

Summary (prev. lecture):

Expansion of one-particle Greens-function:

$$G(a,b) = \sum_{n=0}^{\infty} i^n \int d1d1' \dots dn dn' V(1,1') \dots V(n,n') \times$$

$$\begin{vmatrix} G_0(a_1,b) & \dots & G_0(a_1,n') \\ \vdots & & \vdots \\ G_0(n',b) & \dots & G_0(n',n') \end{vmatrix} \pm \text{connected, topologically inequivalent}$$

- all diagrams to second order

$$G = \uparrow + \overbrace{\mu O} + \overbrace{\mu O}$$

$$+ \overbrace{\mu O} \overbrace{\mu O} + \overbrace{\mu O} \overbrace{\mu O} + \overbrace{\mu O} \overbrace{\mu O} + \overbrace{\mu O} \overbrace{\mu O}$$

$$+ \overbrace{\mu O} \overbrace{\mu O} + \overbrace{\mu O} \overbrace{\mu O}$$

$$+ O(v^3)$$

re-grouping the expansion for G :

$$\text{interacting } G = a \leftarrow e$$

$$\leftarrow = \leftarrow + \text{---} \circlearrowleft \leftarrow$$

$$= \leftarrow + \text{---} \circlearrowleft \leftarrow$$

Note: Expansion

$$G = G_0 + G_0 \Sigma G_0 + \dots$$

new structure of geometric series!

$$G(1,2) = G_0(1,2) + \int d^3 d^4 G_0(1,3) \Sigma(3,4) G_0(4,2)$$

$$= G_0(1,2) + \int d^3 d^4 G_0(1,3) \Sigma(3,4) G_0(4,2)$$

↳ Dyson-equation, which was before ~~was~~ derived
in Chapter IV.4

Special structure of 1st order Σ -terms

$$\Sigma^{(1)}(1,2) = \text{---} \circlearrowleft + \text{---} \circlearrowleft$$

"Hartree" "Fock"

$$= \pm i \delta(1,2) \int d^3 v(1,3) G_0(3,3^+)$$

$$+ i v(1,2) G(1,2)$$

⇒ diagram is local in space and time ($\sim \delta(1,2)$) for
the Hartree part $\hat{=}$ same structure like potential
⇒ mean-field potential multi-particle

- self-energy and Dyson equation

General form of G :

$$G = \text{---} \leftarrow + \text{---} \leftarrow \bullet \text{---} \leftarrow + \text{---} \leftarrow \bullet \text{---} \leftarrow \bullet \text{---} \leftarrow$$

+ ...

where  $\hat{\equiv} \Sigma(t, t') \approx \text{"self-energy"}$

= sum of all parts that break not into pieces by cutting a single G_0 -line
 ("one-particle irreducible")

$$\bullet = \text{---} \leftarrow + \text{---} \leftarrow + \text{---} \leftarrow + \text{---} \leftarrow + \dots$$


Note: in order to write the series like

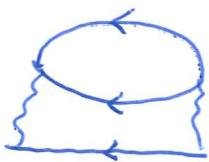
$G = G_0 + G_0 \Sigma G_0 + \dots$, it is essential that the different G -diagrams have no combinatorial factors like $\frac{1}{k!}$.

\Rightarrow Fact is local in time, if the interaction
 $V(t_1, t_2) \sim \delta(t_1 - t_2)$ is local in time

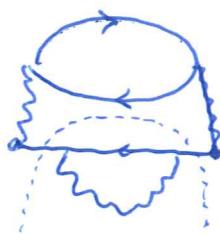
A- Skeleton diagrams

We can get one step further in "reducing the number of diagrams":

- remove all Σ -insertions from the G_0 -lines inside a Σ -diagramme
- (all parts which can be removed by cutting two G_0 -lines)
- replace G_0 -lines in diagram by interacting G lines \Rightarrow



Skeleton
Diagram



no skeleton-diagram

again. Because we do not have to care about combinatorical factors, this skeleton expansion produces the same diagrams.

up to second order:

$$\Sigma = \underbrace{\text{Diagram 1} + \text{Diagram 2} + \text{Diagram 3} + \text{Diagram 4} + \dots}_{\text{End Born approximation}}$$

- Σ becomes functional of the full interacting Greens function and the interaction:

$$\Sigma = \sum_s [G, v] \quad \text{"Skeleton functional"}$$

- Given approximation for $G \Rightarrow$
solution of "self-consistent" Dyson equation

$$G = G_0 + G_0 \cdot \sum_s [G, v] \cdot G$$

("integration over internal indices implied")

\Leftrightarrow self-consistent, non-linear equation

- Is skeleton expansion to given order "better" than bare expansion?
 - Quantitatively, the answer is often "no" (no simple argument)
 - but Skeleton Expansion can be shown to be "conserving", i.e. internally consistent with particle number, energy, momentum conservation laws: $\nabla \vec{j}$ and densities ρ evaluated from the same perturbative expansion satisfy conservation law $\nabla \vec{j} + \frac{\partial \rho}{\partial t} = 0$
 - (\rightarrow Stefanucci, Ch. 9 & 11) Maybe more details later this lecture

IV Quasi-particle dynamics and kinetic equations

IV.1. Quasiparticles, Green's functions & self-energy: overview

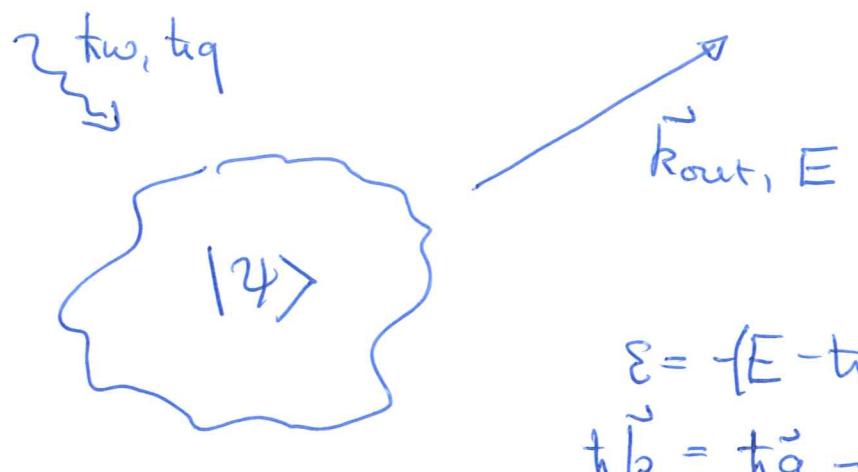
What is the notion of a "particle"

in a many-body system?

When can a many-particle system be described as a collection of entities which behave as independent particles (with Bose or Fermi statistics), in between "occasional collisions"?

• phenomenological indication

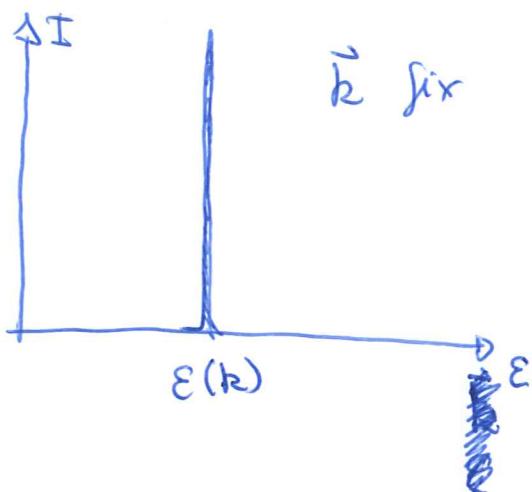
① "ideal photoemission-like experiment": some perturbation, which transfers energy ΔE and momentum \vec{k} and kicks out a particle.



$$\varepsilon = -(E - \text{tw})$$

$$\hbar \vec{k}_2 = \vec{k}_q - \vec{k}_{\text{out}}$$

signal counts:



definite relation between
energy transfer and
momentum transfer \Rightarrow
system consists of particles
with dispersion $\varepsilon(k)$

- 2) Thermodynamic quantities can be described by a function which gives occupation of single-particle levels:

e.g. Fermi system:

$$S = -k_B \sum_k \left[n_k \ln n_k + (1-n_k) \ln (1-n_k) \right]$$

↑
occupation of momentum state

3) Transport:

Dreude formula: $\sigma(\omega) = \frac{\sigma_0}{1-i\omega\tau} \quad \sigma_0 = \frac{ne^2}{m}$

Response of particles with density n , charge e , mass m to ext. field. Scattering rate τ .

τ must be small enough ... if scattering length is comparable to inter-particle distance, quasi-particle description of transport should break down.

- Often good description of solid at low energies: Some "new" exotic (more or less) ground state (Superconductor, Fermi liquid...) + excitations on top of this described in terms of weakly interacting particles (quasi particles) which can be very different from the original electrons and nuclei.

Bosons: phonons, spin-waves, ...

Fermions: ~~are~~ electrons with very different mass, Bogoliubov excitations of superconductors, ...

Important example: Landau Fermi Liquid

Here: just very brief motivation. For more details, see e.g. Negele & Orland, "Quantum Many Particle Systems" for phenomenological theory of Fermi Liquid.

Landau 1957: "System of interacting fermions (first: short range interaction) at low energies

- Behaves like weakly interacting fermions" with renormalized properties (mass, interaction)

"mathematical formulation of "behaves like"?

many-body spectrum of low excitations can be described by spectrum of excitations of independent ~~etc~~ particles (= particle-hole excitations)

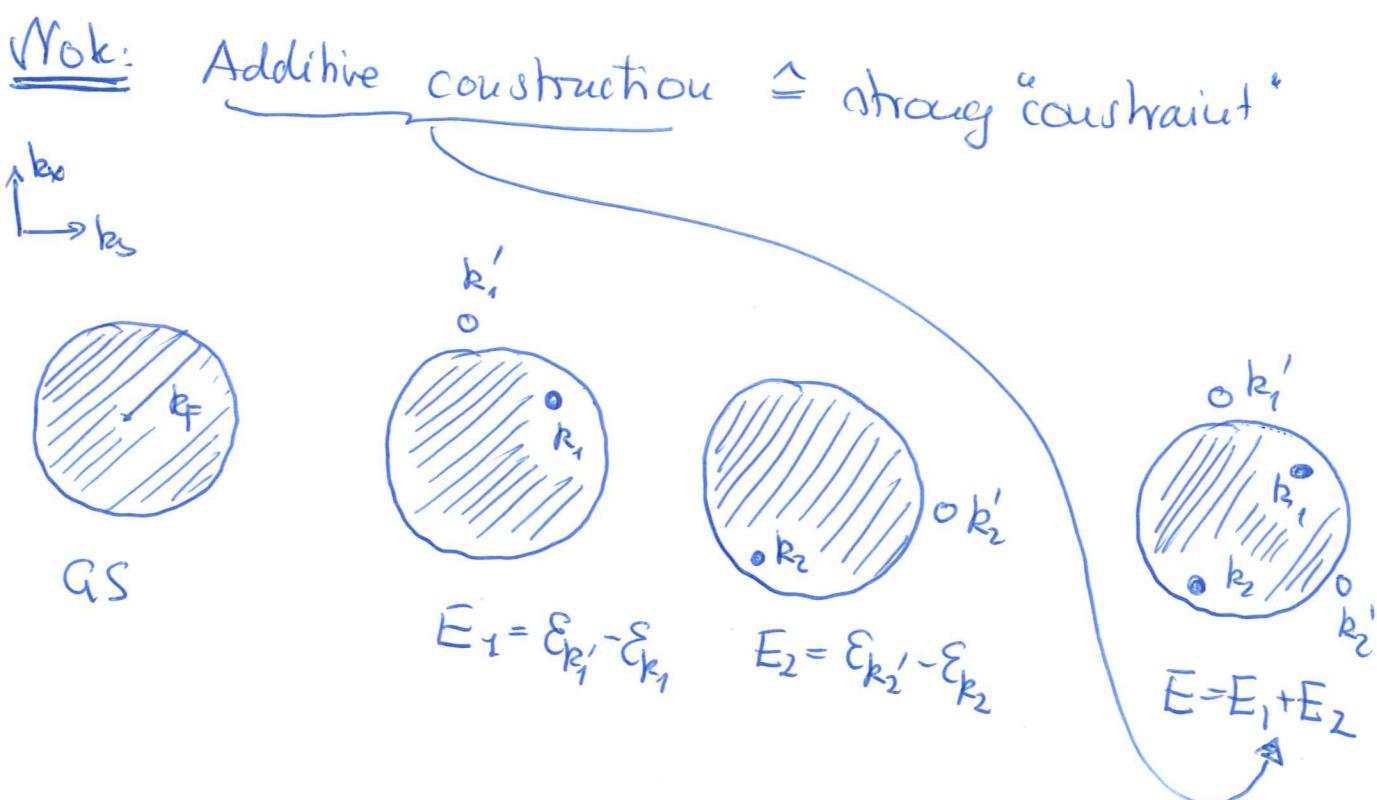
Note: spectrum of noninteracting fermions has a very peculiar form:

$$\textcircled{*} \quad E[\{n_k\}] = \sum_k n_k \epsilon_k \quad n_k \in \begin{cases} 0,1 & k > (k_F) \\ 0,1 & k < (k_F) \end{cases}$$

outside Fermi surface

~~(K) levels for M particles i.e. L states, partially filled~~
~~by many number E_k~~

Imagine you know the spectrum $\{E_n\}$ of many-particle system ... If it can be "fit" in the additive way \oplus , the system behaves like independent particles (Fermions), even though these Fermions can be very different from the original Fermions.



M particles, L single-particle levels :

$${E \choose M} \text{ many body states} \sim \exp(\alpha L) \quad (\text{if } M \text{ is fraction of } L)$$

"fit" by only L parameters and additive rule \oplus !

Landau (1957) :

'95c-

Spectrum of "normal Fermi system" can be asymptotically fit with independent particles.

i.e.,

(1) there is well defined Fermi surface ($k = k_F$ for homogeneous system), so that excitations can be labelled in the same way as for independent particles,

$$\{n_{k\sigma} \mid n_{k\sigma} \in \begin{cases} 0, 1 & k > k_F \\ 0, -1 & k < k_F \end{cases}\} \quad \sigma: \text{spin}$$

$$(2) E[\{n_{k\sigma}\}] = E_0 + \sum_{k\sigma} \epsilon_{k\sigma} n_{k\sigma} + \frac{1}{2} \sum_{\substack{k k' \\ \sigma \sigma'}} f_{k k'}^{(\sigma)} n_{k\sigma} n_{k'\sigma'} + \dots$$

("asymptotically free")

"Proof": (1) \Leftrightarrow adiabatic connection
of interacting and
noninteracting systems

(2) \Leftrightarrow self-consistent argument:

work
here is the hand

if (2) is true, \Rightarrow scattering rate

between low energy states (k close to k_F)

becomes asymptotically small \Rightarrow this justifies description in terms of almost free particles.

- Fermi liquid theory $\hat{=}$ phenomenological theory, e.g., clear that entropy must be

$$S = -k_B \sum_{k\sigma} \left\{ n_{k\sigma} \ln n_{k\sigma} + (1-n_{k\sigma}) \ln (1-n_{k\sigma}) \right\}$$

\hookrightarrow all thermodynamic properties.

(Note: interaction parameters do matter here)

In the following: we discuss microscopic manifestations of well defined quasi-particles in the propagator G and the self energy Σ .