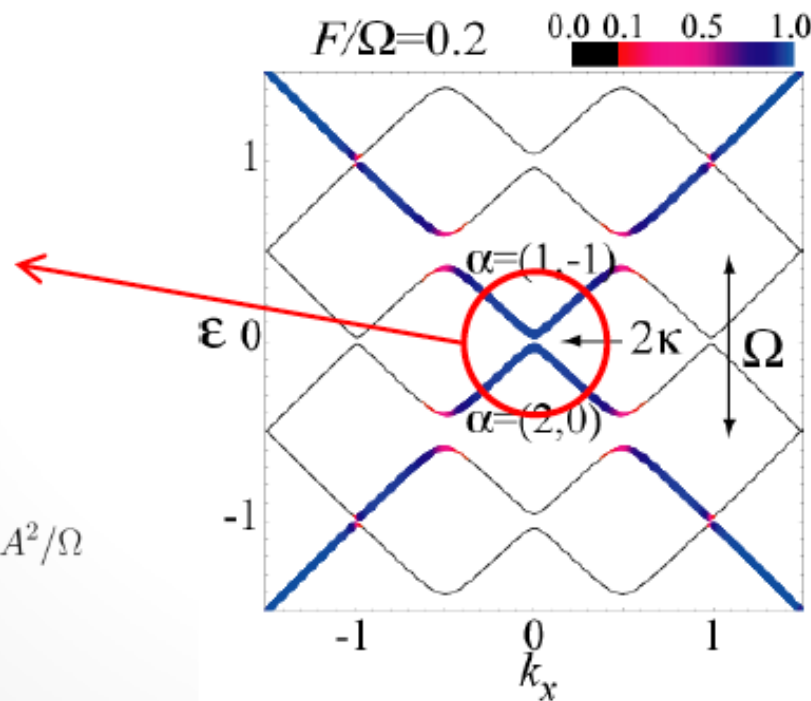
0-photon absorbed state

-1-photon absorbed state



Dirac gap

$$\kappa = \frac{\sqrt{4A^2 + \Omega^2} - \Omega}{2} \sim A^2/\Omega$$



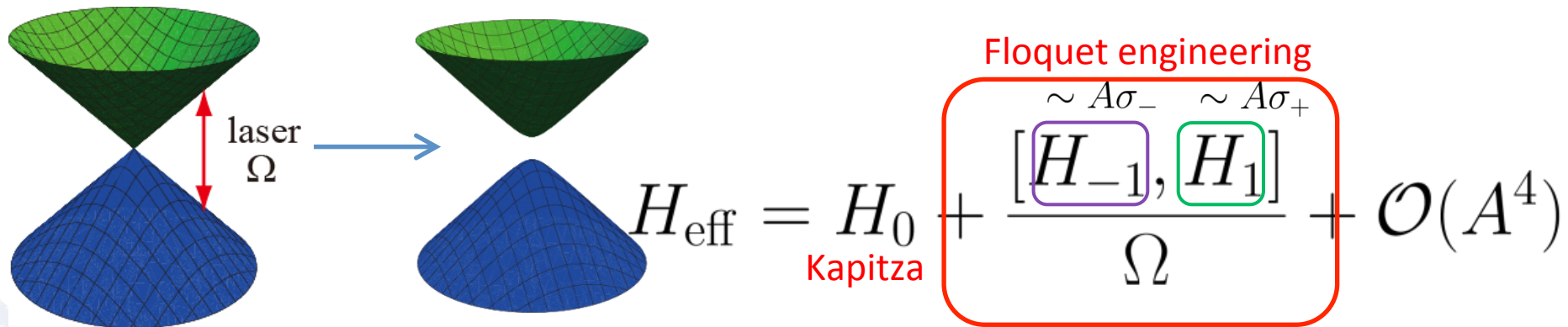
1-photon absorbed state

0-photon absorbed state

-1-photon absorbed state

Dirac fermion + circularly polarized laser

Mass term = energy gap =
synthetic field stemming from a real time-
dependent field $A(t)$


$$H_{\text{eff}} = H_0 + \frac{[\overset{\sim A\sigma_-}{\boxed{H_{-1}}}, \overset{\sim A\sigma_+}{\boxed{H_1}}]}{\Omega} + \mathcal{O}(A^4)$$

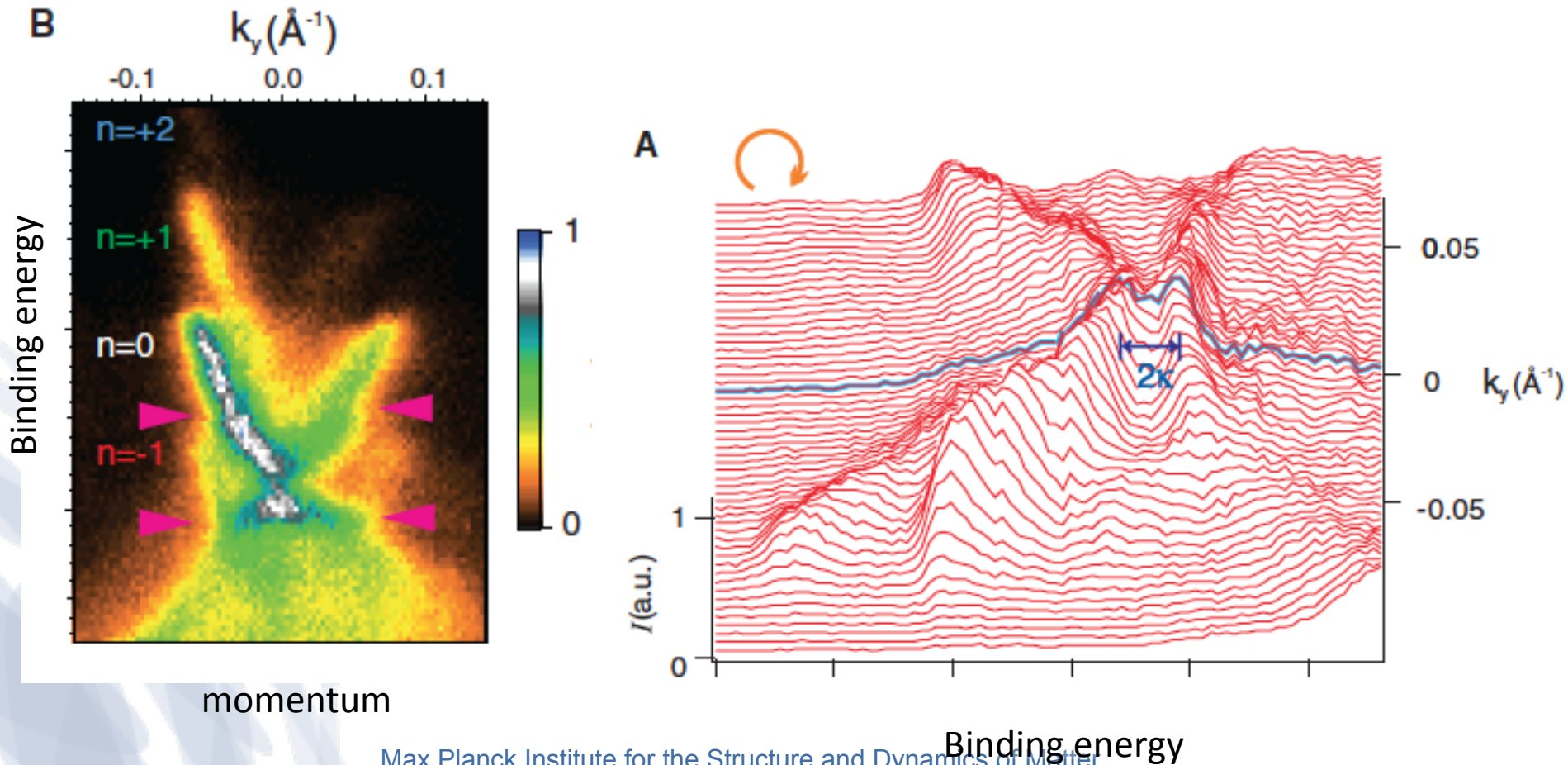
Kapitza

*Oka and Aoki,
PRB 79, 081406 (2009)*

Observation of Floquet-Bloch States on the Surface of a Topological Insulator

Bi_2Se_3
Science **342**, 453 (2013)

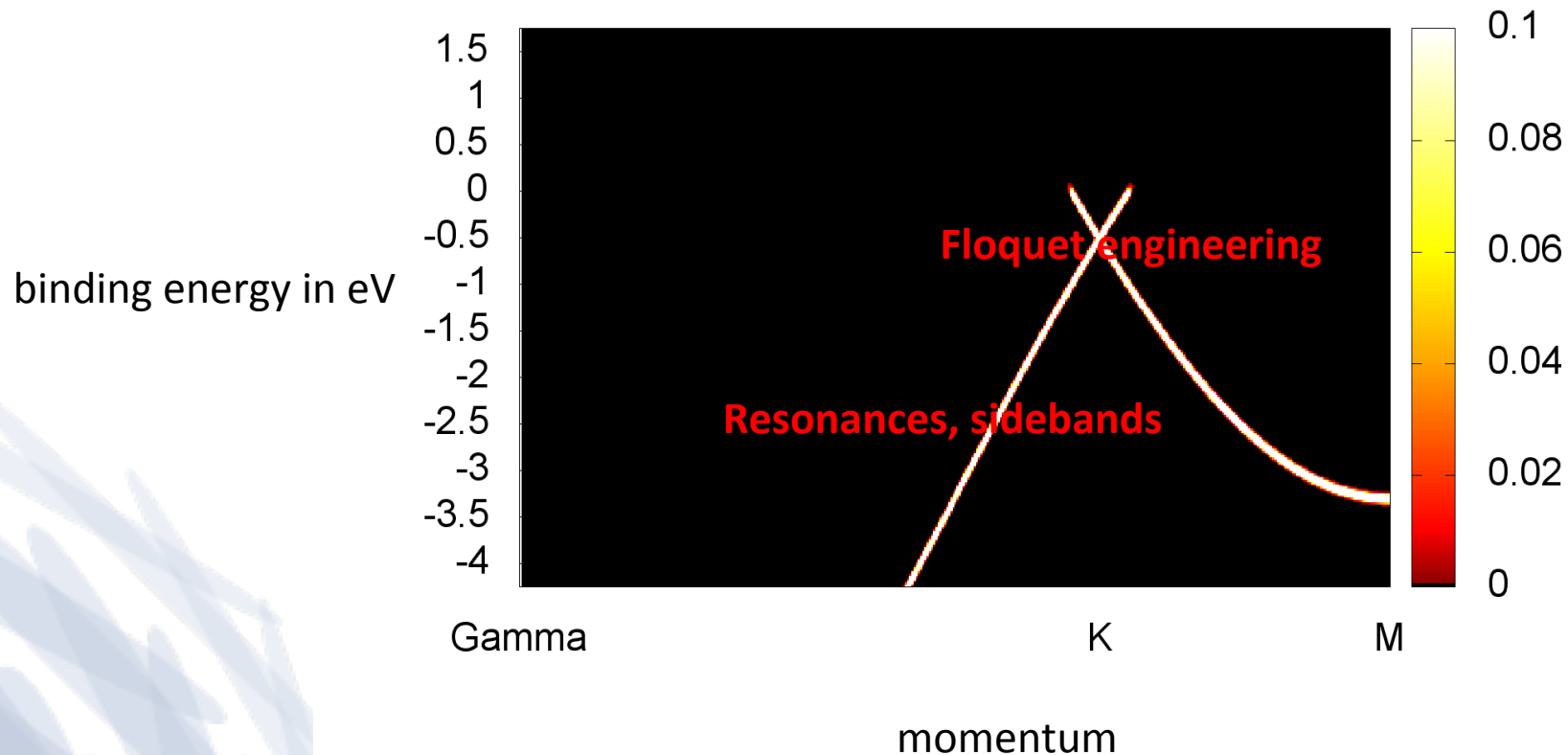
Y. H. Wang,^{*} H. Steinberg, P. Jarillo-Herrero, N. Gedik[†]



Floquet-Bloch states in graphene

Tight-binding model + nonequilibrium Keldysh formalism:
Time-resolved ARPES during 1.5 eV circularly polarized laser pulse
Turning graphene into a Chern insulator

$$A = 4.12179\text{e-}05$$



M. A. Sentef, M. Claassen, A. F. Kemper, B. Moritz, T. Oka, J. K. Freericks, and T. P. Devereaux, Nature Commun. 6, 7047 (2015)

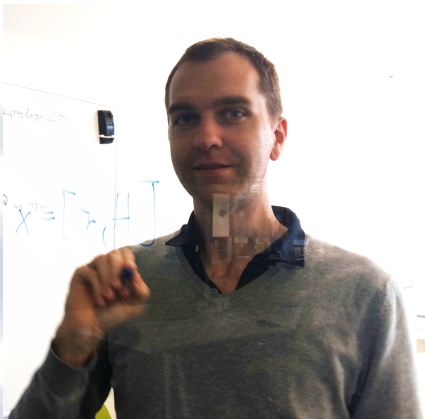
Floquet-Weyl semimetal

Ab initio: TDDFT + Floquet theory

Creating stable Floquet-Weyl semimetals
by laser-driving of 3D Dirac materials

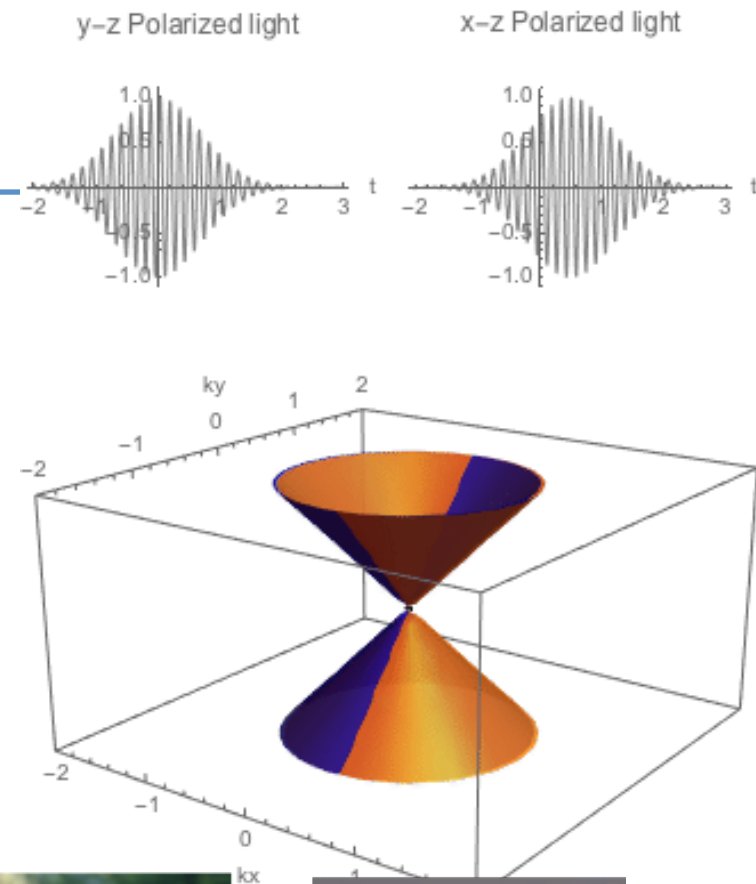
Hannes Hübener✉, Michael A. Sentef, Umberto De Giovannini, Alexander F. Kemper & Angel Rubio✉

Nature Commun. 8, 13940 (2017)



Hannes Hübener Umberto de Giovannini Alexander Kemper Angel Rubio

Max Planck Institute for the Structure and Dynamics of Matter
(NE State)



Summary I

- Floquet engineering: tuning effective parameters and changing materials properties by laser driving

Nature Commun. 6, 7047 (2015)

Nature Commun. 8, 13940 (2017)



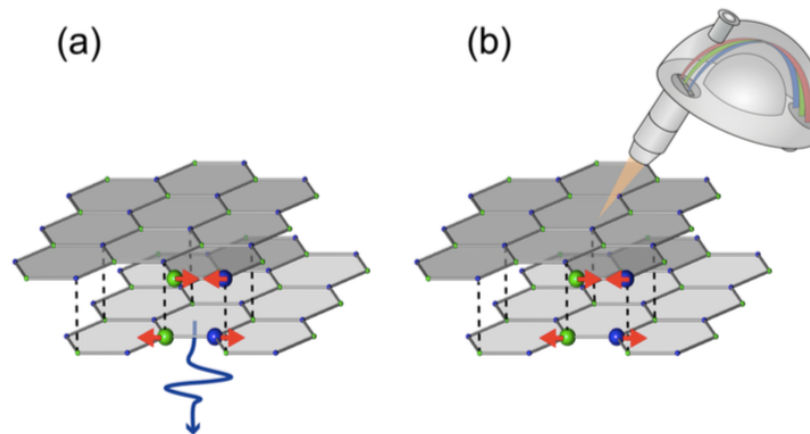
II Dynamically enhanced coupling

Enhanced electron-phonon coupling in graphene with periodically distorted lattice

E. Pomarico, M. Mitrano, H. Bromberger, M. A. Sentef, A. Al-Temimy, C. Coletti, A. Stöhr, S. Link, U. Starke, C. Cacho, R. Chapman, E. Springate, A. Cavalleri, and I. Gierz
Phys. Rev. B **95**, 024304 – Published 13 January 2017

PRB 95, 024304 (2017)

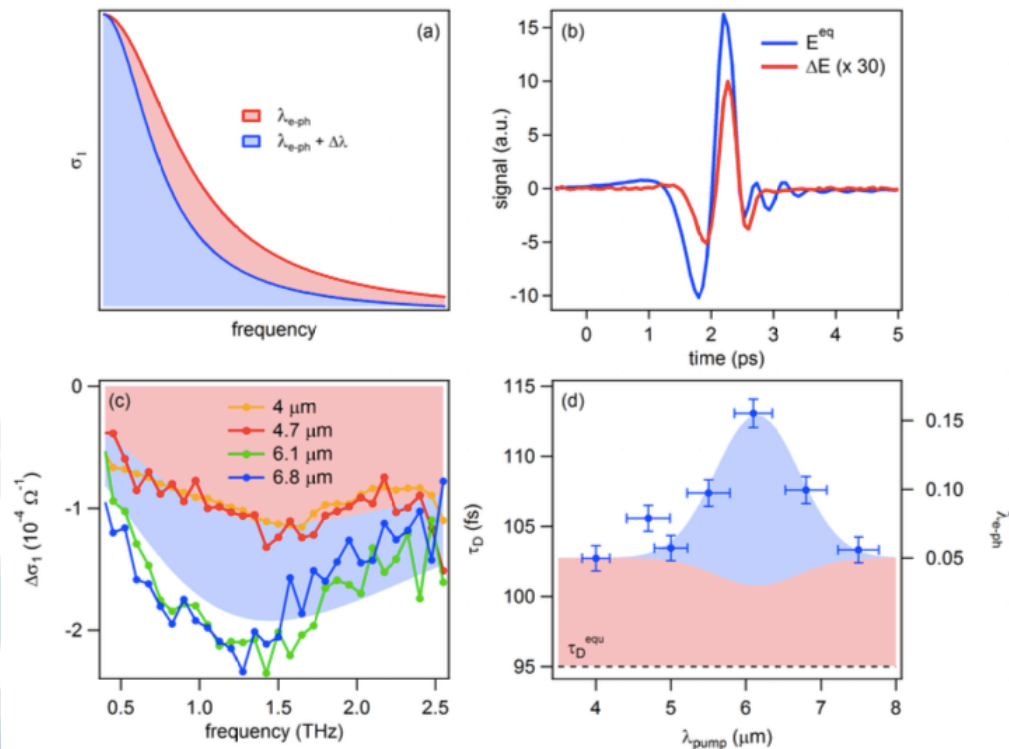
enhanced electron-phonon for pump on resonance with IR phonon



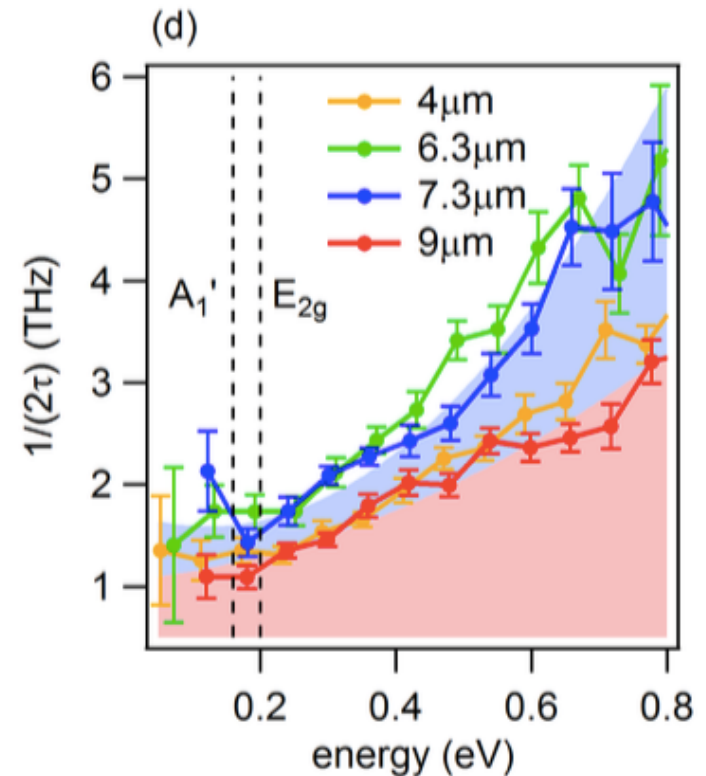
Dynamically enhanced coupling?

Enhanced electron-phonon coupling in graphene with periodically distorted lattice *PRB 95, 024304 (2017)*

transient reduction of Drude weight



enhanced tr-ARPES relaxation



3-fold enhancement of effective $\lambda_{\text{el-ph}}$! Why?

2-site model with nonlinear coupling

arXiv:1702.00952

$$\begin{aligned}\hat{H}(t) = & -J \sum_{\sigma} (c_{1,\sigma}^{\dagger} c_{2,\sigma} + c_{2,\sigma}^{\dagger} c_{1,\sigma}) \\ & + g_2 \sum_{\sigma, l=1,2} \hat{n}_{l,\sigma} (b_l + b_l^{\dagger})^2 \\ & + \Omega \sum_{l=1,2} b_l^{\dagger} b_l + F(t) \sum_{l=1,2} (b_l + b_l^{\dagger}),\end{aligned}$$

also cf.
Kennes et al.,
Nature Physics (2017),
1609.03802

Idea: Drive nonlinearly coupled phonon and look at electronic response

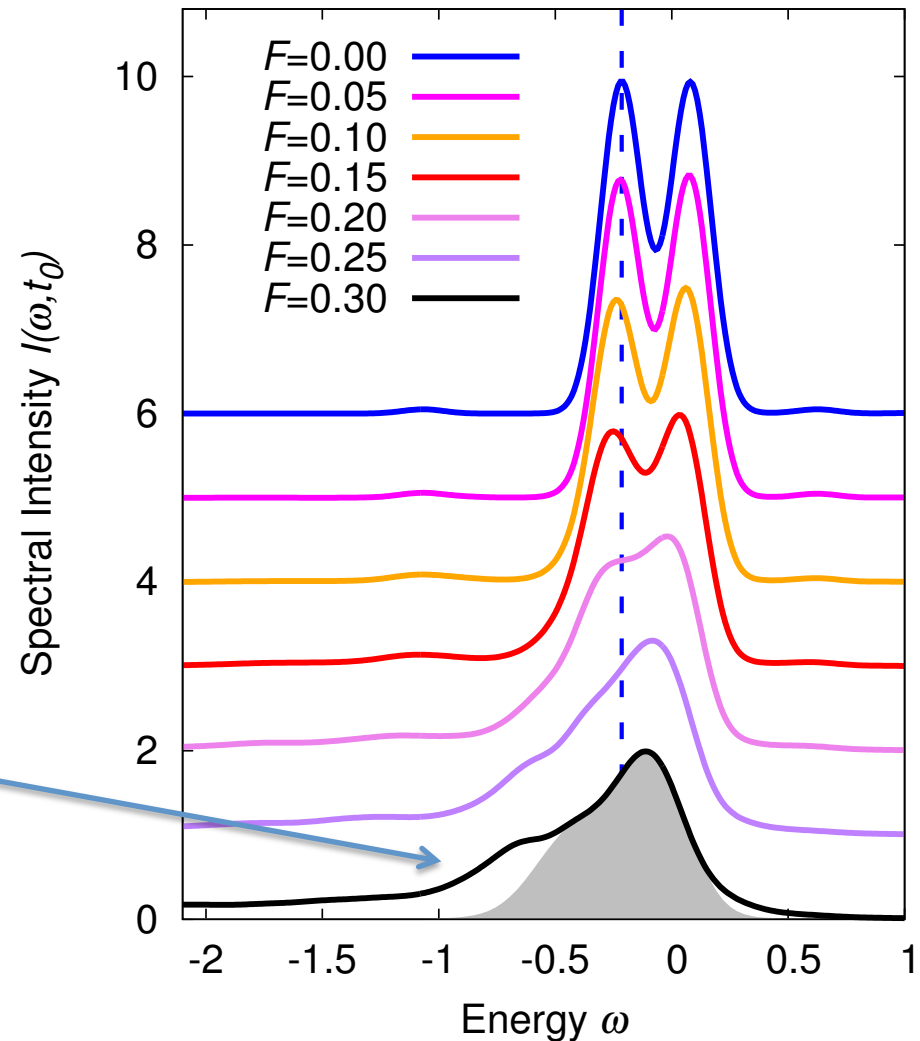
Drive: $F(t) = F \sin(\omega t),$

Response:
$$I(\omega, t_0) = \text{Re} \int dt_1 dt_2 e^{i\omega(t_1-t_2)} s_{t_1, t_2, \tau}(t_0) \\ \times \left[\langle \psi(t_2) | c_{1,\uparrow}^{\dagger} \mathcal{T} e^{-i \int_{t_1}^{t_2} H(t) dt} c_{1,\uparrow} | \psi(t_1) \rangle + \right. \\ \left. + \langle \psi(t_1) | c_{1,\uparrow} \mathcal{T} e^{-i \int_{t_2}^{t_1} H(t) dt} c_{1,\uparrow}^{\dagger} | \psi(t_2) \rangle \right],$$

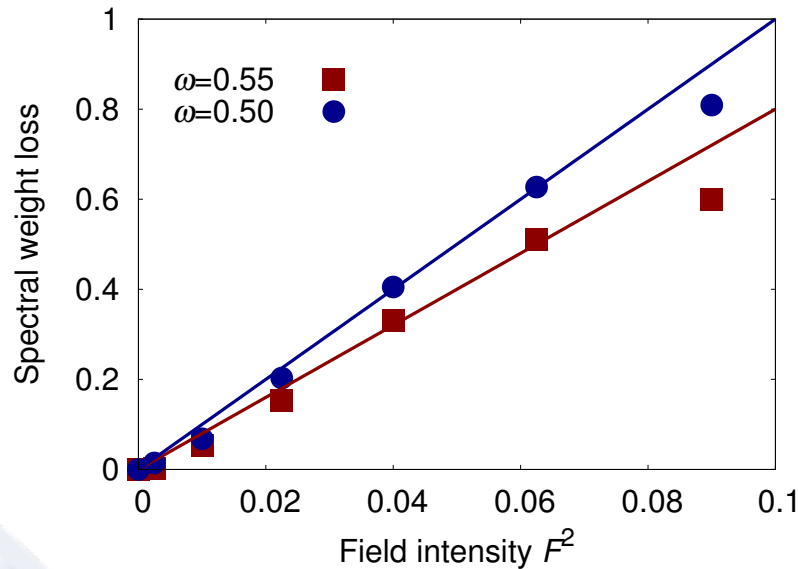
Reduced coherence peaks
with stronger driving

Looks like enhanced el-ph
coupling

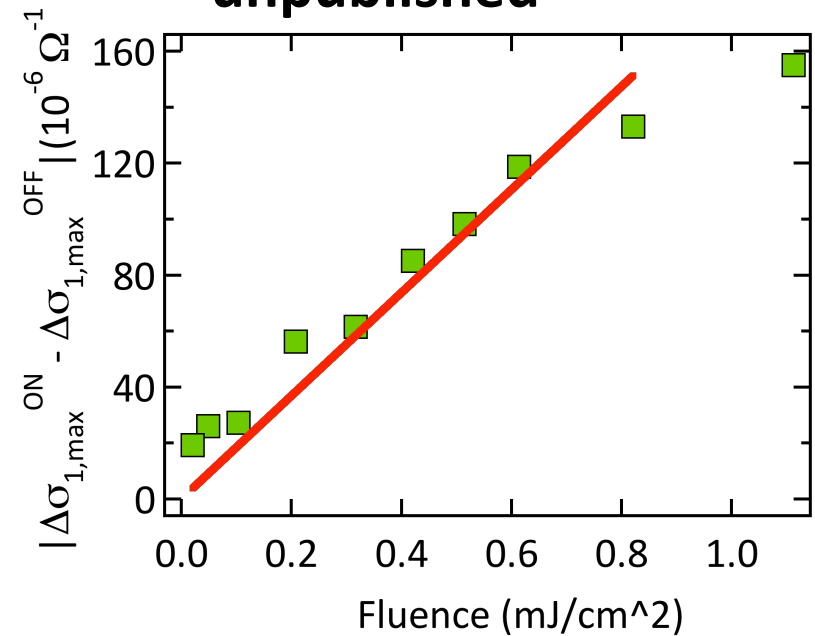
light-induced polaron formation



Theory



Data by E. Pomarico, unpublished



Scaling of coherent spectral weight loss: proportional to field intensity **consistent with experiments**

- enhanced electron-phonon coupling in phononically driven bilayer graphene

PRB 95, 024304 (2017)



E. Pomarico



I. Gierz



A. Cavalleri

Exact solution of (small) electron-phonon model system:

- theoretical proposal: nonlinear el-ph coupling as mechanism behind this enhancement

arXiv:1702.00952

Summary

From models to materials:

- theoretical simulations enable us to reveal mechanisms behind ultrafast dynamics in solids

THANK YOU!



III Theory of laser-controlled competing orders

Phys. Rev. Lett. 118, 087002 (2017)

Why?

- **understand** ordering mechanisms
- **control** ordered states: ultrafast switching
- **induce** new states of matter

How?

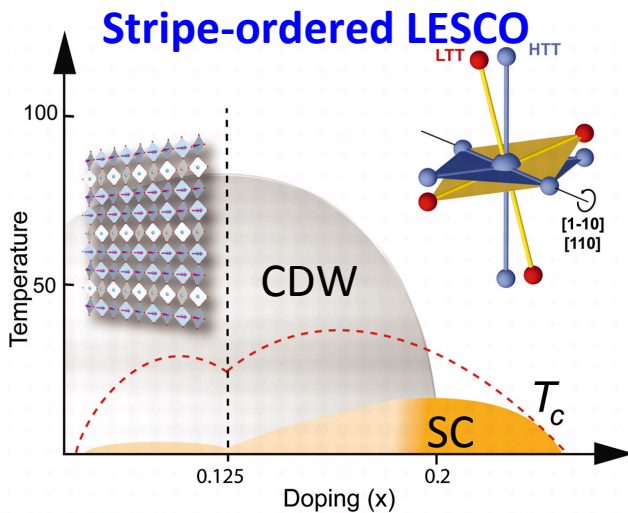
- **laser near resonance** with collective modes

Generic mechanism to control competing orders with light?

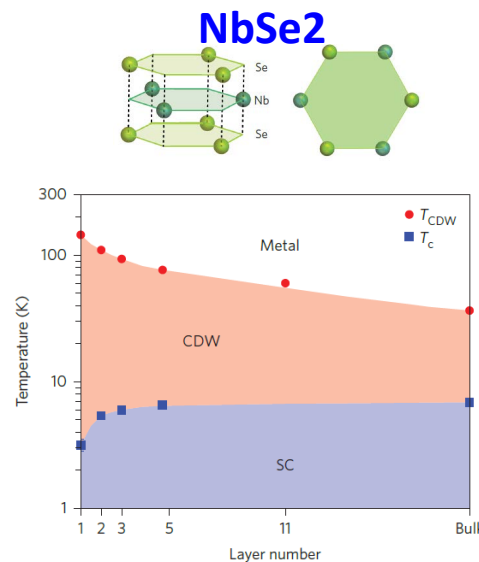
Recent theories on laser-controlled couplings and competing orders:

Akbari et al., EPL 101, 17003 (2013); Moor et al., PRB 90, 024511 (2014); Fu et al., PRB 90, 024506 (2014); Dzero et al., PRB 91, 214505 (2015); Tsuji&Aoki, PRB 92, 064508 (2015); Cea et al., PRB 93, 180507 (2016); Kemper et al., PRB 92, 224517 (2015); Sentef et al., PRB 93, 144506 (2016); Krull et al., Nat. Commun. 7, 11921 (2016); Patel&Eberlein, PRB 93, 195139 (2016); Knap et al., PRB 94, 214504 (2016); Komnik&Thorwart EPJB 89, 244 (2016); Coulthard et al., 1608.03964; Kennes et al., Nat. Physics (2017), doi:10.1038/nphys4024; Sentef, 1702.00952; Babadi et al. 1702.02531; Murakami et al., 1702.02942; Mazza&Georges, 1702.04675; Dehghani&Mitra, 1703.01621

Experimental motivation: competing orders

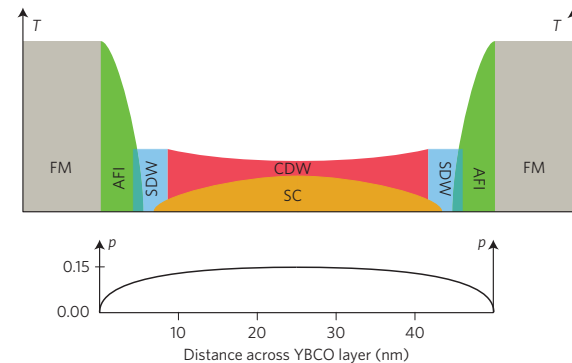


*D. Fausti et al.,
Science, 331, 189 (2011)*

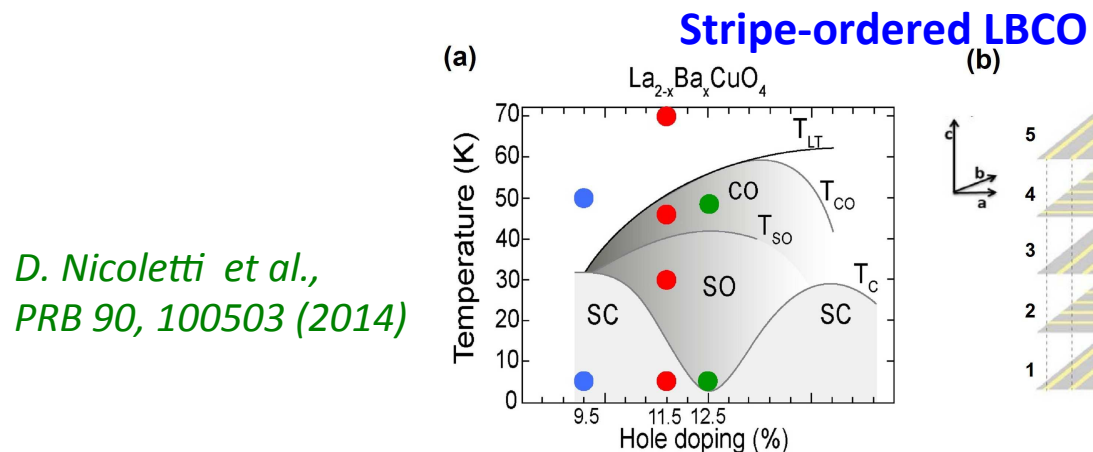


*X. Xi et al., Nat. Nanotechnol. 10,
765 (2015)*

YBCO-LCMO heterostructure

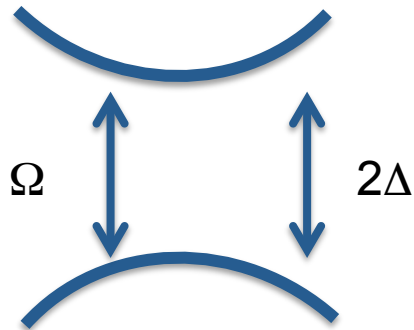


*A. Frano et al.,
Nat. Mater. 15, 831 (2016)*



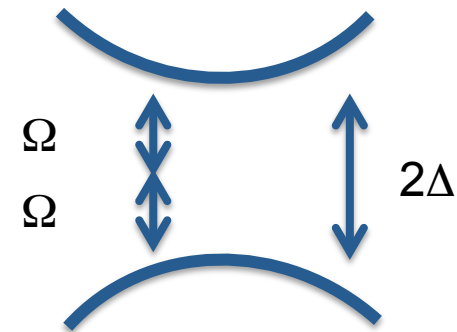
*D. Nicoletti et al.,
PRB 90, 100503 (2014)*

CDW $\sim A$
1-photon resonance

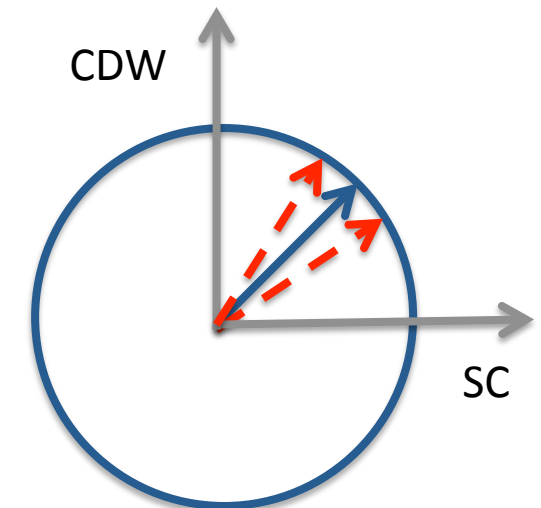


SC $\sim A^2$
2-photon resonance

Tsuji&Aoki, PRB 92, 064508 (2015)
Cea et al., PRB 93, 180507 (2016)

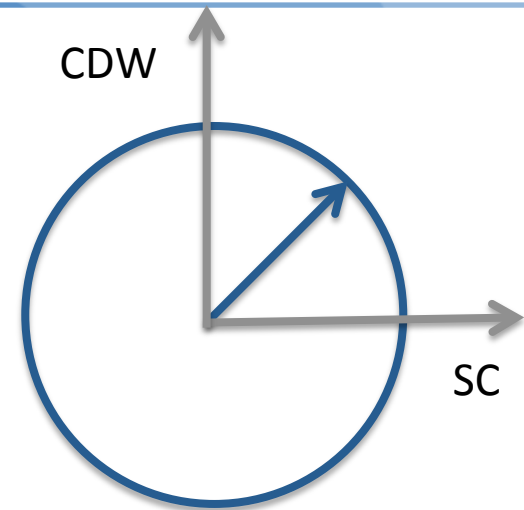


... laser lifts SC/CDW degeneracy
... Goldstone-like collective mode?



Competing orders

- attractive $-U$ Hubbard model
- degeneracy of SC and CDW at perfect nesting
- $SO(4)$ symmetry (SC, CDW, eta pairing)



VOLUME 63, NUMBER 19

PHYSICAL REVIEW LETTERS

6 NOVEMBER 1989

η Pairing and Off-Diagonal Long-Range Order in a Hubbard Model

Chen Ning Yang



C. N. Yang



S.-C. Zhang

Reprinted from Mod. Phys. Lett. B4 (1990) 759–766
© World Scientific Publishing Company

SO_4 SYMMETRY IN A HUBBARD MODEL

CHEN NING YANG

*Institute for Theoretical Physics, State University of New York,
Stony Brook, NY 11794-3840, USA*

and

S. C. ZHANG

*IBM Research Division, Almaden Research Center,
San Jose, CA 95120-6099, USA*

$$H = \sum_{k\sigma} \epsilon(k) n_{k\sigma} + U \sum_i n_{i\uparrow} n_{i\downarrow} = H_J + H_U,$$
$$\epsilon(k) = -2J(\cos(k_x) + \cos(k_y)),$$

2D square lattice + attractive U + mean-field decoupling

$$\begin{aligned} \Delta_{SC} &= U \sum_k f_k, & f_k &\equiv \langle c_{-k\downarrow} c_{k\uparrow} \rangle && \text{(SC)}, \\ \Delta_{CDW} &= U \sum_k g_k, & g_k &\equiv \frac{1}{2} \sum_{\sigma} \langle c_{k\sigma}^{\dagger} c_{k+Q\sigma} \rangle && \text{(CDW)}, \\ \Delta_{\eta} &= U \sum_k \eta_k. & \eta_k &\equiv \langle c_{-(k+Q)\downarrow} c_{k\uparrow} \rangle && (\eta \text{ pairing}). \end{aligned}$$

Equations of motion for electronic driving:

$$\begin{aligned}
 i\partial_t n_k &= -\Delta_{SC}(f_k - f_k^*) + \Delta_{CDW}(g_k - g_k^*) - \Delta_\eta^* \eta_k + \Delta_\eta \eta_k^*, & \text{eta pairing provides coupling} \\
 i\partial_t f_k &= \Delta_{SC}(1 - (n_k + n_{-k})) + (\epsilon(k - A) + \epsilon(k + A))f_k + \Delta_{CDW}(\eta_k + \eta_{k+Q}) - \Delta_\eta(g_k^* + g_{-k}^*), \\
 i\partial_t g_k &= \Delta_{CDW}(n_k - n_{k+Q}) - 2\epsilon(k - A)g_k + \Delta_{SC}(\eta_k^* - \eta_{k+Q}) + \Delta_\eta f_k^* - \Delta_\eta^* f_{k+Q}, \\
 i\partial_t \eta_k &= \eta_k(\epsilon(k - A) - \epsilon(k + A)) + \Delta_{CDW}(f_k + f_{k+Q}) - \Delta_{SC}(g_{-k} + g_k^*) - \Delta_\eta(n_k + n_{-(k+Q)} - 1).
 \end{aligned}$$

nonlinear equations:
self-consistency in real time

$$\begin{aligned}
 \Delta_{SC} &= U \sum_k f_k, \\
 \Delta_{CDW} &= U \sum_k g_k, \\
 \Delta_\eta &= U \sum_k \eta_k.
 \end{aligned}$$

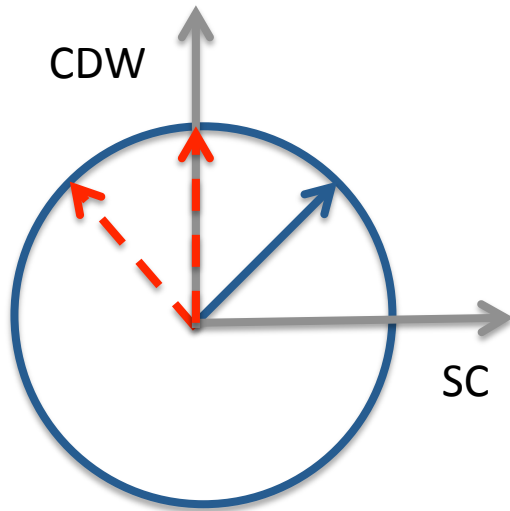
Nonequilibrium:

Periodic driving field: $A(t) = A_{\max} \sin(\omega t) (\mathbf{e}_x + \mathbf{e}_y)$

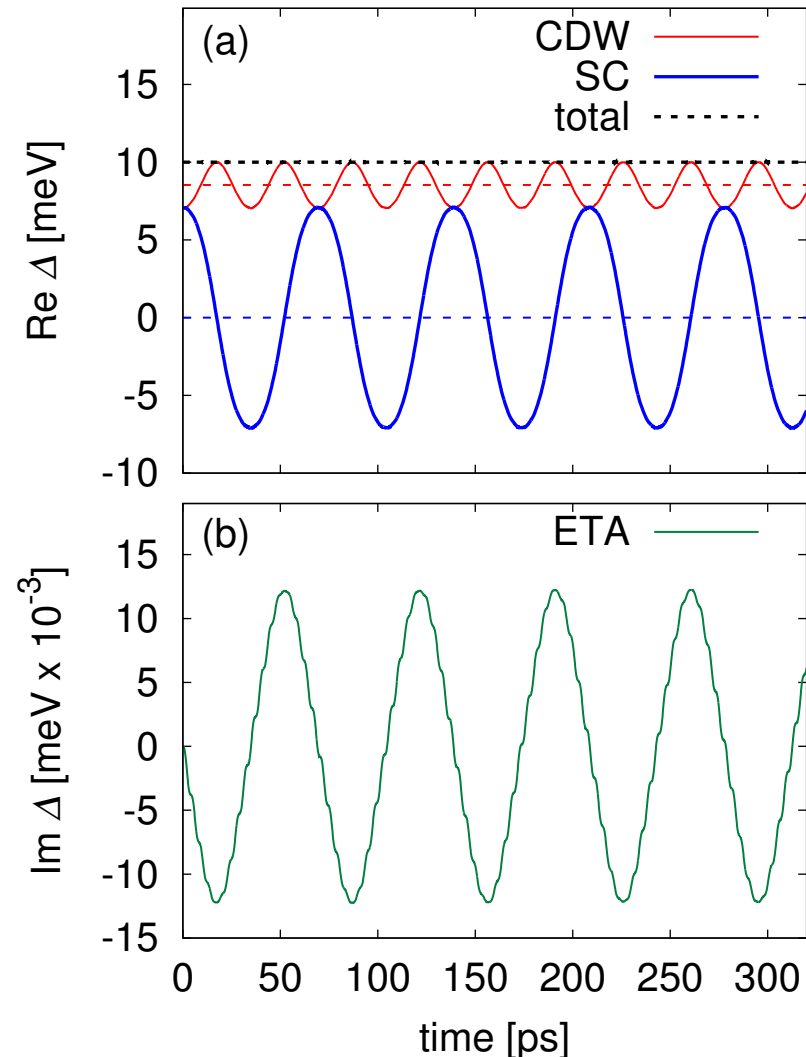
$A_{\max} = 5 \times 10^{-5}$, $E_{\max} \sim 10\text{-}100 \text{ V/cm}$ – **weak fields!**

Gap resonance – coexisting initial state

Below resonance:
SC down, CDW up

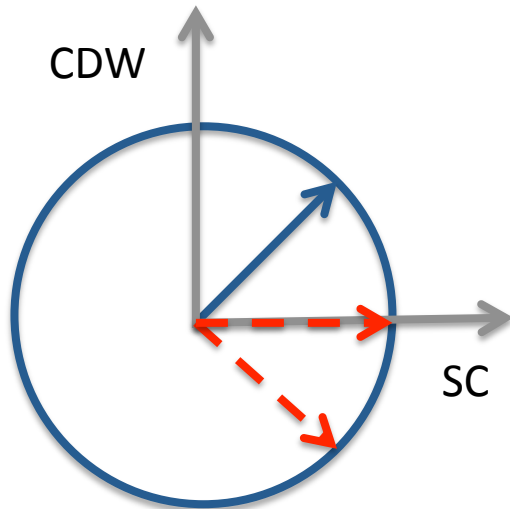


$\omega = 19$ meV, below resonance

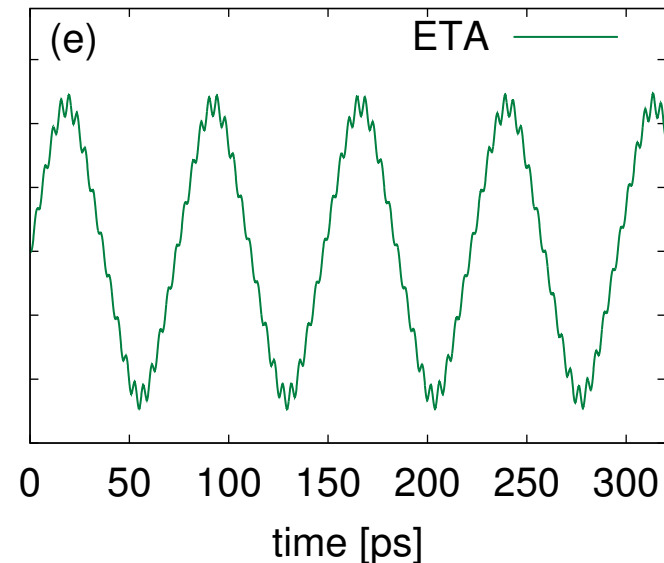
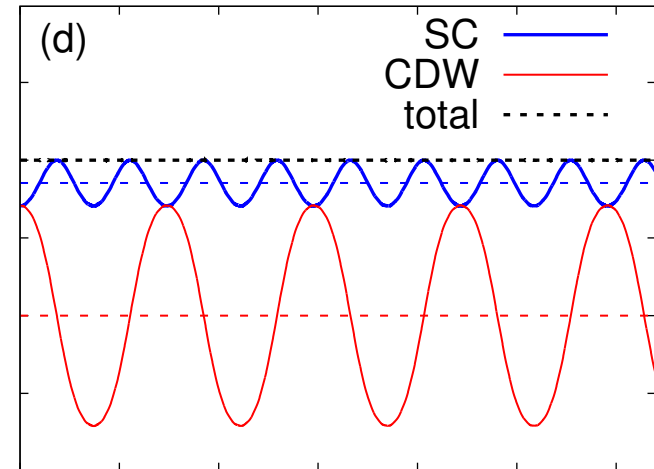


Gap resonance – coexisting initial state

Above resonance:
SC up, CDW down



$\omega = 21$ meV, above resonance

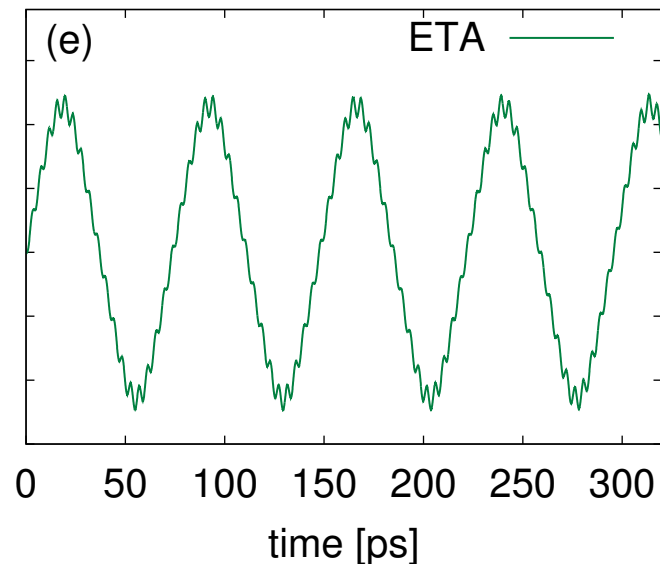
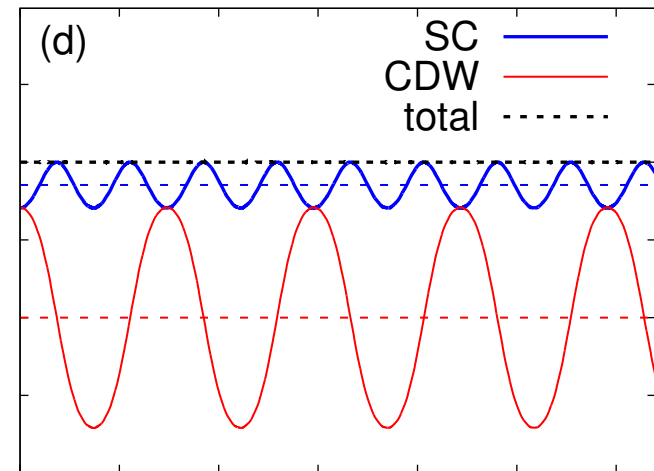


Gap resonance – coexisting initial state

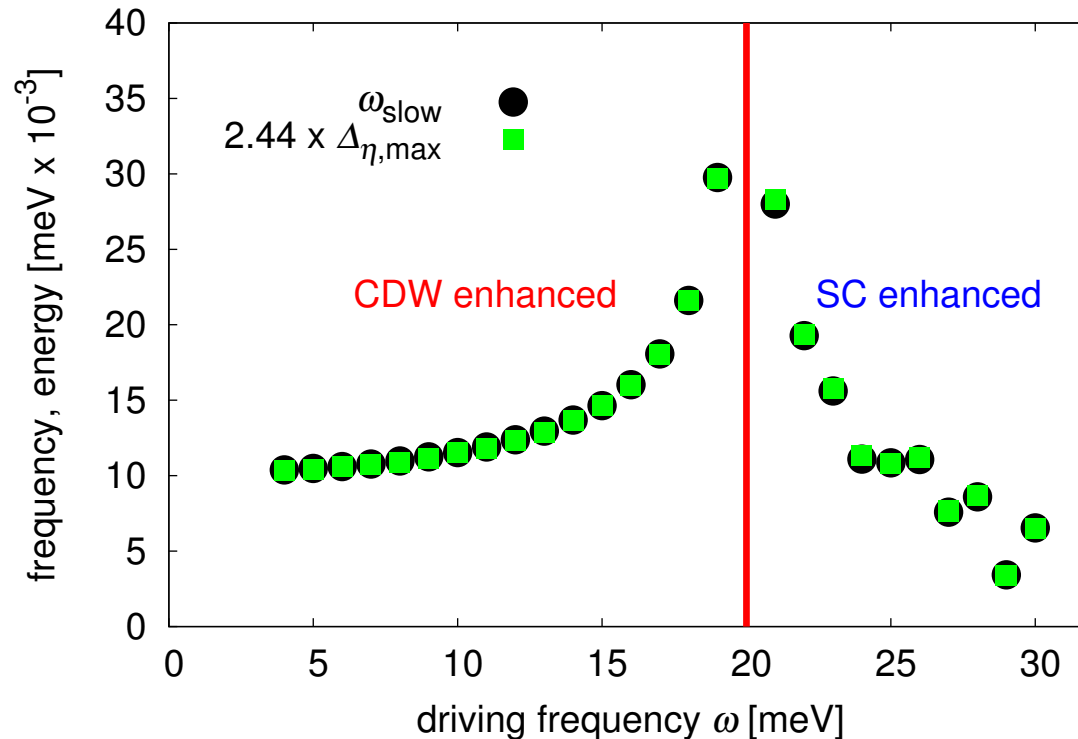


„Floquet time crystal“??

$\omega = 21$ meV, above resonance



Gap resonance



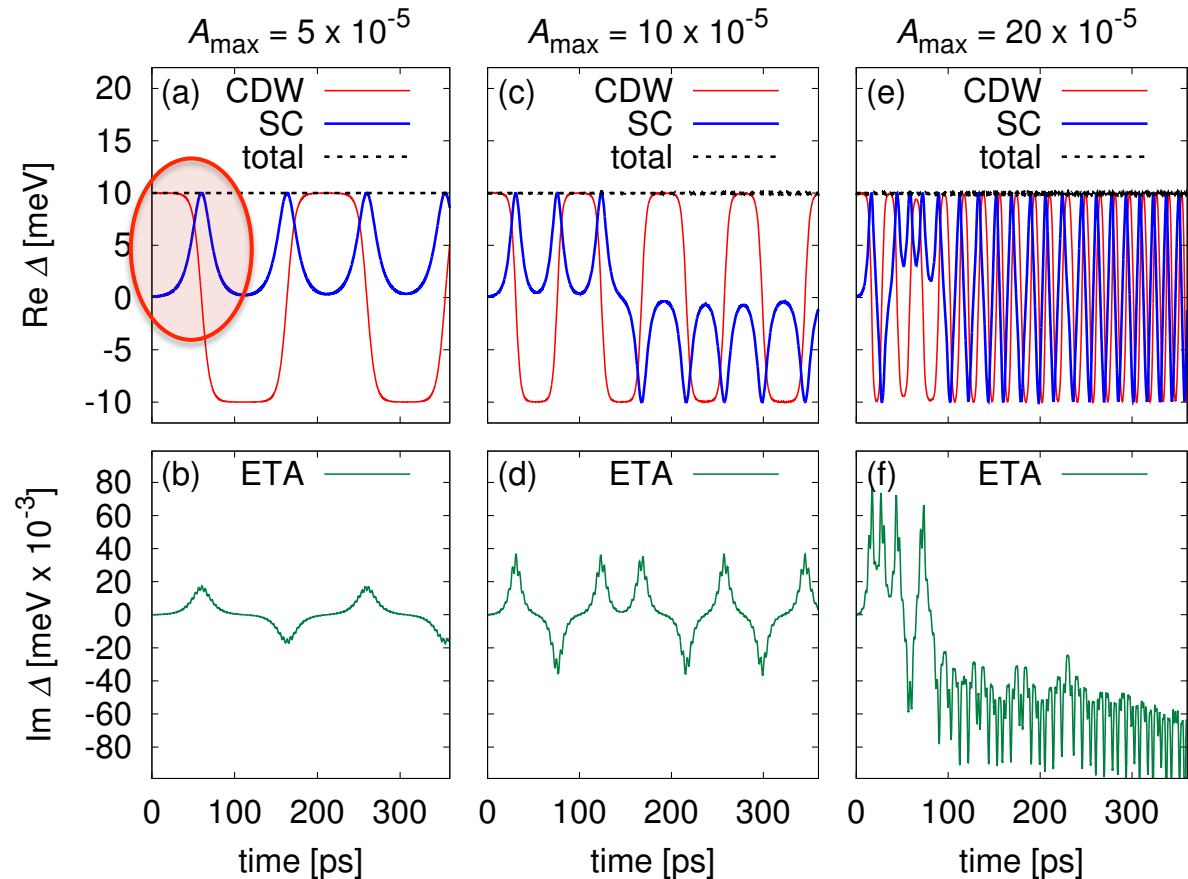
oscillation frequency set by light-induced eta pairing amplitude, which gives „mass“ to collective mode

resonant behavior at $\Omega=2\Delta$ = single-particle gap

Inducing superconductivity

99% CDW initial state
Drive slightly above gap

SC comes alive!
Irregular behavior for
stronger driving



Summary III

Tight-binding model + time-dependent mean-field theory:

- laser-controlled switching between SC/CDW
- path to understanding of light-induced superconductivity in systems with competing orders?

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