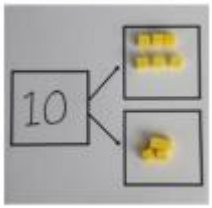
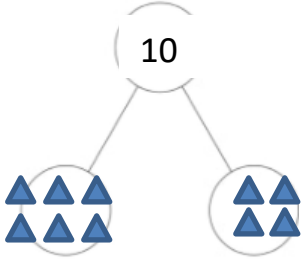
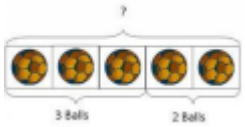
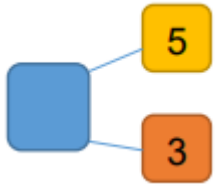
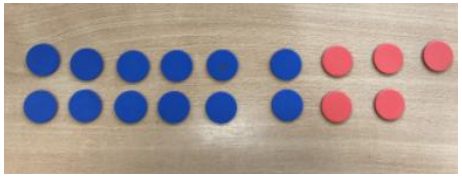
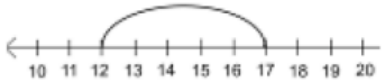
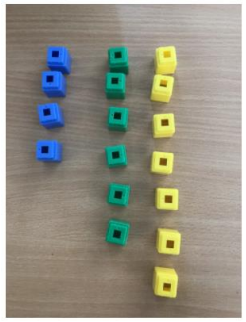
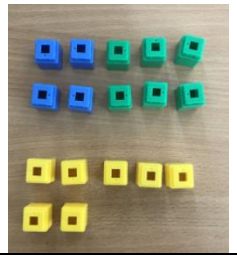
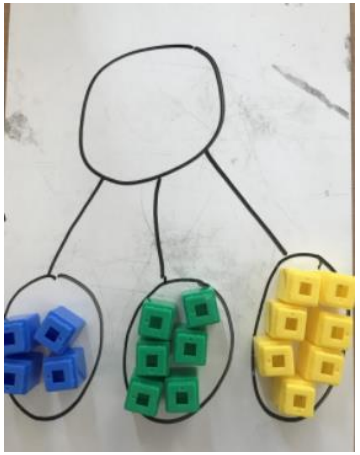
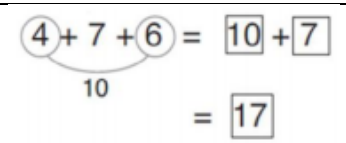
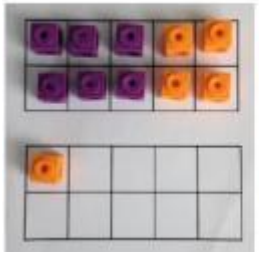
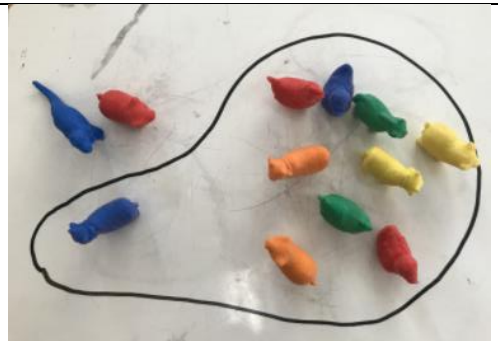
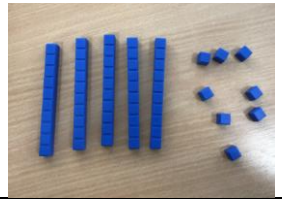
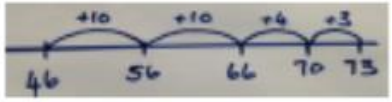
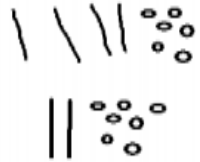
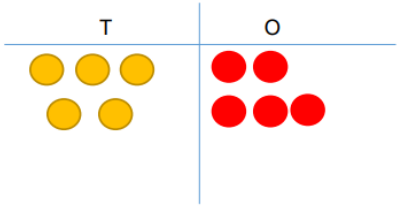
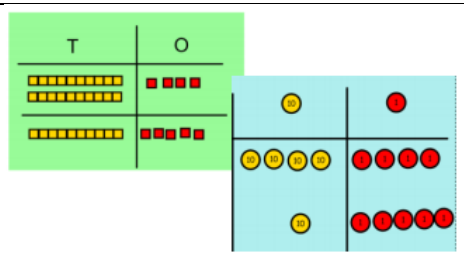
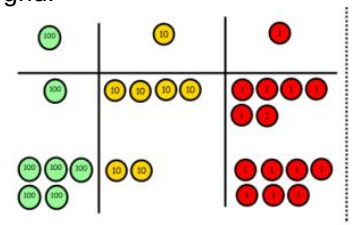
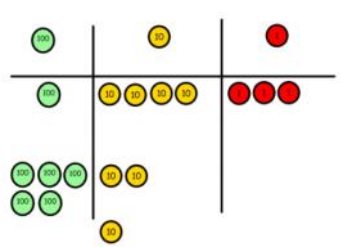
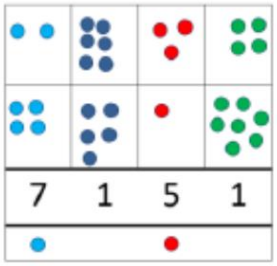
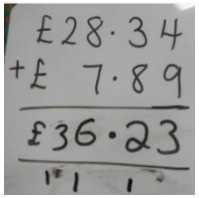
Progression in Calculations

Addition

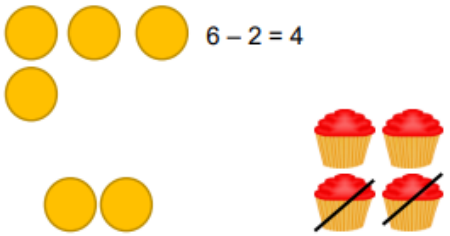
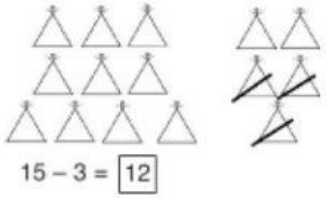
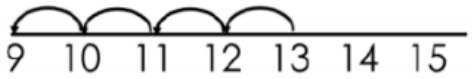
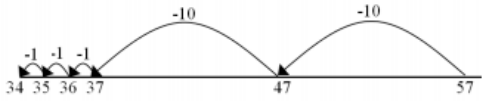
Objective and Strategies	Concrete	Pictorial	Abstract	Manipulatives	Vocabulary
Combining two parts to make a whole: part-part - whole model Reception/ Y1/Y2	   Use cubes/other objects to add two numbers together as a group or in a bar.  	  Use pictures to add two numbers together as a group or in a bar. 	4 + 3 = 7 10 = 6 + 4  Use the part-part-whole diagram as shown above to move into the abstract.	Cubes Natural objects Counters Numicon	Number bonds Add, more, plus, make, sum, total, total, altogether Equals, is the same as (including equals sign)

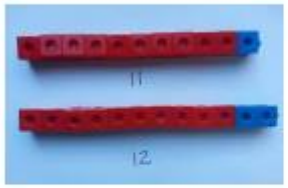
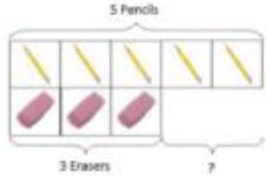
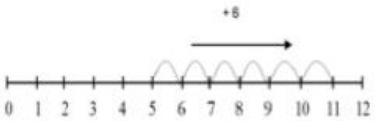
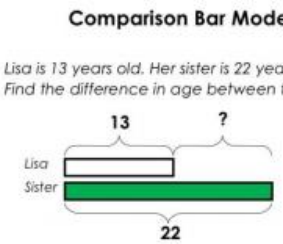
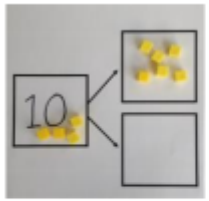
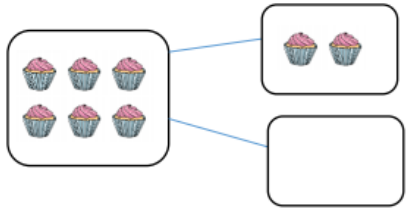
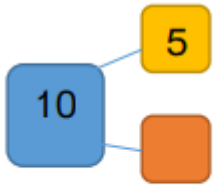
Starting at the bigger number and counting on Y1/2	 Start with the larger number on the bead string and then count on to the smaller number 1 by 1 to find the answer. 	12 + 5 = 17  Start at the larger number on the number line and count on in ones or in one jump to find the answer.	5 + 12 = 17 Place the larger number in your head and count on th smaller number to find you answer.	Bead strings Counters	Number line Count on Add, more, plus, make, sum, total, total, altogether Equals, is the same as (including equals sign)
Adding three single digits Y1/Y2	4 + 6 + 7 = 17  		 Combine the two numbers that make 10 and then add on the remainder.	Cubes Natural objects Counters Numicon	Number bonds Add, more, plus, make, sum, total, total, altogether Equals, is the same as (including equals sign)

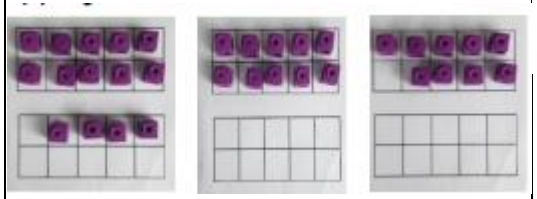
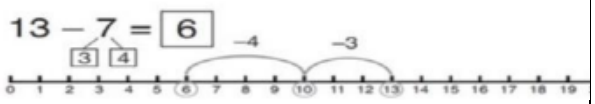
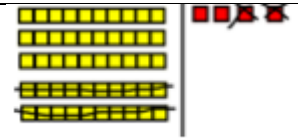
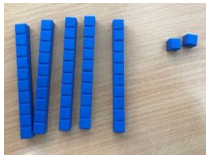
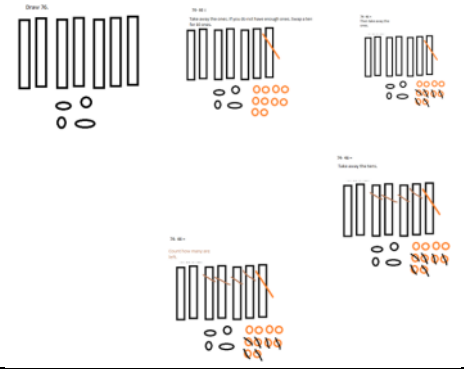
Regrouping to make 10. Y2	6 + 5 = 11  Start with the bigger number and use the smaller number make 10.	 9 + 5 = 14 1 4 +1 +4 0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18	7 + 4= 11 If I am at seven, how many more do I need to make 10? How many more do I add on now?	Tens frame Cubes	Regroup How many more to make . . ?
Adding two 2 digit numbers. Y2/3	35 + 23 = 58  	 46 + 27= 	Using number bonds and facts to add two 2 digit numbers mentally 46 + 27= 40 + 20 = 60 6 + 7 = 13 60 + 13 = 73 OR 46 + 20 = 66 66 + 7 = 73	Dienes Place value chart Number lines Blank number lines 100 square	Number bonds Hundreds, tens, ones Partition, recombine
Column method – no regrouping Y3/4/5/6	24 + 15= Add together the ones first then add the tens. Use the Dienes blocks first before moving onto place value counters	After practically using the Dienes blocks and place value counters, children can draw the counters to help them to solve additions.  T O	Calculations 21 + 42 = 21 + 42	Dienes (Y3) Place value counters Decimal place value counters Coloured counters	Partition Column addition and subtraction

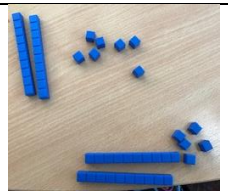
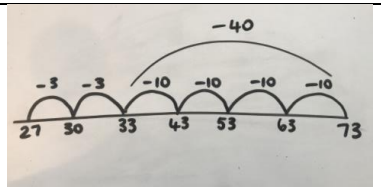
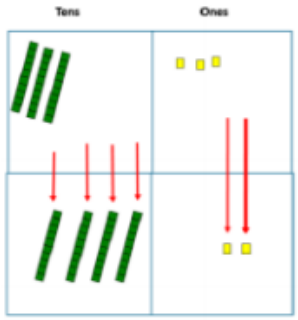
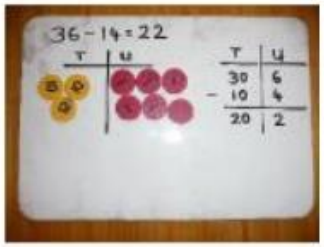
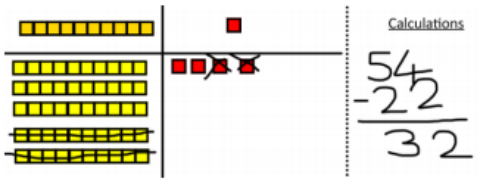
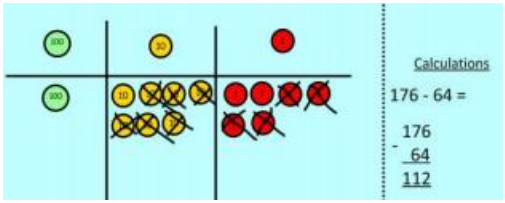
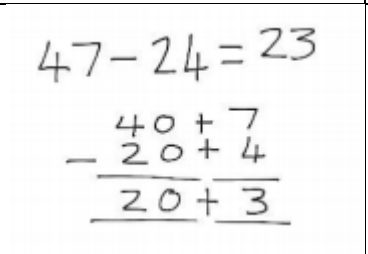
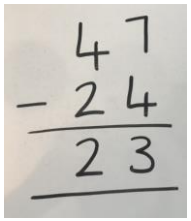
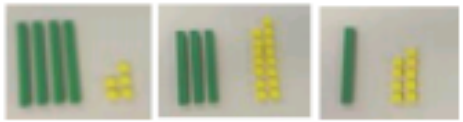
					
Column method – regrouping Y3/4/5/6	Make both numbers on a place value grid.  Add up the units and exchange 10 ones for one 10.  Add up the rest of the columns, exchanging the 10 counters from one column for the next place value column until every column has been added. This can also be done with Dienes (Year 3) to help children clearly see that 10 ones equal 1 ten and 10 tens equal 100. As children move on to decimals, money and decimal place value counters can be used to support learning.	Children can draw a pictorial representation of the columns and place value counters to further support their learning and understanding. 	Start by partitioning the numbers before moving on to clearly show the exchange below the addition. 20 + 5 40 + 8 60 + 13 = 73 **7 8 9** **+ 6 4 2** **1 4 3 1** 1 1 As the children move on, introduce decimals with the same number of decimal places and different. Money can be used here. 72.8 + 54.6 127.4 1 1 	Dienes Place value counters Decimal place value counters Coloured counters	**Carry over – refer to correct value** Partition, exchange Column addition and subtraction

Subtraction

Objective and Strategies	Concrete	Pictorial	Abstract	Manipulatives	Vocabulary
Taking away ones Reception/Y 1	Use physical objects, counters, cubes etc to show how objects can be taken away. 	Cross out drawn objects to show what has been taken away. 	6 – 2 = 4 15 – 3 = 12	Cubes Natural objects Counters	Subtract, take away, minus How many are left?
Counting back Y1/2	Make the larger number in your subtraction. Move the beads along your bead string as you count backwards in ones.  Use counters and move them away from the group as you take them away counting backwards as you go. 	Count back on a number line or number track  Start at the bigger number and count back the smaller number showing the jumps on the number line.  This can progress all the way to counting back using two 2 digit numbers.	Put 13 in your head, count back 4. What number are you at? Use your fingers to help.	Bead string Counters	Number line Count back

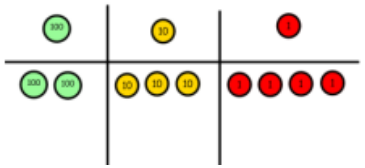
Find the difference Y1/Y2	Compare amounts and objects to find the difference.  Use cubes to build towers or make bars to find the difference  Use basic bar models with items to find the difference 	 Count on to find the difference. Draw bars to find the difference between 2 numbers.  Comparison Bar Models *Lisa is 13 years old. Her sister is 22 years old. Find the difference in age between them.* 13 ? Lisa Sister 22	Hannah has 23 sandwiches, Helen has 15 sandwiches. Find the difference between the number of sandwiches.	Cubes Numicon	Number line, bar model Difference between How many fewer is . . . than . . ?
Part Part Whole Model Y1/Y2	 Link to addition- use the part whole model to help explain the inverse between addition and subtraction. If 10 is the whole and 6 is one of the parts. What is the other part? 10 - 6 =	Use a pictorial representation of objects to show the part part whole model. 	 Move to using numbers within the part whole model	Cubes Natural objects Counters	Part, whole, inverse

Subtraction with bridging across the ten Y2	14-5 = 9  Make 14 on the ten frames. Take away the four first to make 10 and then take away one more so you have taken away 5. You are left with the answer of 9.	 Start at 13. Take away 3 to reach 10. Then take away the remaining 4 so you have taken away 7 altogether. You have reached your answer.	16 – 8= How many do we take off to reach the next 10? How many do we have left to take off?	Tens frame Cubes Counters	Tens, ones Bridging
Subtracting two 2-digit numbers no regrouping Y2	 54- 22 = Make first number with dienes. Look at the second number. Take away the tens. Take away the ones. How many left?	Repeat but with drawing rather than concrete apparatus. 	54- 22 = 54- 20 = 34 34- 2 = 32	Dienes Place value counters	Subtract, take away, minus
Subtracting two 2-digit numbers regrouping Y2/3	52 – 25 =  Exchange one ten for ten ones   Subtract 25	Repeat but with drawings 	Subtract mentally 75-46 =	Place value counters Dienes	Subtract, take away, minus Tens, ones **Exchange**

					
Column method without regrouping Y3/4/5/6	 Use Dienes to make the bigger number then take the smaller number away. Show how you partition numbers to subtract. Again make the larger number first. 	Draw Dienes or place value counters alongside the written calculation to help to show working out.  	 This will lead to a clear written column subtraction. 	Dienes Place value counters Coloured counters Place value chart	Subtract, take away, minus
Column method with regrouping Y3/4/5/6	Use Dienes to start with before moving on to place value counters. Start with one exchange before moving onto subtractions with 2 exchanges. Column method (using Dienes to exchange) 45 – 26 = 	Draw the counters onto a place value grid and show what you have taken away by crossing the counters out as well as clearly showing the exchanges you make.	 Children can start their formal written method by partitioning the number into clear place value columns.	Dienes Place value counters Coloured counters Place value chart	Subtract, take away, minus **Exchange**

1) Start by partitioning 45
2) Exchange one ten for ten ones
3) Subtract the ones, then the tens

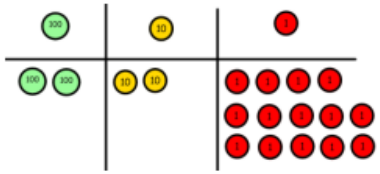
Make the larger number with the place value counters.



Calculations

234
- 88

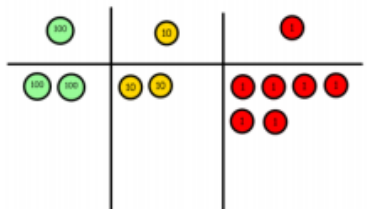
Start with the ones, can I take away 8 from 4 easily? I need to exchange one of my tens for ten ones.



Calculations

234
- 88

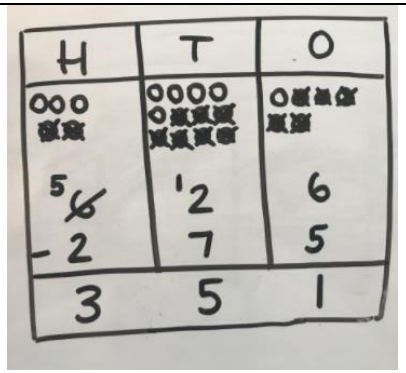
Now I can subtract my ones.



Calculations

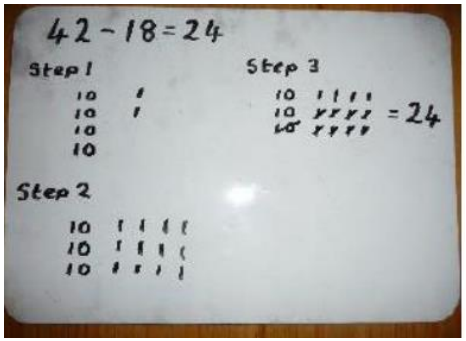
234
- 88

Now look at the tens, can I take away 8 tens easily? I need to exchange one hundred for ten tens.



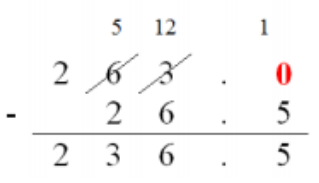
When confident, children can find their own way to record the exchange/ regrouping.

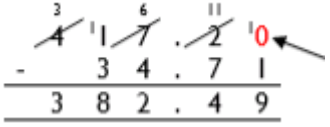
Just writing the numbers as shown here shows that the child understands the method and knows when to exchange/ regroup.



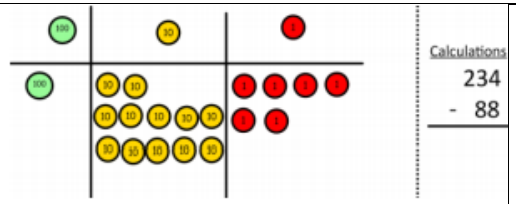
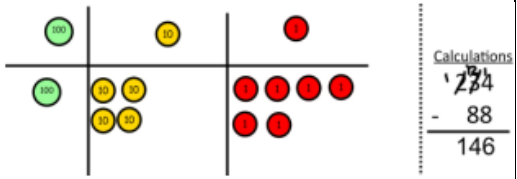
Moving forward the children use a more compact method.

This will lead to an understanding of subtracting any number including decimals.

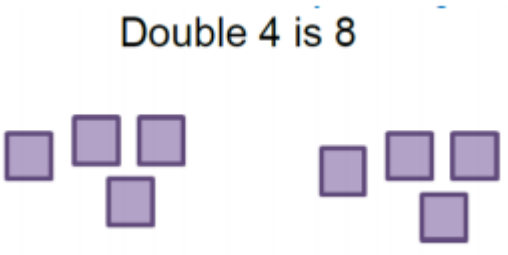
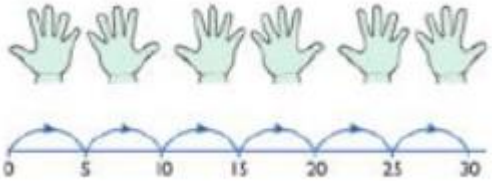


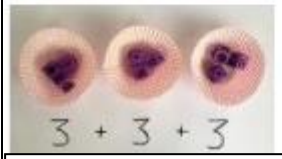
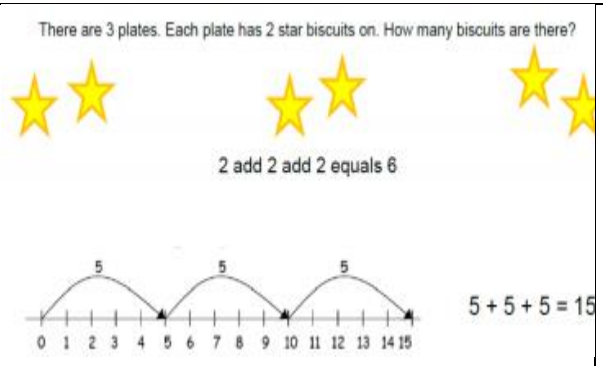
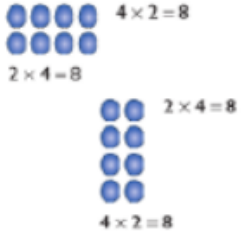
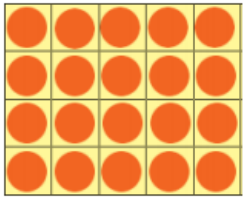
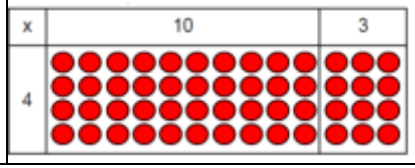


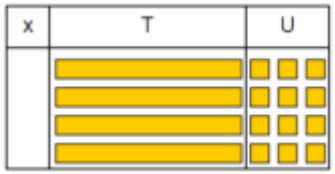
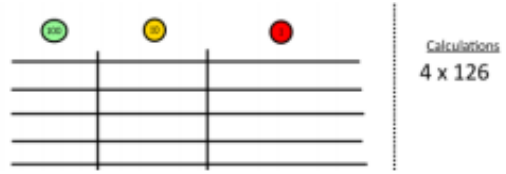
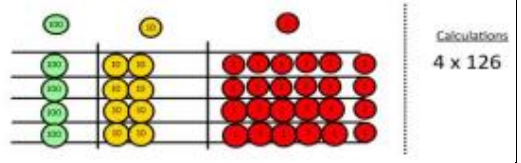
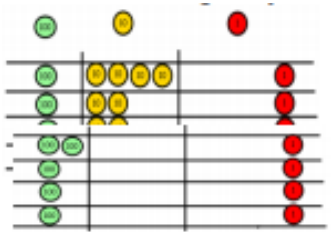
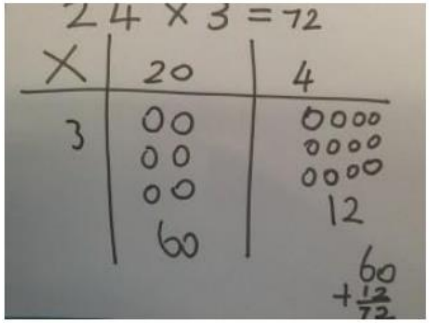
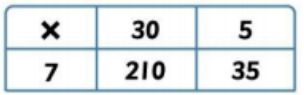
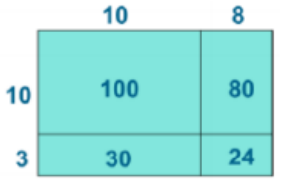
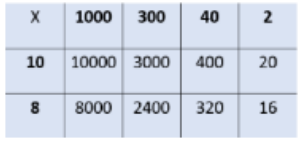
When subtracting decimals with different numbers of decimal places, children should be taught and encouraged to make them the same through identification that 2 tenths is the same as 20 hundredths, therefore, 0.2 is the same value as 0.20

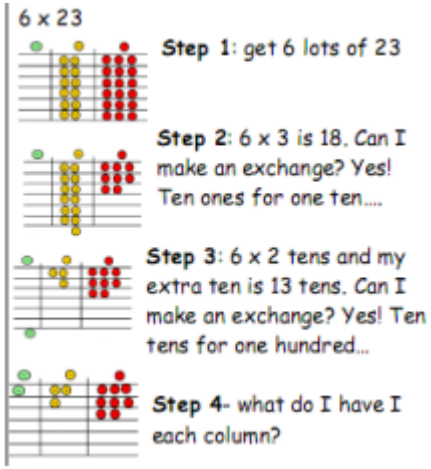
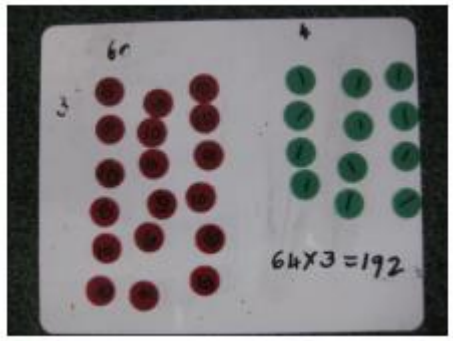
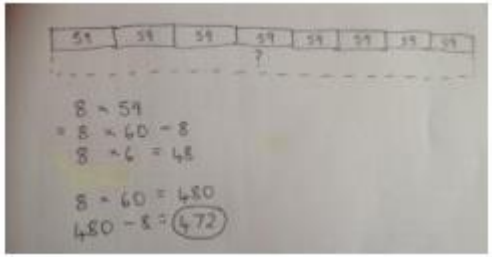
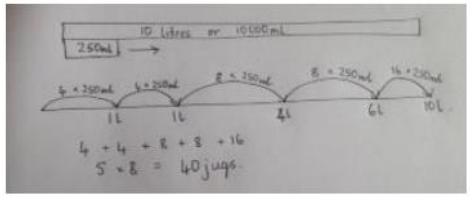
	 Calculations 234 - 88 Now I can take away eight tens and complete my subtraction  Calculations ~~2~~ ~~3~~ 4 - 88 146 Show children how the concrete method links to the written method alongside your working. Cross out the numbers when exchanging and show where we write our new amount.				

Multiplication

Objective and Strategies	Concrete	Pictorial	Abstract	Manipulatives	Vocabulary
Doubling Reception/Y 1	Use practical activities to show how to double a number.  double 4 is 8 4 × 2 = 8 	Draw pictures to show how to double a number. 	4 + 4 = You've got two of the same number.	Cubes Natural objects Counters Numicon	Double, doubling, twice Groups of, repeated addition
Counting in multiples Reception/Y 1	  Count in multiples supported by concrete objects in equal groups	 Use a number line or pictures to continue support in counting in multiples	Count in multiples of a number aloud. Write sequences with multiples of numbers. 2, 4, 6, 8, 10 5, 10, 15, 20, 25 , 30	Cubes Bead string Natural objects Counters Numicon	Multiple, count in twos, fives, tens (forwards from/ backwards from)

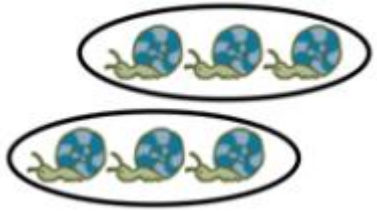
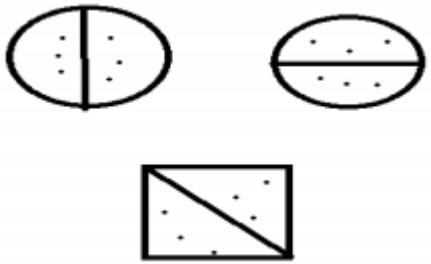
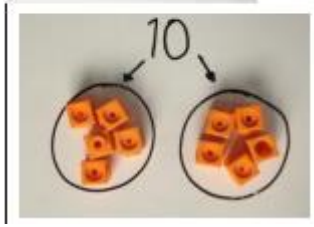
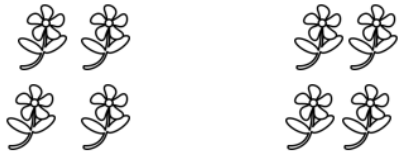
Repeated addition Y1/Y2	  Use different objects to add equal groups.  This could be written or said as 3 groups of 3.	 This could be written or said as 3 groups of 5.	Write addition sentences to describe objects and pictures.  	Cubes Bead string Natural objects Counters Numicon Dishes for groups of e.g. cupcake cases/ yogurt pots	Repeated addition, groups of, total, plus, add, altogether
Arrays – showing commutative multiplication Y2/Y3/Y4	Create arrays using counters/cubes to show multiplication sentences.   This could be written or said as 3 groups of 4 or 4 groups of 3.	Draw arrays in different rotations to find commutative multiplication sentences.  Link arrays to area of rectangles. 	Use an array to write multiplication sentences and reinforce repeated addition.  5 + 5 + 5 = 15 3 + 3 + 3 + 3 + 3 = 15 5 x 3 = 15 3 x 5 = 15	Counters Cubes Array grids	Array, row, column Multiply by, times Commutative
Grid method Y3/Y4	 Show the link with arrays to first	Children can represent the work they have done with place value counters in a way that they understand. They can draw the counters, using colours to	Start with multiplying by one digit numbers and showing the clear addition alongside the grid.	Place value counters Coloured counters	Array, row, column, exchange

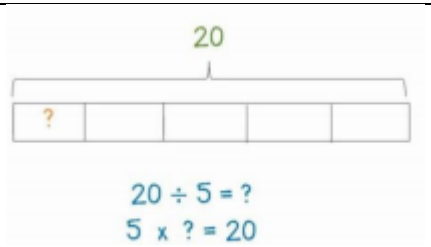
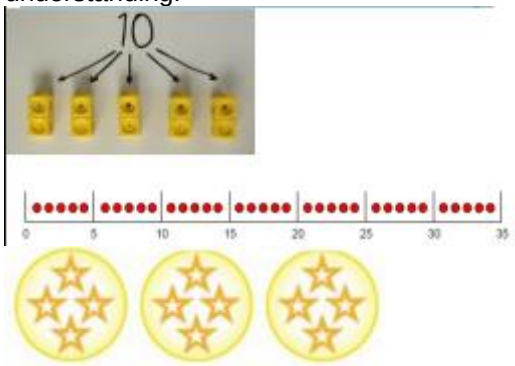
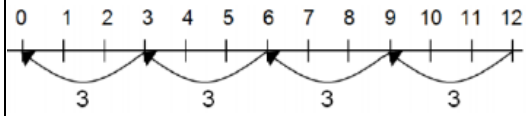
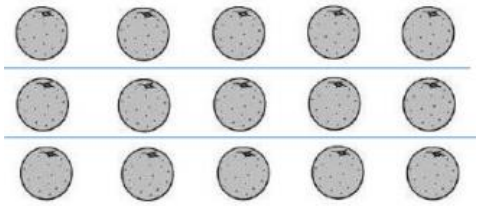
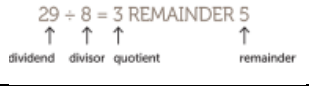
	introduce the grid method. 4 rows of 10 4 rows of 3 Move on to using Dienes to move towards a more compact method.  4 rows of 13 Move on to place value counters to show how we are finding groups of a number. We are multiplying by 4 so we need 4 rows.  Fill each row with 126.  Add up each column, starting with the ones making any exchanges needed.  Then you have your answer.	show different amounts or just use circles in the different columns to show their thinking as shown below.  	 210 + 35 = 245 Moving forward, multiply by a 2 digit number showing the different rows within the grid method.   		

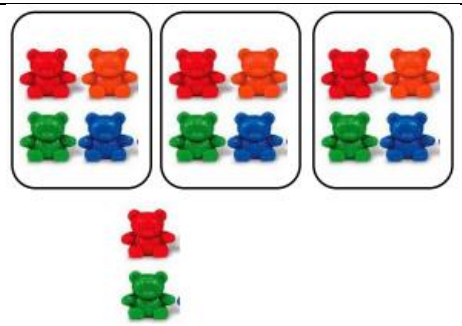
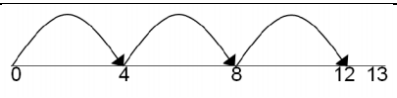
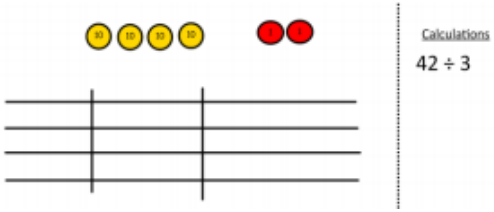
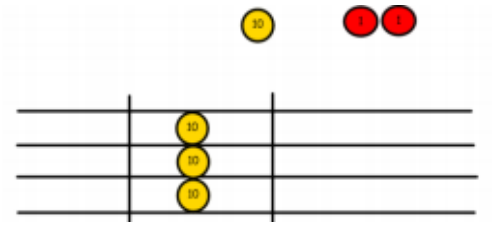
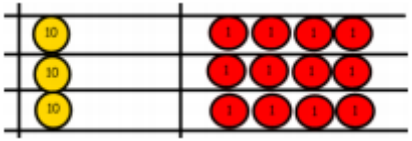
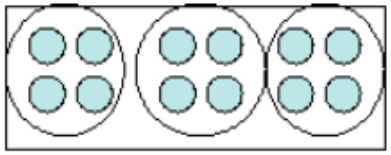
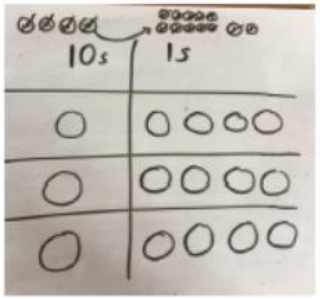
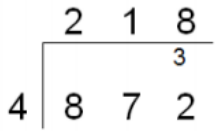
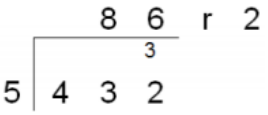
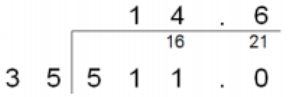
Column multiplication Y4/Y5/Y6	Formal column method with place value counters (children need this stage, initially, to understand how the column method works).   It is important at this stage that they always multiply the ones first and note down their answer followed by the tens which they note below.	Children to represent the counters/Dienes, pictorially e.g. the image below.  Bar modelling and number lines can support learners when solving problems with multiplication alongside the formal written methods.   	The aim is to get to the formal method but the children need to understand how it works. 6 x 23 = 23 × 6 138 1 1 With long multiplication, remind the children about lining up their numbers clearly in columns. If it helps, children can write out what they are solving next to their answer. 32 x 24 8 (4 x 2) 120 (4 x 30) 40 (20 x 2) 600 (20 x 30) 768 This moves to the more compact method.	Place value counters Coloured counters	Regrouping Multiplier, multiplicand, product

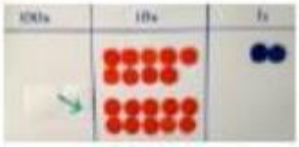
			96 32 x 192 ⇦ this is 96 x 2 2880 ⇦ this is 96 x 30 3072 ⇦ this is 96 x 32		

Division

Equal groups Reception/Y1			Are the groups equal? Why? Can you make them equal?	Cubes Natural objects Counters	Equal groups, share
Halving Reception/Y1	  	 Reinforce not always a vertical line through the middle.	Half of 6 = 3 Mental recall of halves to 10	Cubes Natural objects Counters Food	Halve, halving, share, share equally, equal groups of
Sharing objects into groups Y1/2	 I have 10 cubes, can you share them equally in 2 groups? 	Children use pictures or shapes to share quantities.  8 ÷ 2 = 4 Think of the bar as a whole. Split it into the number of groups you are dividing by and work out how many would be within each group.	Share 9 buns between three people. 9 ÷ 3 = 3	Cubes Natural objects Counters	share, share equally, equal groups of

					
Division as grouping Y2	Divide quantities into equal groups. Use cubes, counters or objects to aid understanding. 	Use a number line to show jumps in groups. The number of jumps equals the number of groups. 	28 ÷ 7 = 4 Divide 28 into groups of 7. How many are in each group?	Cubes Counters	Number line Groups of, divided by, divide, equal groups of
Division within arrays Y2	Link division to multiplication by creating an array and thinking about the number sentences that can be created.  Eg 15 ÷ 3 = 5 5 x 3 = 15 15 ÷ 5 = 3 3 x 5 = 15	 Draw an array and use lines to split the array into groups to make multiplication and division sentences.	Find the inverse of multiplication and division sentences by creating four linking number sentences. 7 x 4 = 28 4 x 7 = 28 28 ÷ 7 = 4 28 ÷ 4 = 7	Counter Cubes Array grids	Array, row, column Groups of
Division with a remainder Y2/Y3	14 ÷ 3 = Divide objects between groups and see how much is left over.	Jump forward in equal jumps on a number line then see how many more you need to jump to find a remainder. 13 ÷ 4 =	Complete written divisions and show the remainder using r 	Counters Cubes	Left, left over, remainder

		 Draw dots and group them to divide an amount and clearly show a remainder. 14 ÷ 4 = 	29 ÷ 8 = 3 r 5		
Short division Y3/4/5/6	Use place value counters to divide with written method alongside.  42 ÷ 3= Starting with the biggest place value, we are sharing 40 into three groups. We can put 1 ten in each group and we have 1 ten left over.  We exchange this ten for ten ones and then share the ones equally among the groups. 	Children can continue to use drawn diagrams with dots or circles to help them divide numbers into equal groups.  Children to represent the counters pictorially e.g. the image below. 42 ÷ 3 =  Encourage them to move towards counting in multiples to divide more efficiently.	Begin with divisions that divide equally with no remainder.  Move onto divisions with a remainder.  Finally move into decimal places to divide the total accurately. 	Coloured counters Place value counters	Short division, divisor, dividend, quotient

	We look how much in 1 group so the answer is 14.				
Long division Y5/Y6	192 ÷ 6 using place value counters to support written method  Exchange 100 for ten 10s  19 tens into groups of 6  3 groups so that is 30 x 6, exchange remaining 10 for ten 1s   So 192 ÷ 6 = 32 	Divide numbers up to 4-digits by a 2-digit whole number using the formal written method of short division where appropriate for the context. 564 ÷ 13 4 3 r 5 13 ⟌ 5 6 4 Using known multiplication facts 1 13 2 26 4 52 5 130 8 104 10 260 564 ÷ 13 = 43 r 5 = $43\frac{5}{13}$ = 43.38... 4 3 . 3 8 ... 13 ⟌ 5 6 4 . 5 11 0 Divide numbers up to 4-digits by a 2-digit whole number using the formal written method of long division, and interpret remainders as whole number remainders, fractions or by rounding, as appropriate for the context. 564 ÷ 13 4 3 . 3 8 ... 13 ⟌ 5 6 4 . 0 0 ... 5 2 4 4 - 3 9 5 0 - 3 9 1 1 0 - 1 0 4 6 = 43 r 5 = $43\frac{5}{13}$ = 43.4 (to 1dp)		Counters Place value chart	Long division, divisor, dividend, quotient

		432 ÷ 15 becomes