Ph 458 Evidence and Policy

WEEK 2: Evidence in ‘Hard Science’; The Elements of Probability Theory; and “Inverse Probability”

READING AND STUDY QUESTIONS

1. **Reading: Karl Popper ‘Conjectures and Refutations’; Imre Lakatos ‘Science and Pseudoscience**

Contrast Lakatos’s view about how scientists react to ‘refutations’ with those of Popper.

2. **My notes on ‘Probability Theory’; *sections 1 and 3***

What I hope will be some fairly straightforward exercises (do get others to help you with them if they seem a bit tricky):

1. Consider again the ‘game’ of drawing a card at random from a well-shuffled pack
   (a) What’s the probability of getting *either* a heart or a spade (i) in terms of basic events and (ii) by applying the addition law?
   (b) What’s the probability of getting *either* a heart or an ace (i) in terms of basic events and (ii) by applying the addition law?
   (c) What’s the probability of getting a card that is, at the same time, a spade and a face card (i) in terms of basic events and (ii) by applying the law for joint probabilities?
   (d) (i) Are getting an ace and getting a spade probabilistically independent events? How about the events of getting a king and a face card?
   (e) What are the *conditional probabilities* P(ace/spade), P(spade/ace), P(king/face card), P(face card/king)?
   (f) What’s the probability of getting a card that is, at the same time, a king and a face card (i) in terms of basic events and (ii) by applying the law for joint probabilities?

2. Let’s modify the example in my ***Notes*** slightly: suppose again that there are two, physically indistinguishable coins in a box, again one is fair but the other is totally biased: it *always* produces a head whenever you toss it. You select one coin from the box and toss it 4 times and observe $r = 4$ (i.e. all 4 tosses turn up heads)
   (a) Using Bayes’ theorem, what’s the probability, given this outcome, that the coin selected was the fair one?
   (b) Using Bayes’ theorem, what’s the probability, given this outcome, that the coin selected was the one maximally biased in favour of heads?
   (c) Identify the ‘priors’ and ‘posteriors’ of the two hypotheses (coin fair, coin biased) and discuss whether the changes induced by the observation $r = 4$ seem intuitively right.
   (d) Suppose that the outcome of the experiment (of tossing the coin 4 times) had been $r = 3$ (i.e. 3 heads, 1 tail). (i) what should intuitively happen to the posteriors of the two hypotheses? (ii) what happens according to Bayes’ theorem?