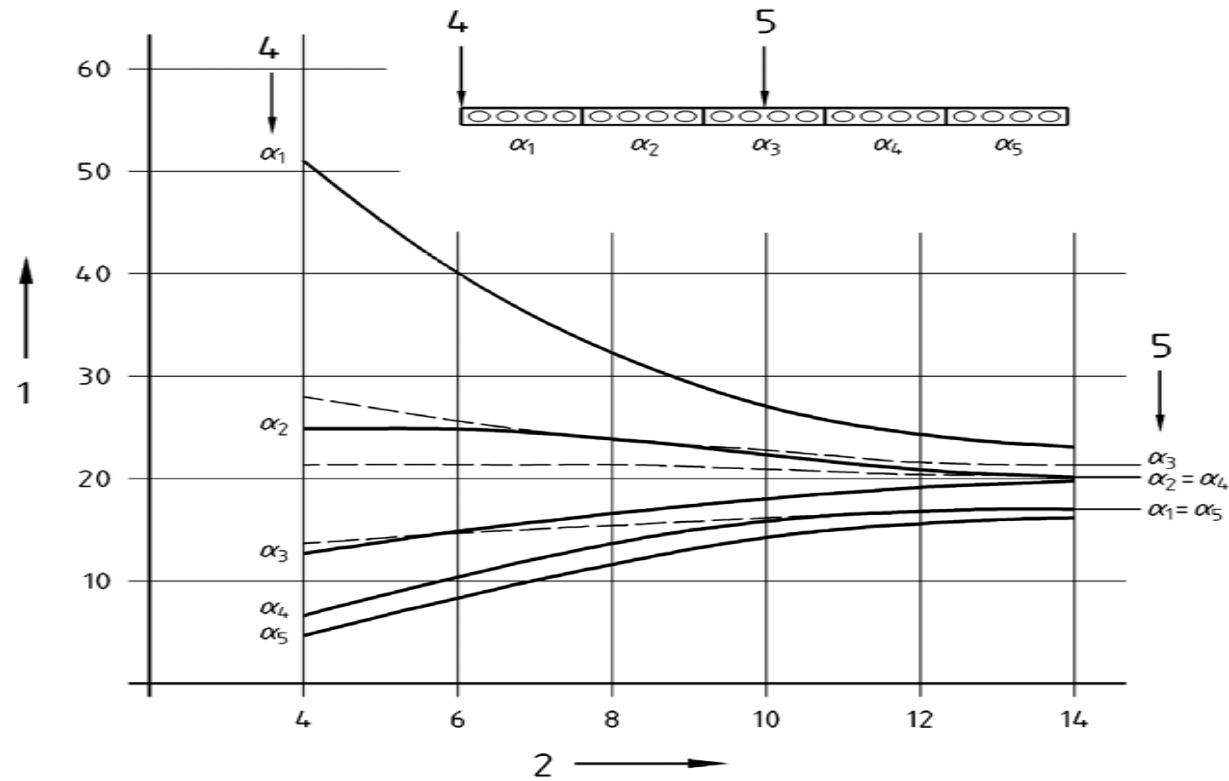




Holcotors results

- EN 1168 Distribution factors
- EN 1168 3 line support
- Shear & torsion interaction

Gösta Lindström, Dr. Sc, Technical Manager, Strängbetong

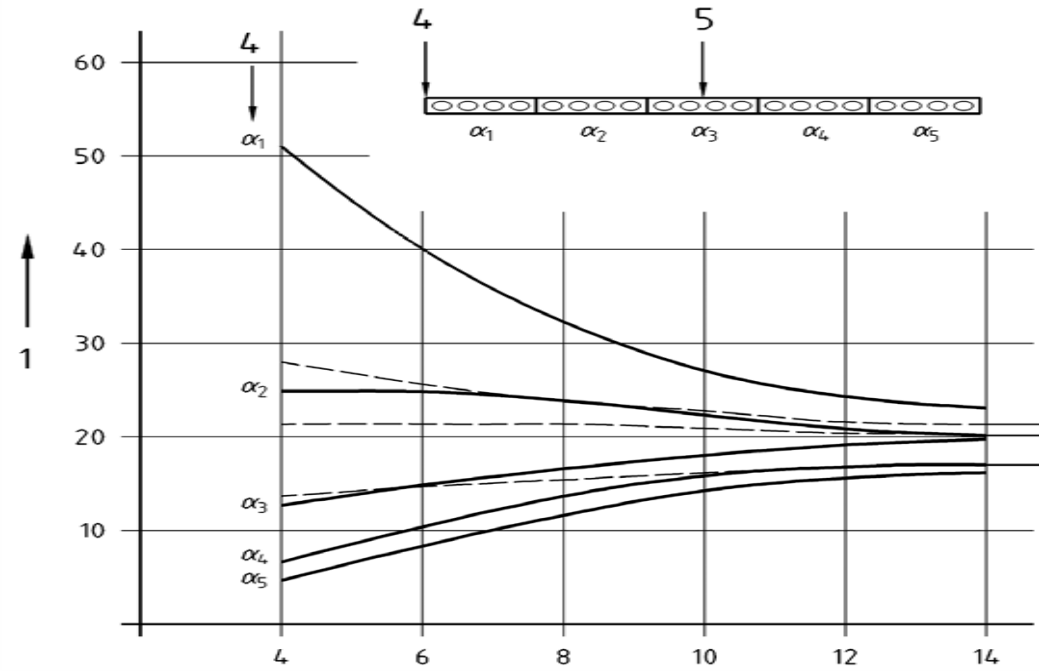


Key

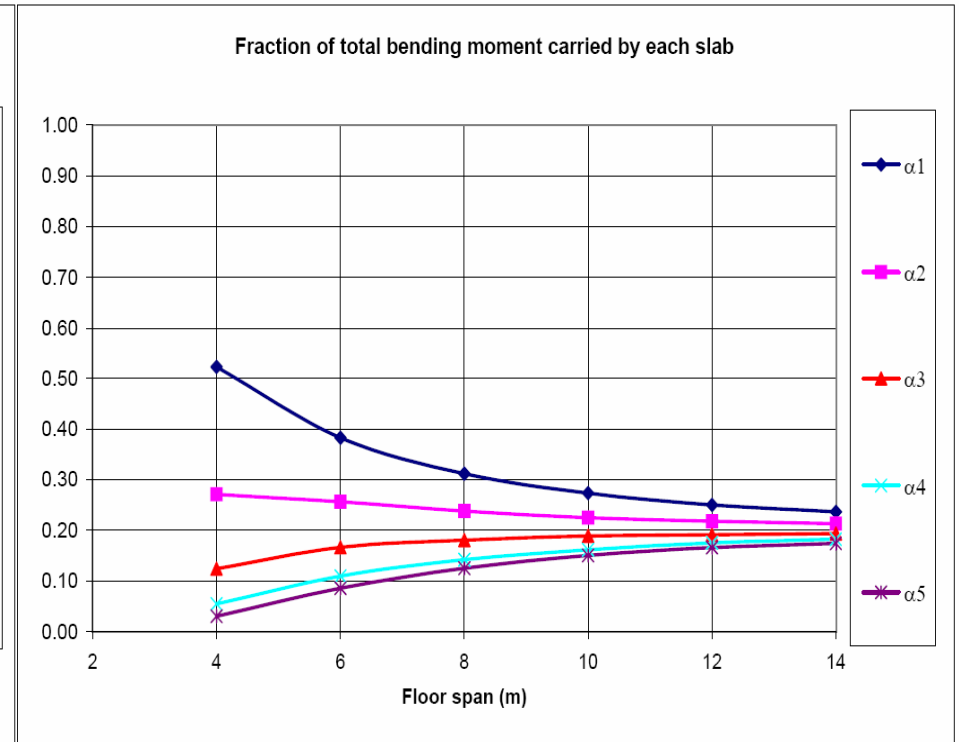
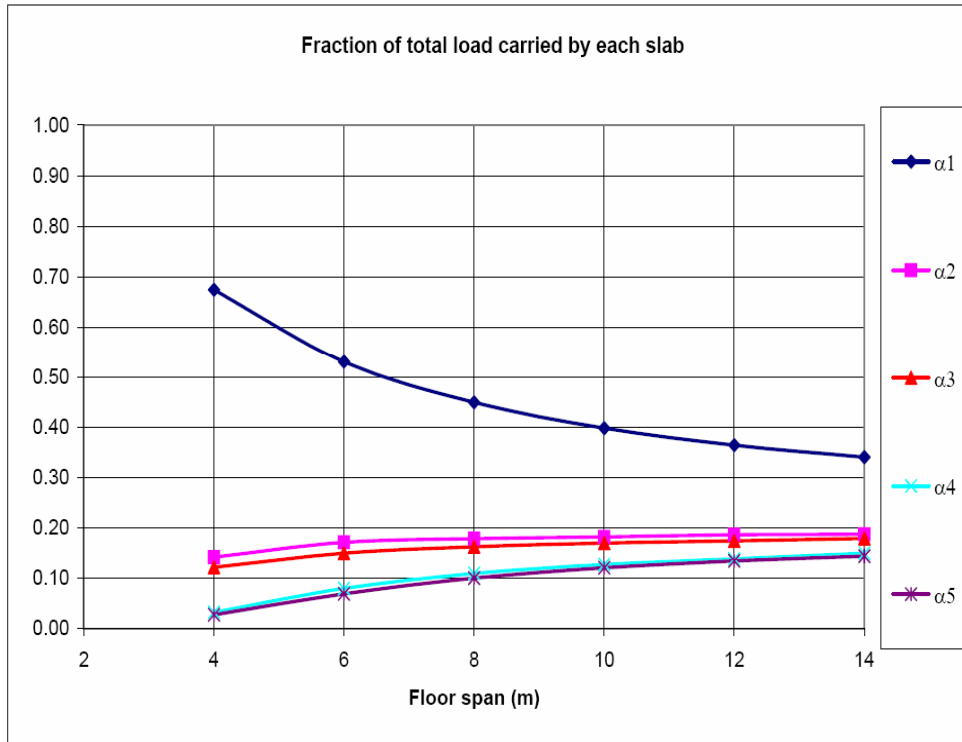
- 1 Loading percentage (%)
- 2 Span (*l*) in m
- 3 Linear loads
- 4 Edge
- 5 Centre

Figure C.1 — Load distribution factors for linear loads

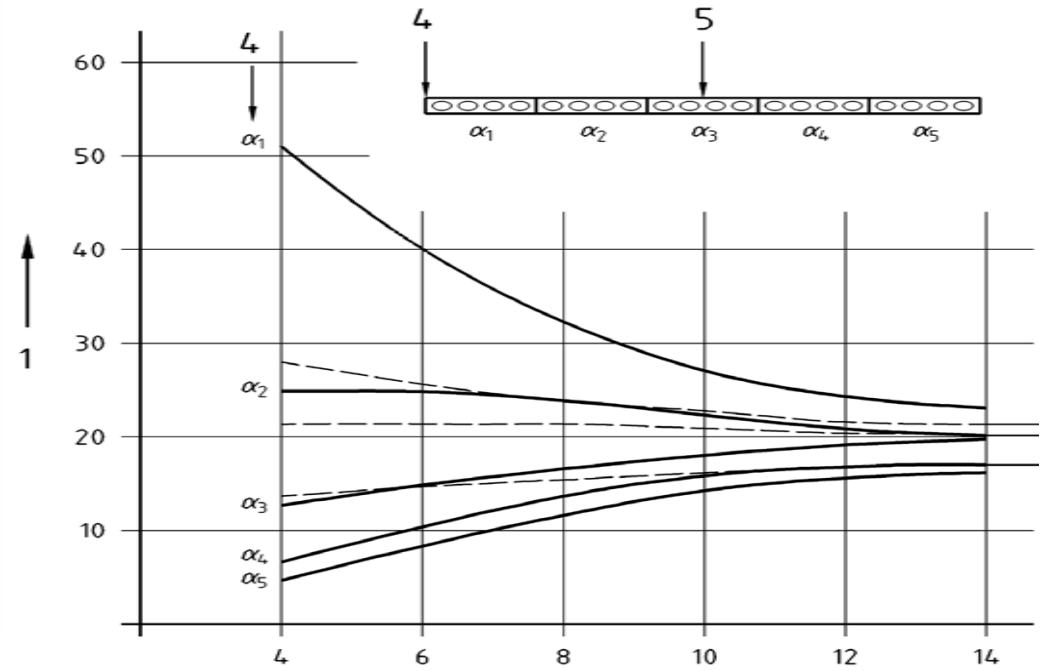
EN 1168 distribution factors



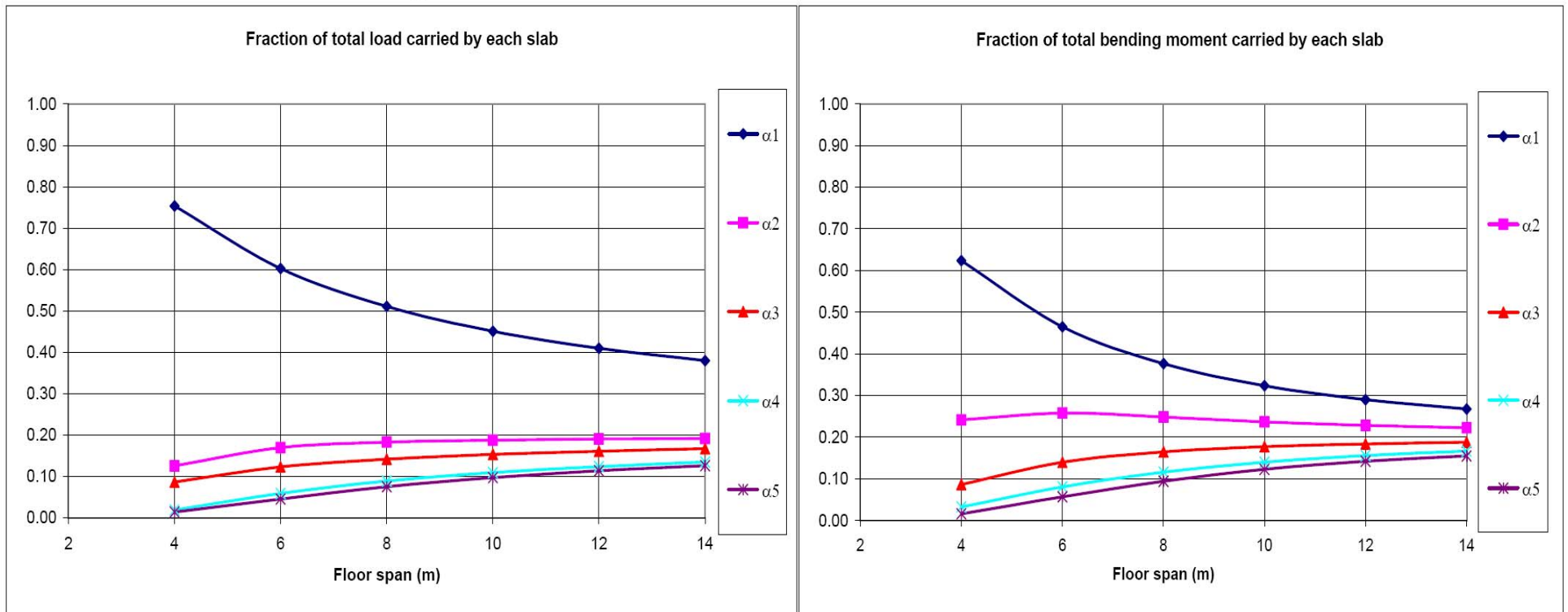
$EI/GK_v = 0.5$ Line load on edge



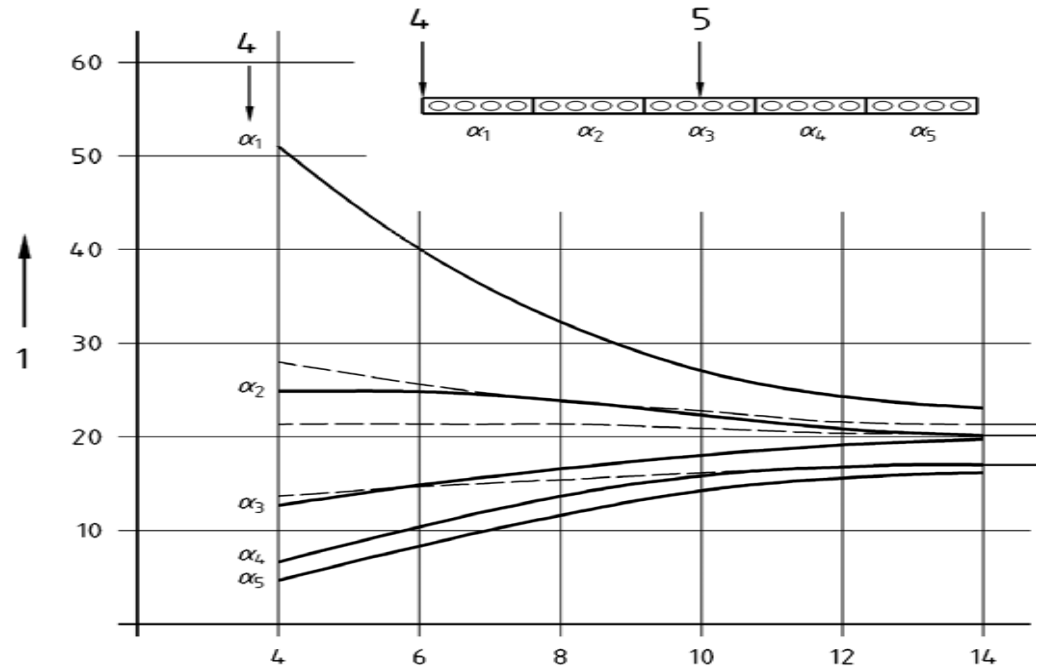
EN 1168 distribution factors



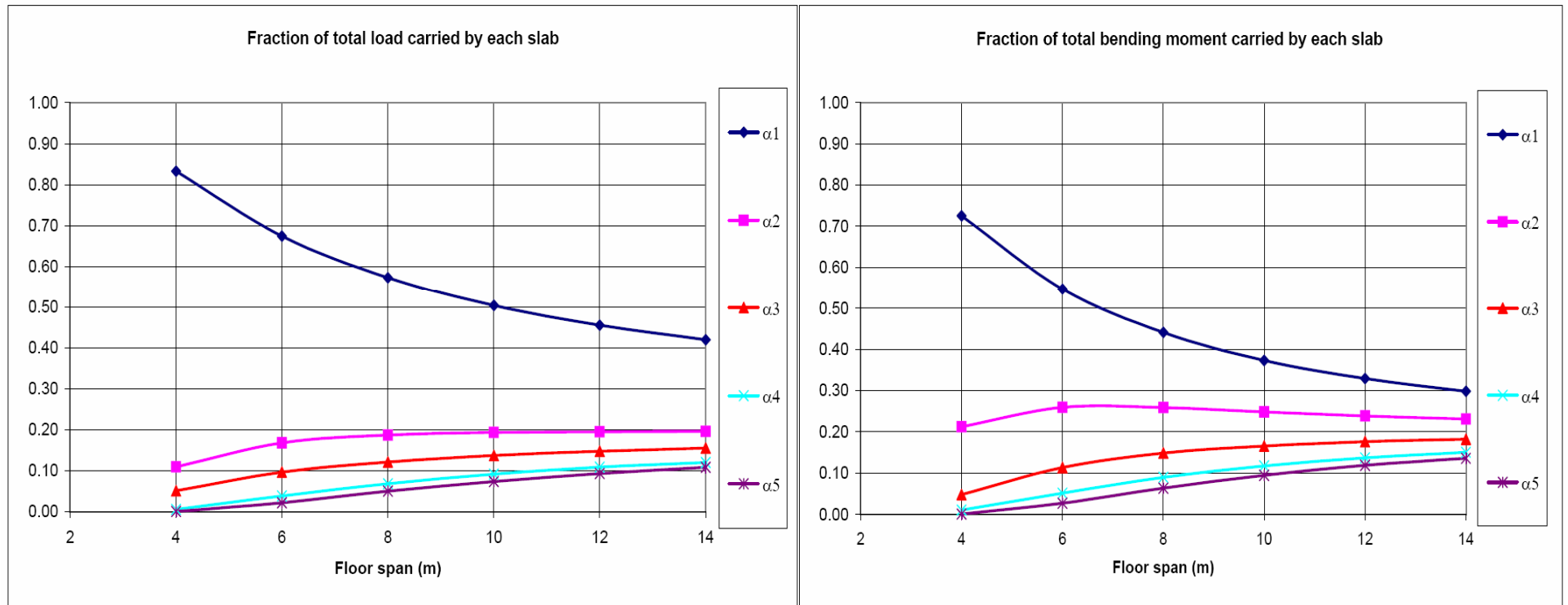
EI/GKv = 1.0 Line load on edge



EN 1168 distribution factors



EI/GKv = 1.5 Line load on edge

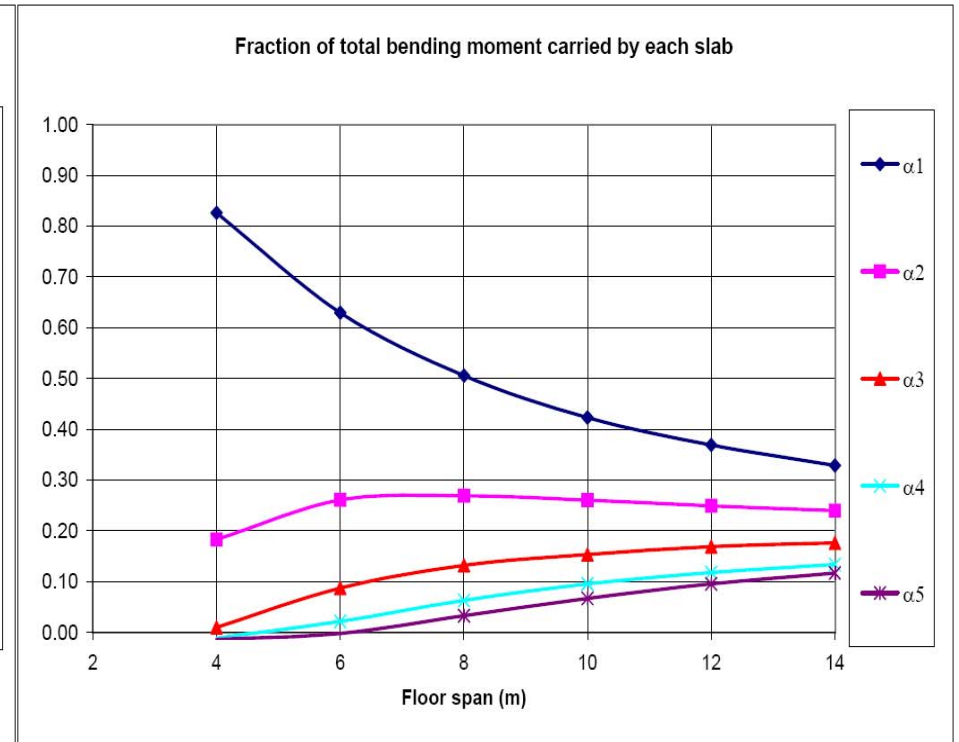
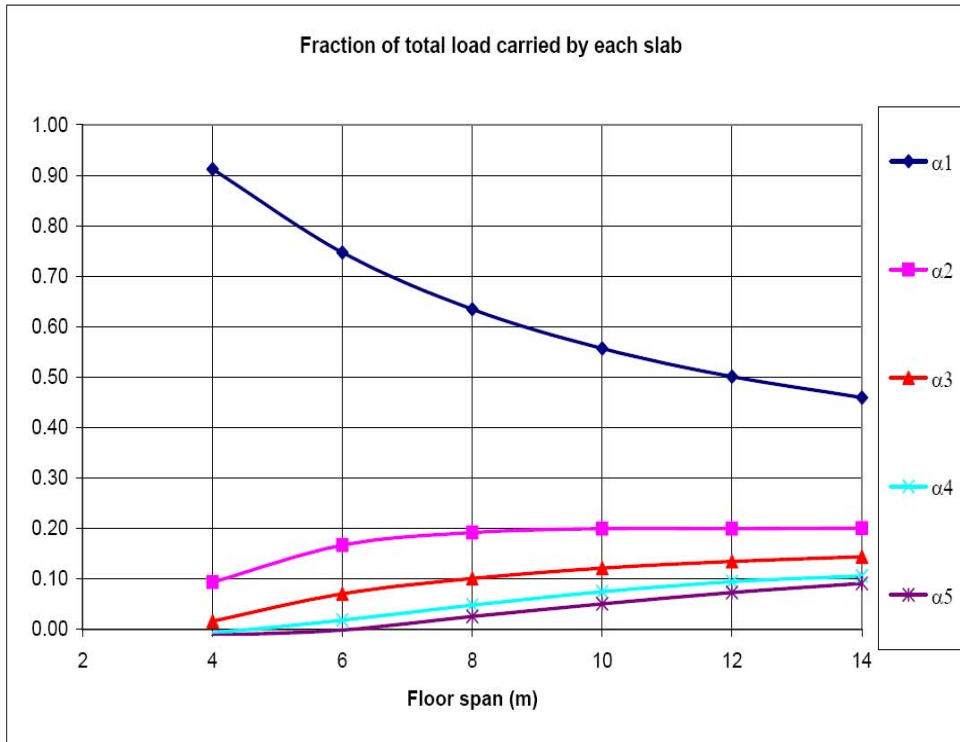
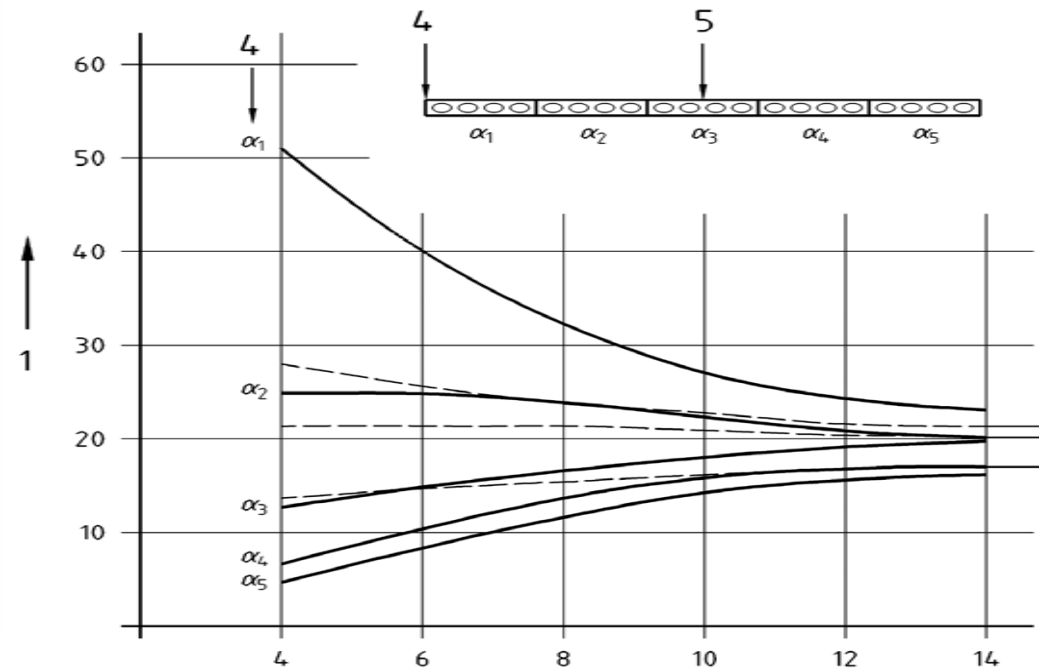


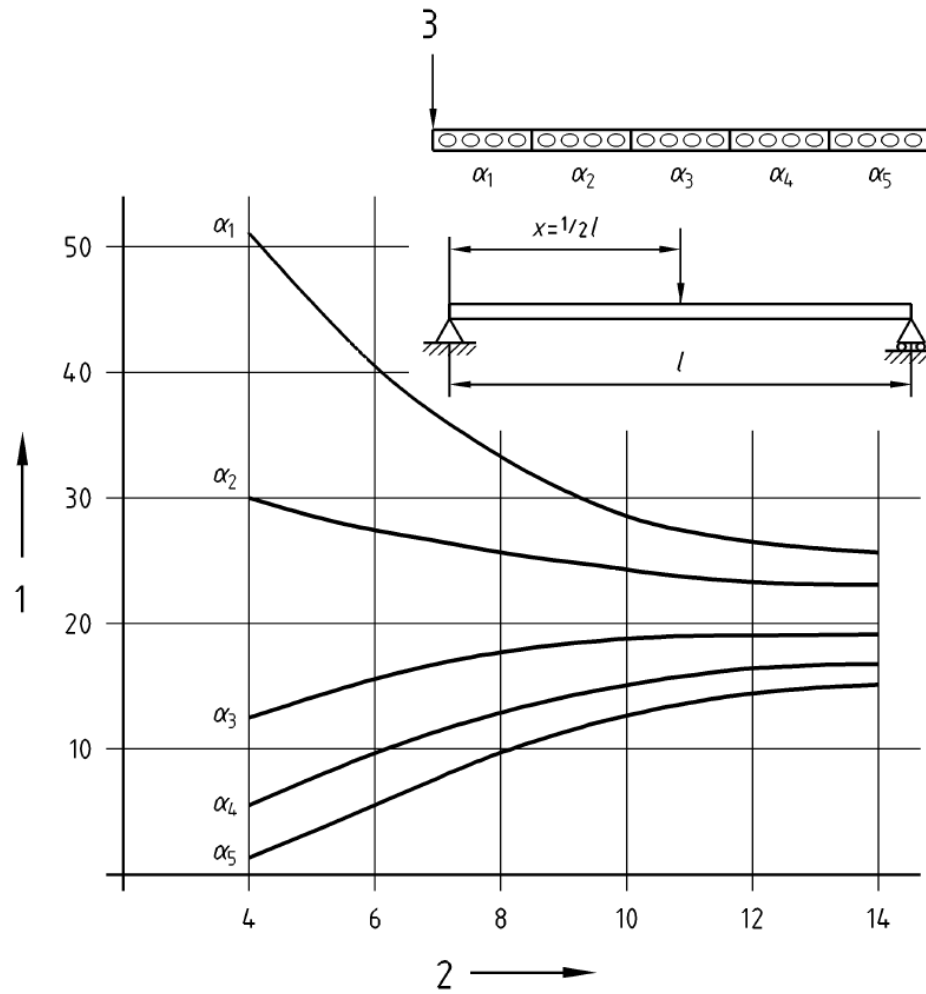
EN 1168 distribution factors

Conclusion for line load

α -factors in EN 1168 defines the fraction of total bending moment using $EI/GK_v = 0.6$

$EI/GK_v = 2.0$ Line load on edge



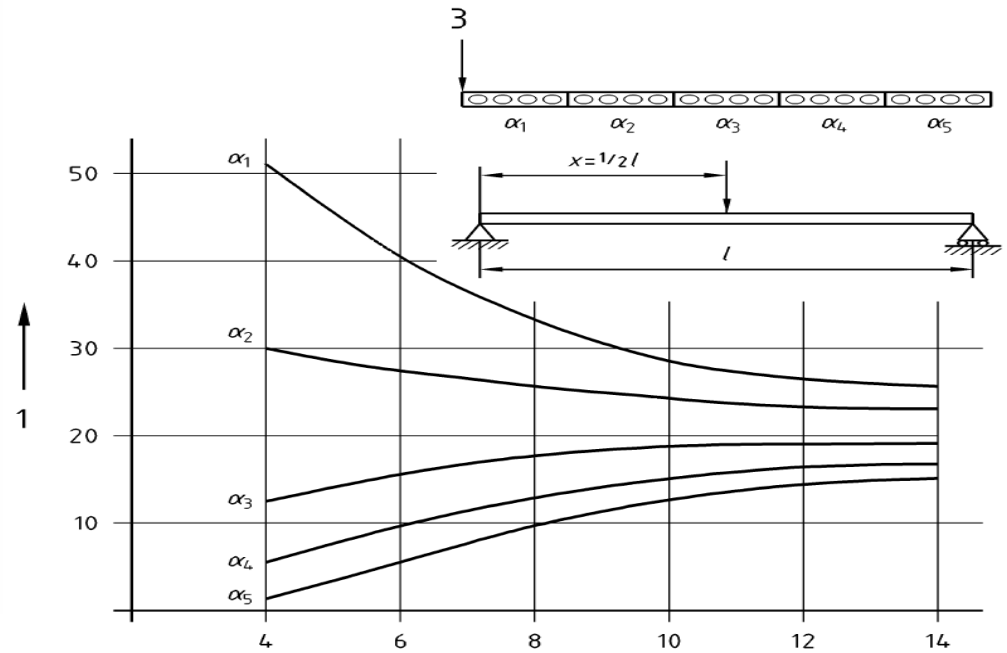


Key

- 1 Loading percentage (%)
- 2 Span (l) in m
- 3 Point load

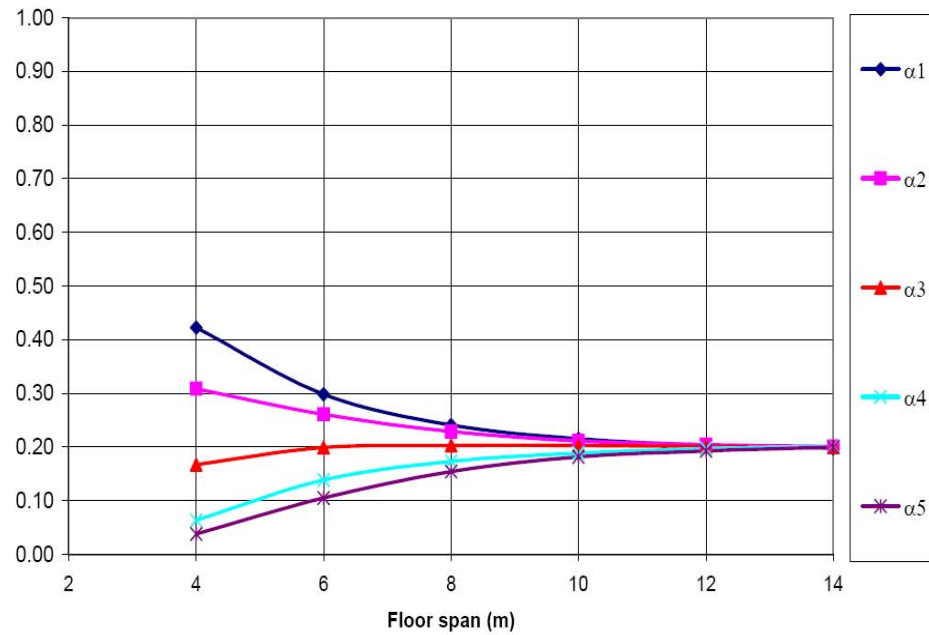
Figure C.3 — Load distribution factor for point loads at edge

EN 1168 distribution factors

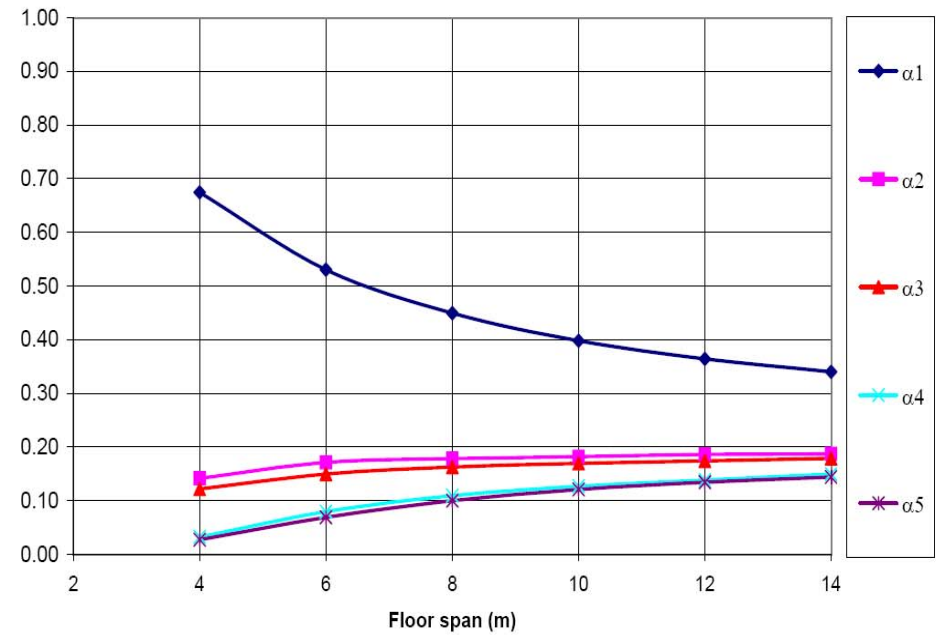


$EI/GK_v = 0.5$ Point load on edge

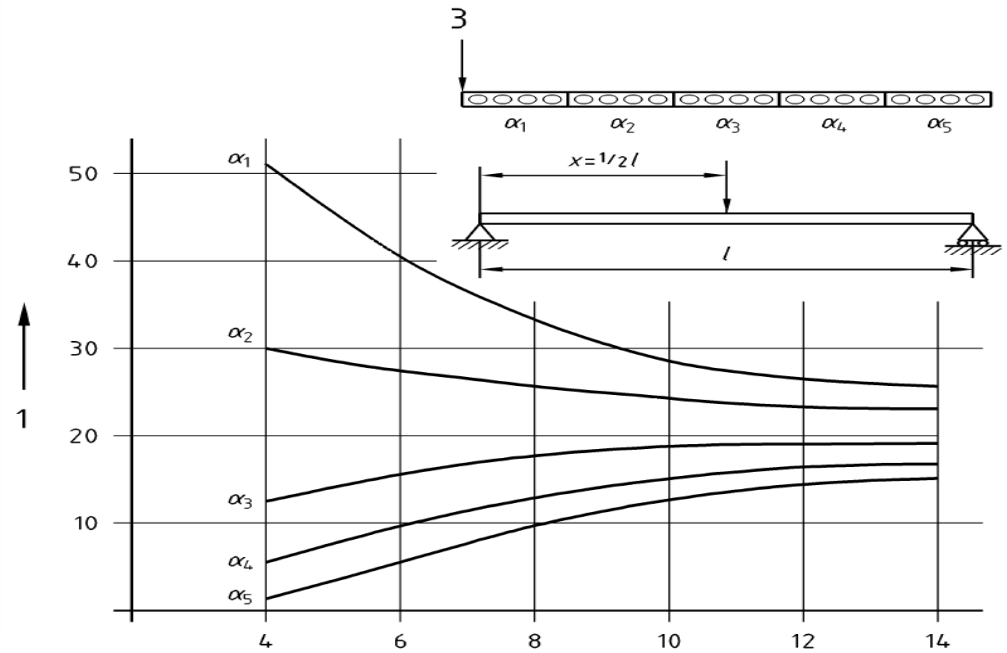
Fraction of total load carried by each slab



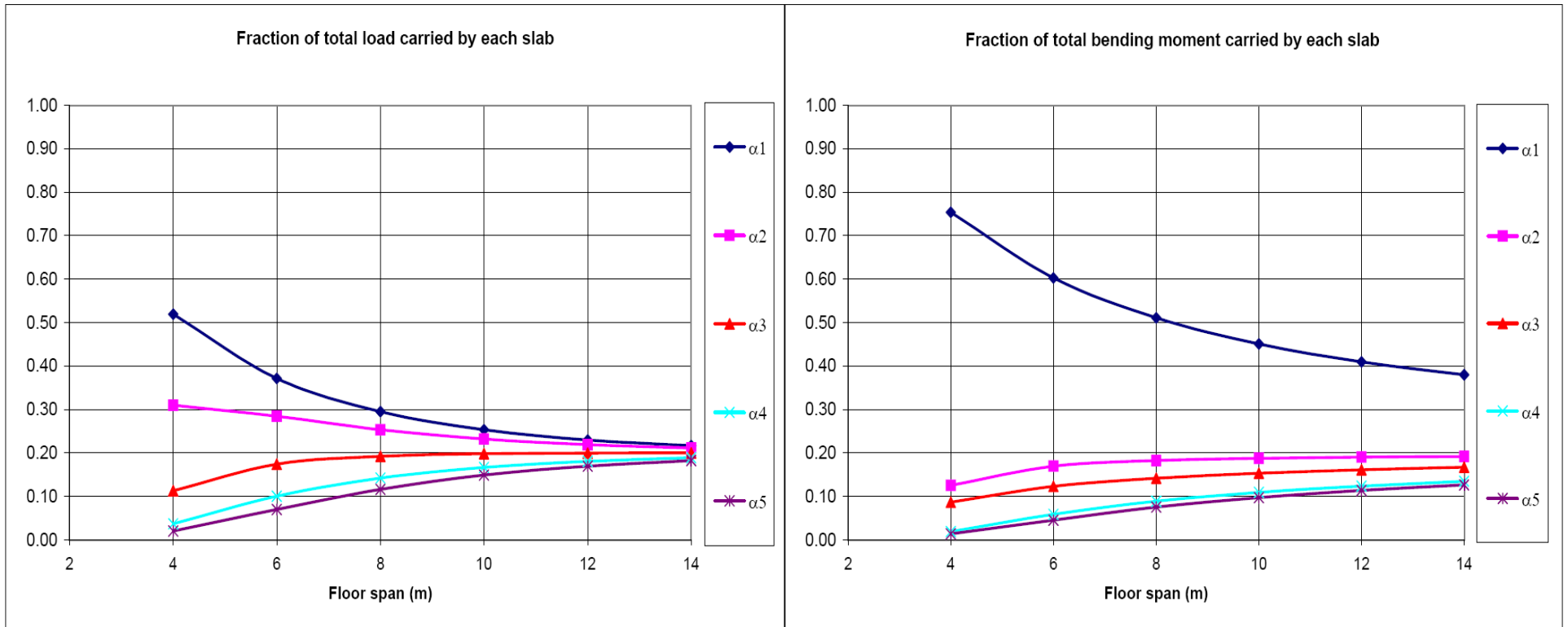
Fraction of total bending moment carried by each slab



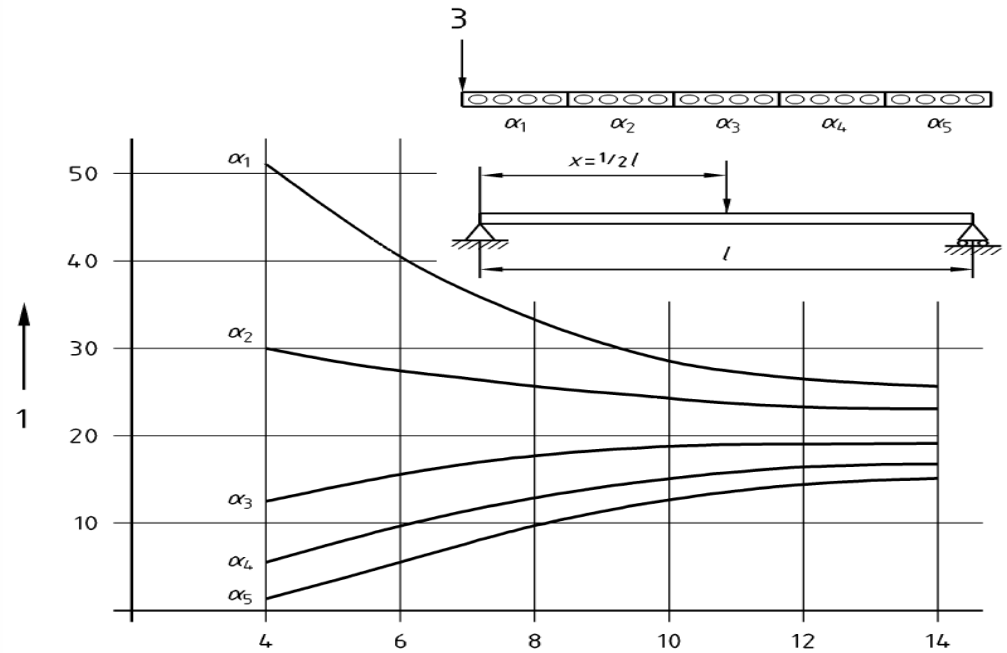
EN 1168 distribution factors



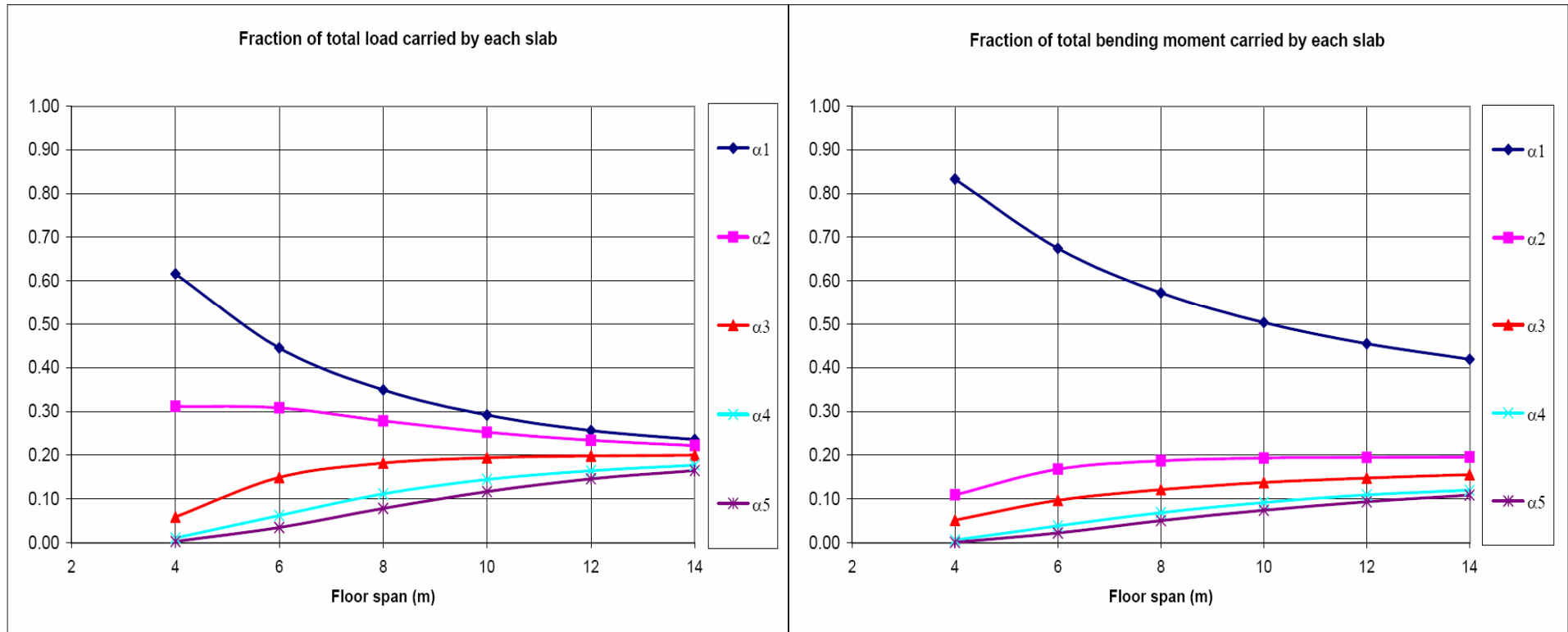
$EI/GK_v = 1.0$ Point load on edge



EN 1168 distribution factors



$EI/GK_v = 1.5$ Point load on edge

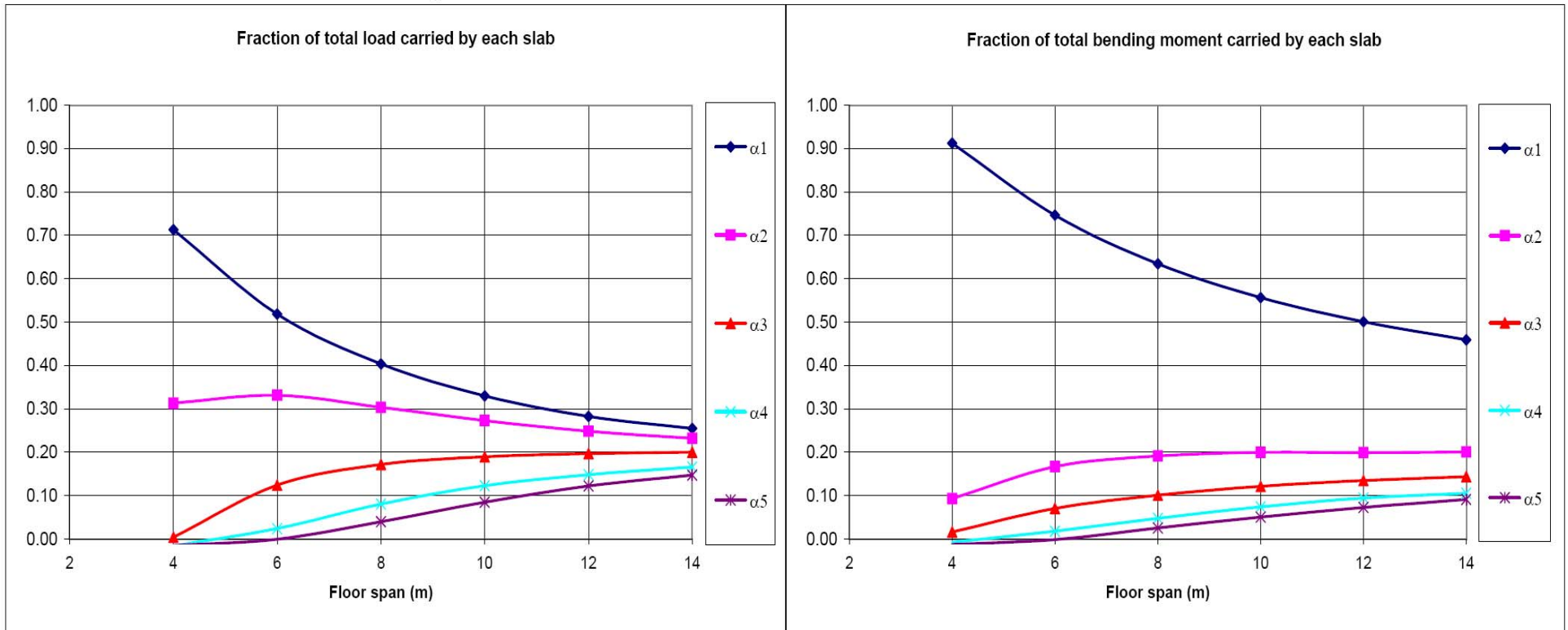
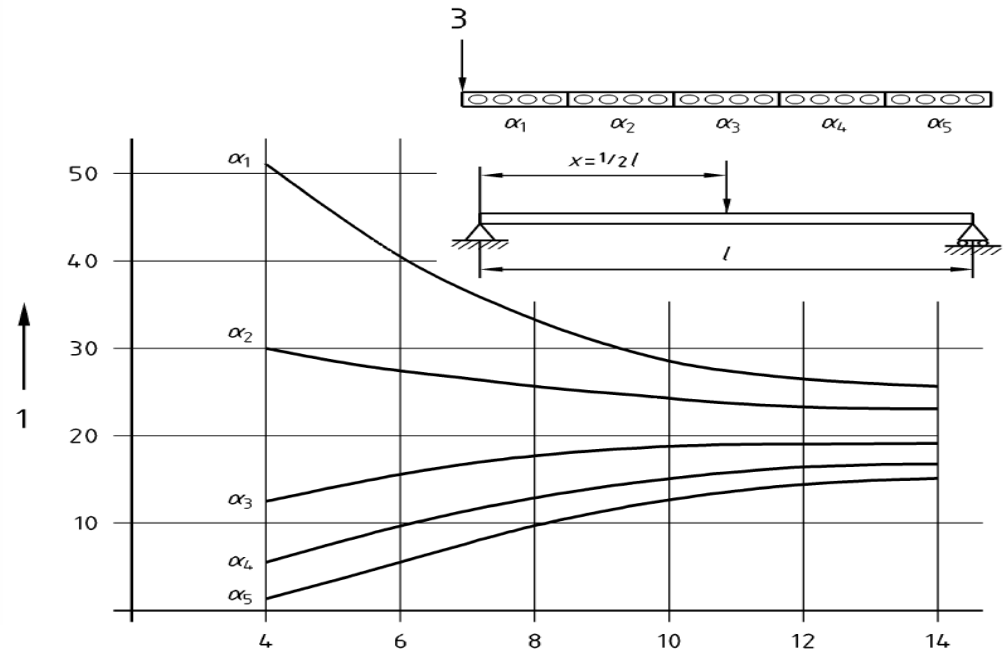


EN 1168 distribution factors

Conclusion for point load

α -factors in EN 1168 defines the fraction of total load using $EI/GK_v = 1$

$EI/GK_v = 2.0$ Point load on edge



Bending / Torsional stiffness

EI/GK_v

$$G = 0.4 E \quad (\nu=0.25)$$

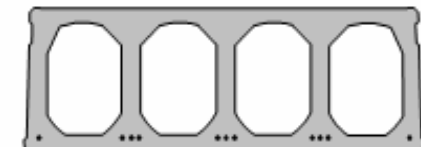
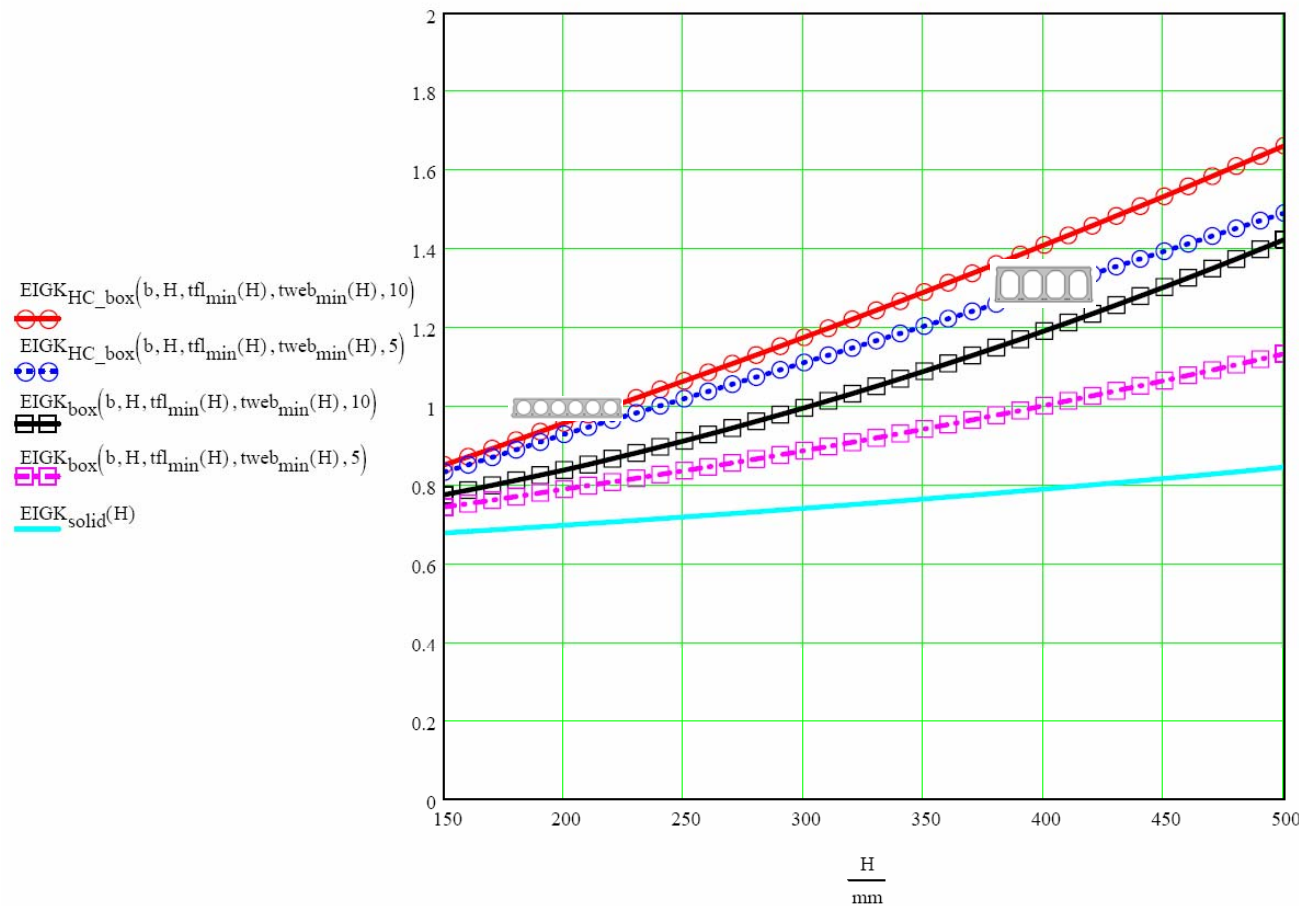


Figure The ratio between bending and torsional stiffness for some X-sections of total width 1200 mm and total depth H in the range 150 to 500 mm.

The flange and web thickness are selected as minimum according to EN 1168.

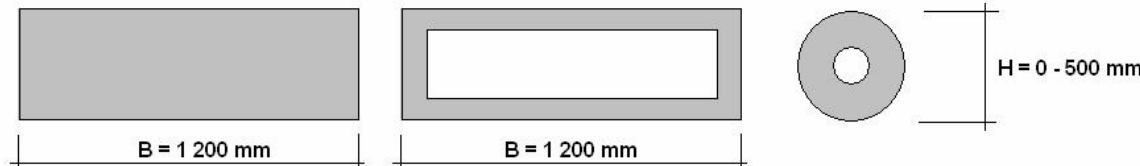
Shear modulus $G = 0.4 E$

The X-sections curves represents the following from top down:

- Hollow core section with 10 webs
- Hollow core section with 5 webs
- Thin walled box section with 10 webs
- Thin walled box section with 5 webs
- Homogeneous cross section

For torsional stiffness of thin walled box only the contribution from outer webs are included

Some Torsion basics



Maximum shear stress for different X-sections can be expressed as

$$\tau = G \cdot d\Phi/dx \cdot K_v / W_v =$$

const · SectionProperty

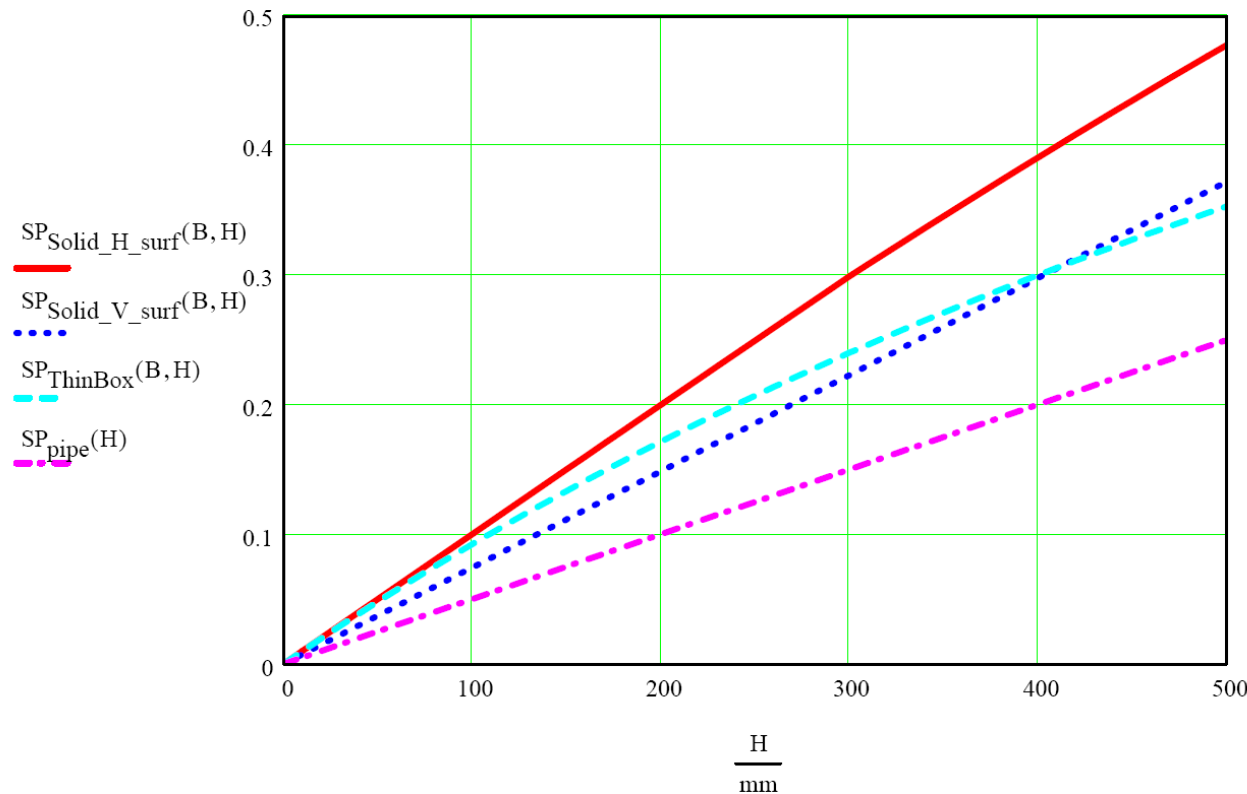


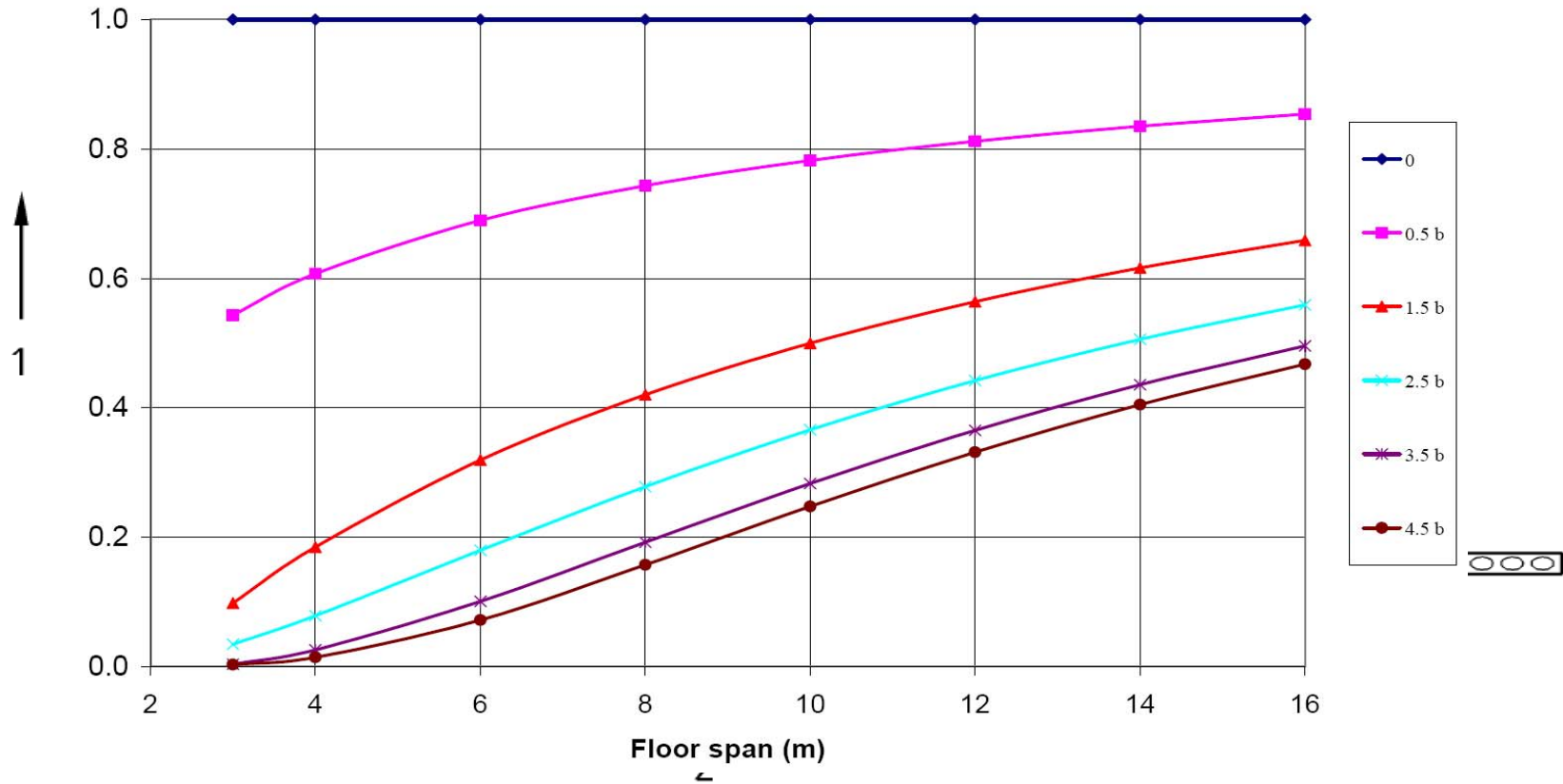
Figure For a constant torsional gradient the relative magnitude of shear stress for different cross-sections are presented as a function of the cross-section depth H.

The sections from top down are:

- Solid rectangular section, width 1.2 m and depth H, stress on horizontal surface (top or bottom)
- Solid rectangular section, width 1.2 m and depth H, stress on vertical sides
- Thin walled box of const thickness, width 1.2 m and depth H, stress on all walls
- Pipe section of diameter H of any wall thickness



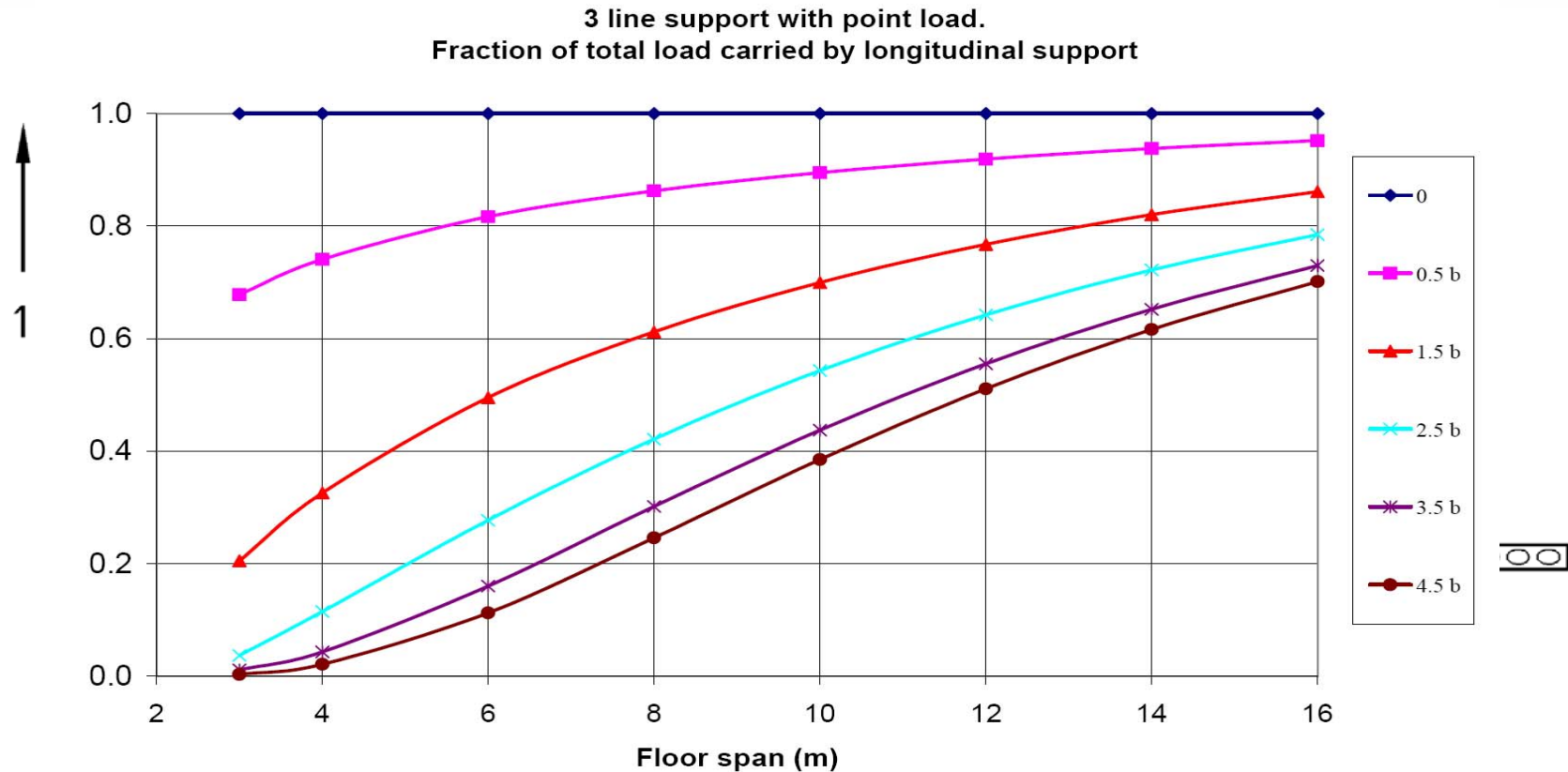
3 line support with line load.
 Fraction of total load carried by longitudinal support



Key

- 1 Reaction force/linear load
- 2 Span (l) in m
- 3 Linear load
- 4 Reaction force

Figure C.5 — Reaction force at longitudinal support due to linear load



Key

- 1 Reaction force x span/point load
- 2 Span (l) in m
- 3 Point load
- 4 Reaction force

Figure C.6 — Reaction force at longitudinal support due to a point load at midspan



Can the improved interaction be utilized without FE analysis ?

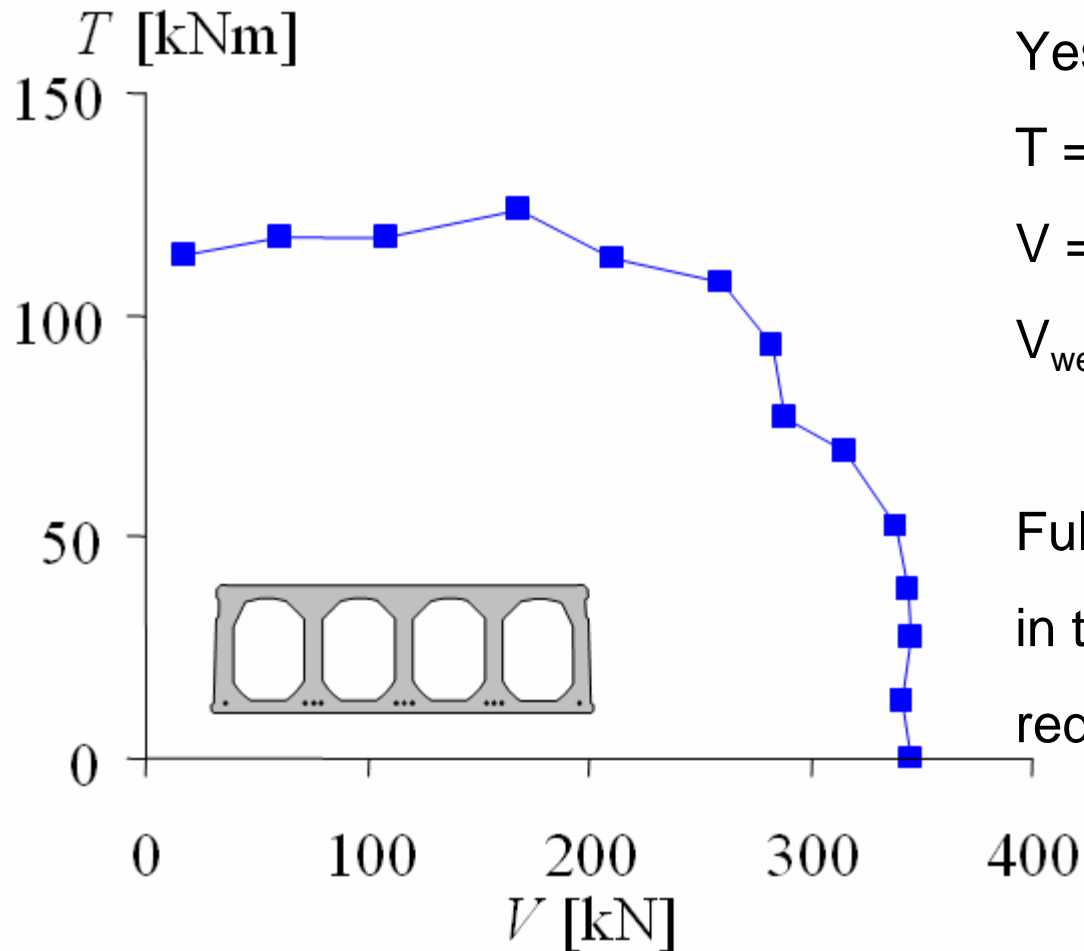
Yes, assume:

$T = 100 \text{ kNm}$ (pure torsion)

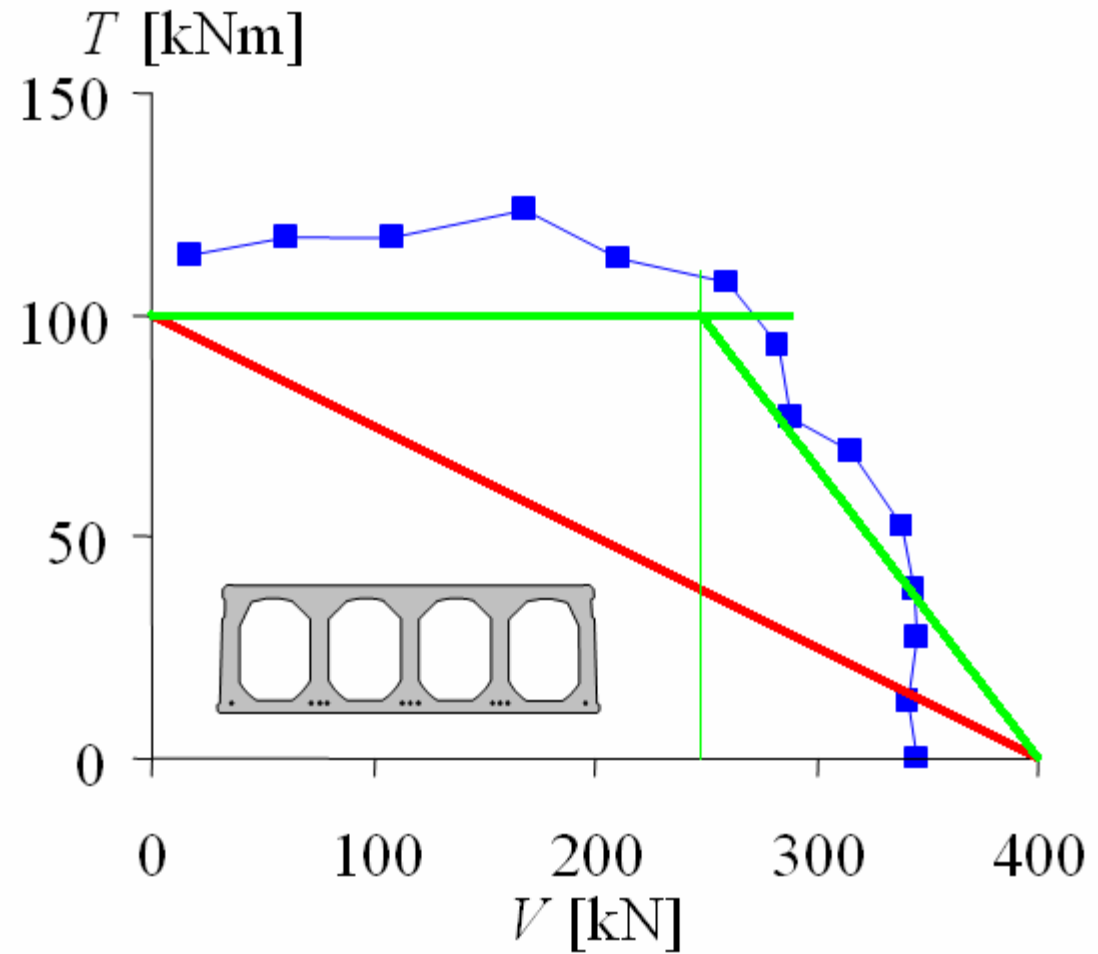
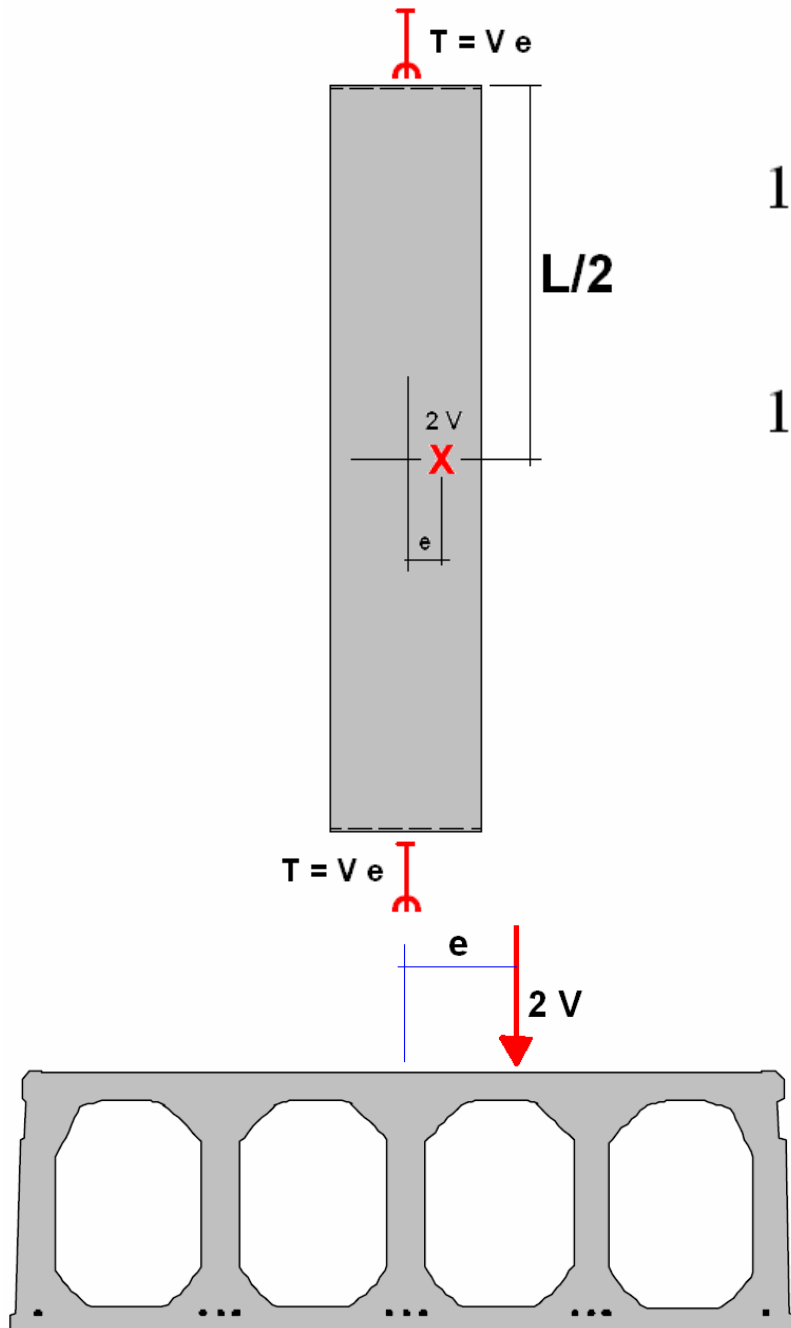
$V = 400 \text{ kN}$ (pure shear tension)

$V_{\text{web}} = 400 / 5 = 80 \text{ kN}$ each

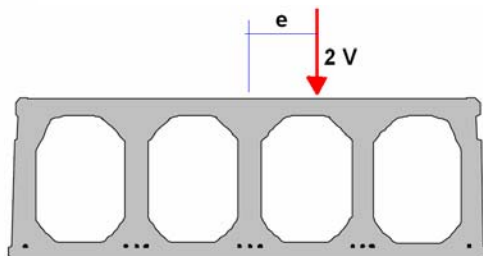
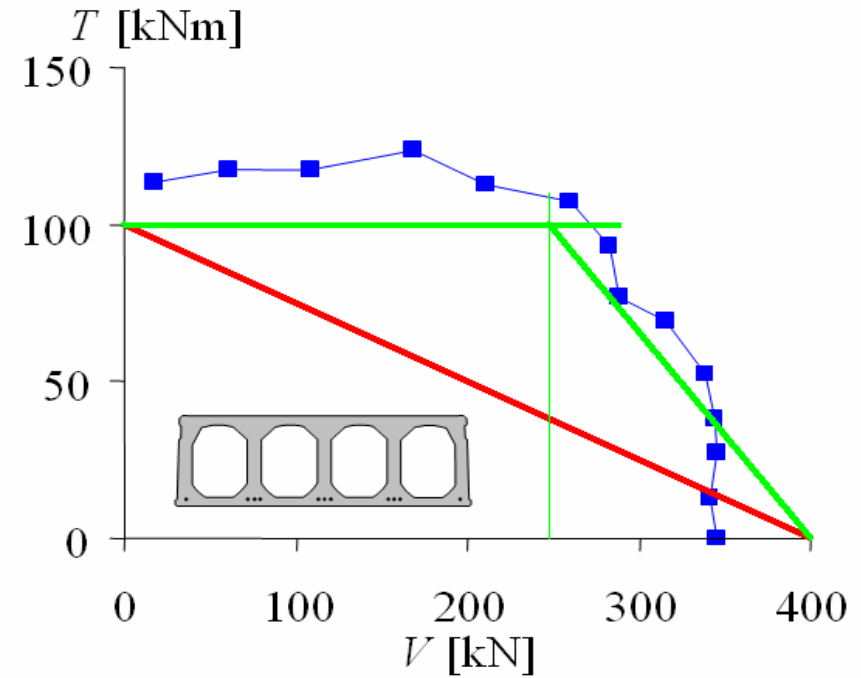
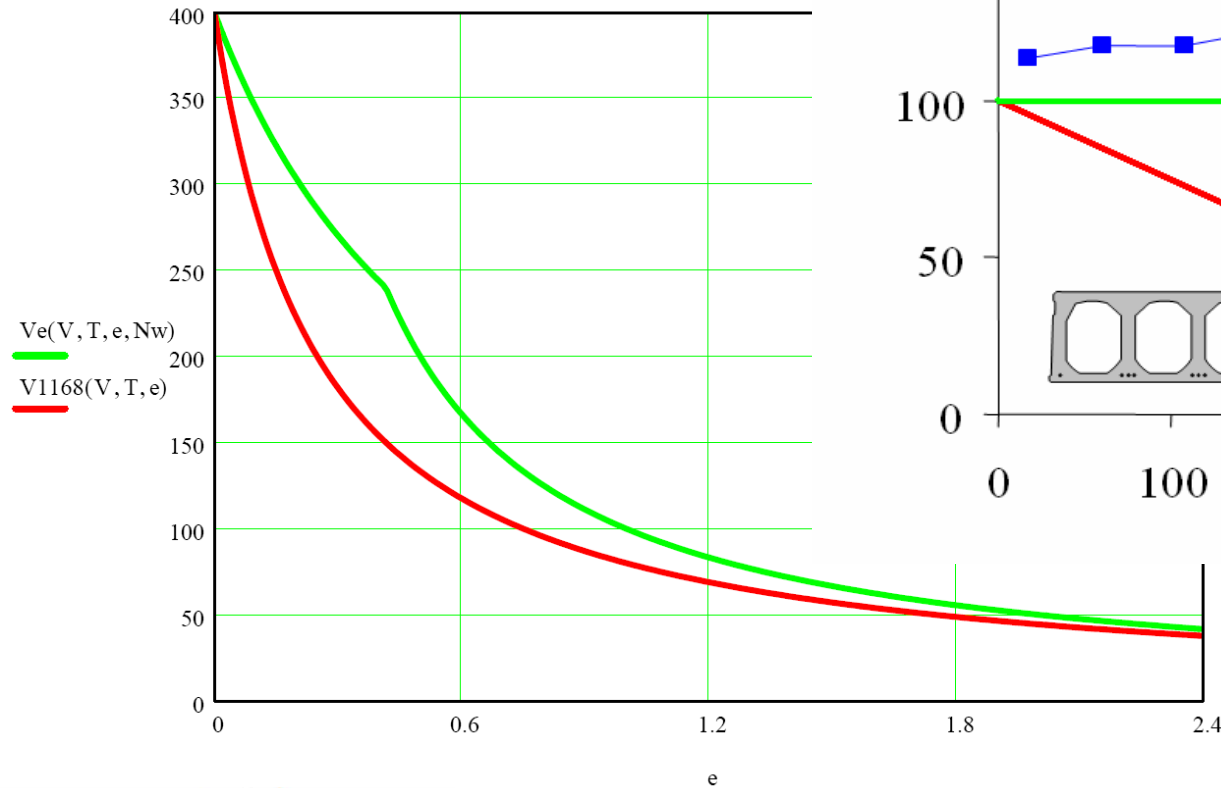
Full torsional shear flow
in thin walled box without
reduction of shear in inner webs



Shear torsion interaction formula

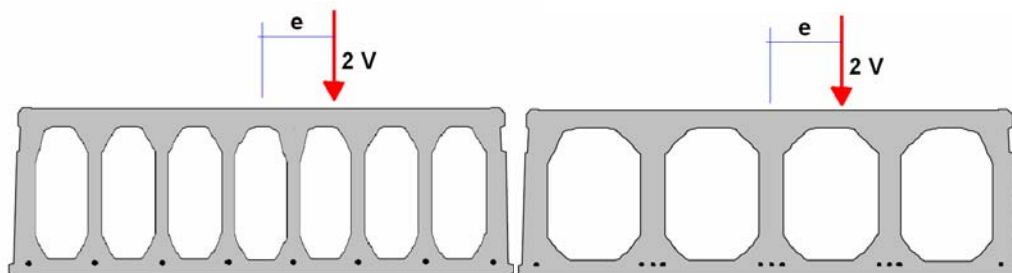
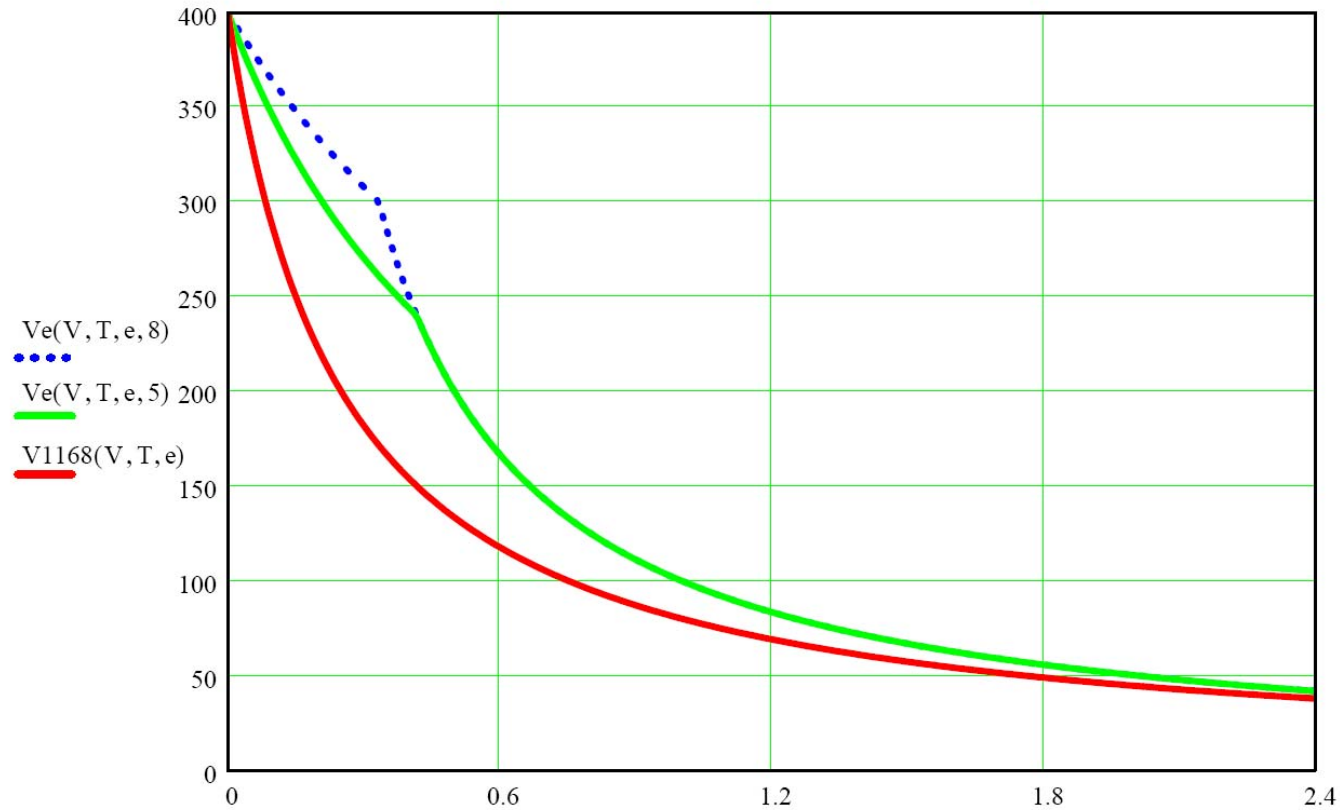


Load capacity V (kN) with different eccentricity e due to the two interaction curves

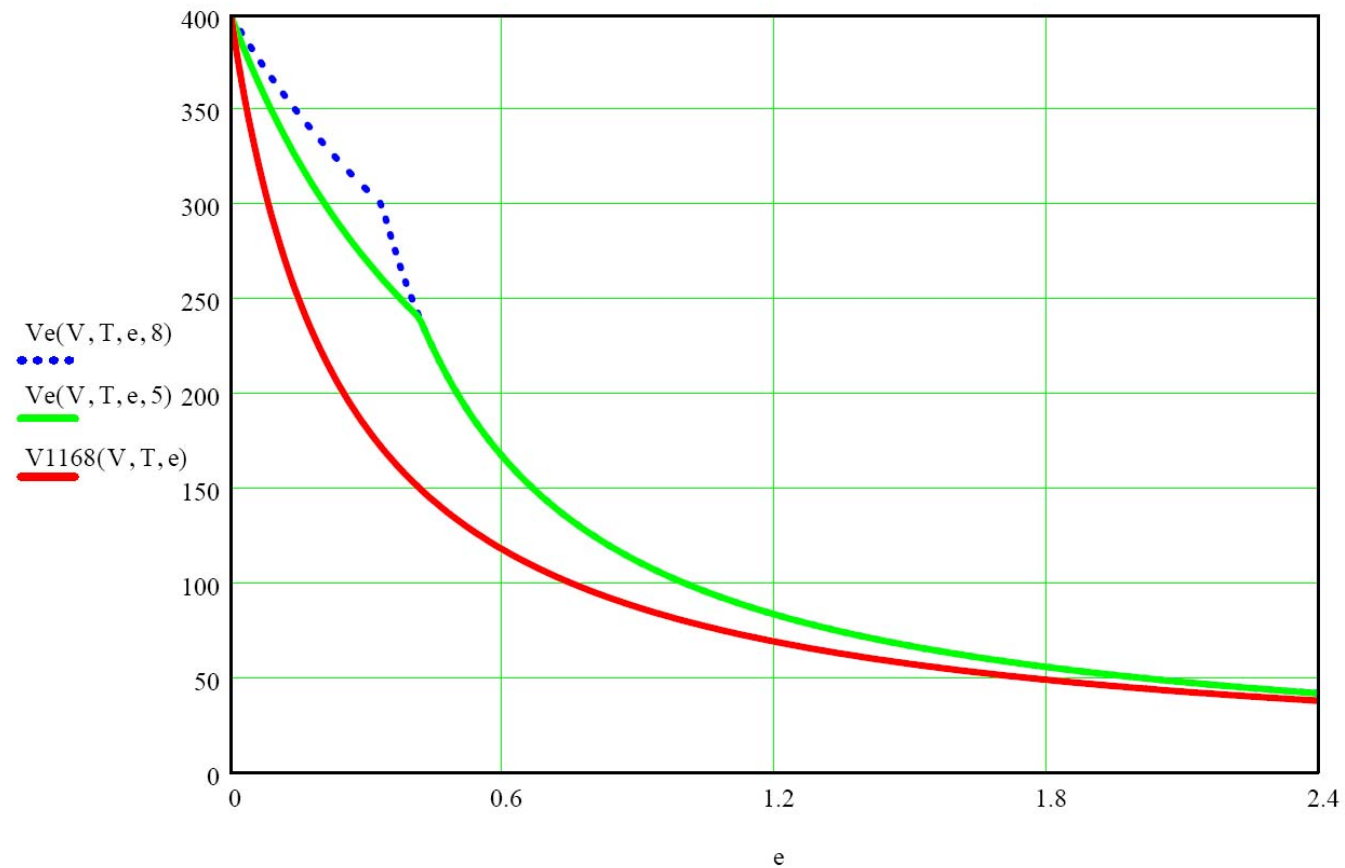


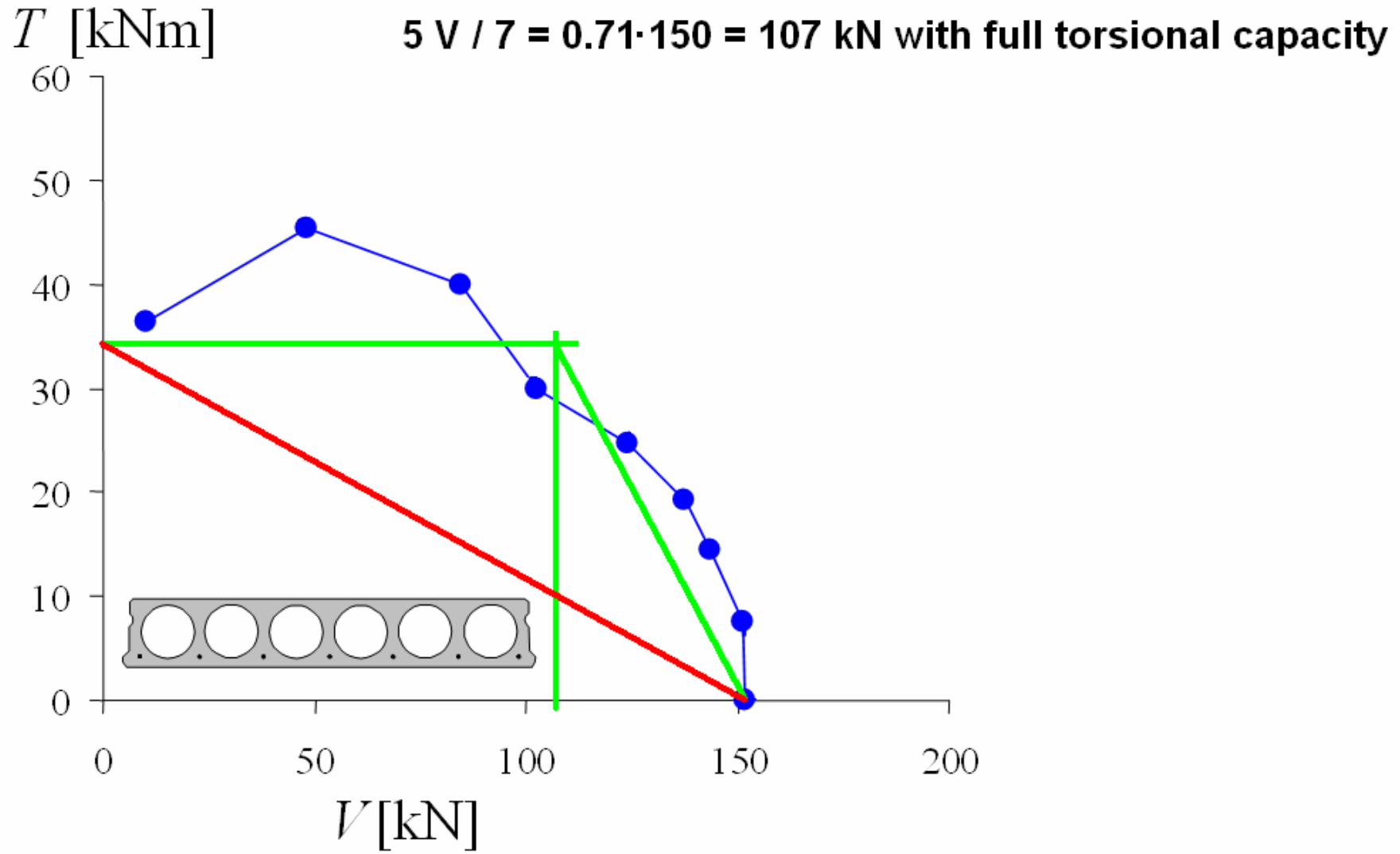
Load capacity V (kN) with different eccentricity e due to the two interaction curves

Capacity for cross-section with 8 or 5 webs



EN 1168 distribution factors, comparison





Shear torsion interaction



For Shear capacity calculation a method including the shear stress in the transfer zone is recommended. HOLCOTORS project, Lin Yang formula was tested

2.5 Comparison with FE-analysis

The case with slab HD/F 380 and prestressing reinforcement 1100 mm^2 has also been modelled in a FE-analysis to compare the shear stresses calculated with the suggested model.

The figure below shows the principal stresses over the cross section height. The transfer of prestress is assumed linearly along the transfer length 403 mm.

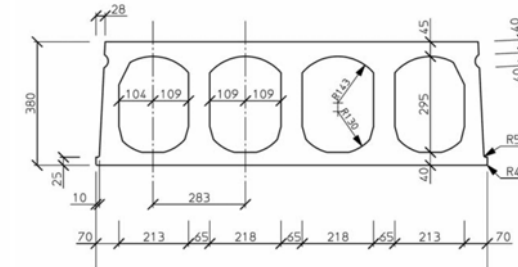


Figure 1.3:3 Cross section HD/F 380, $A_p = 1100 \text{ mm}^2$, $z_p = 37 \text{ mm}$

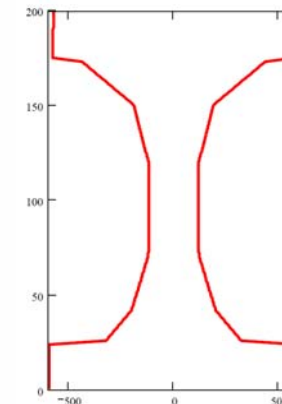
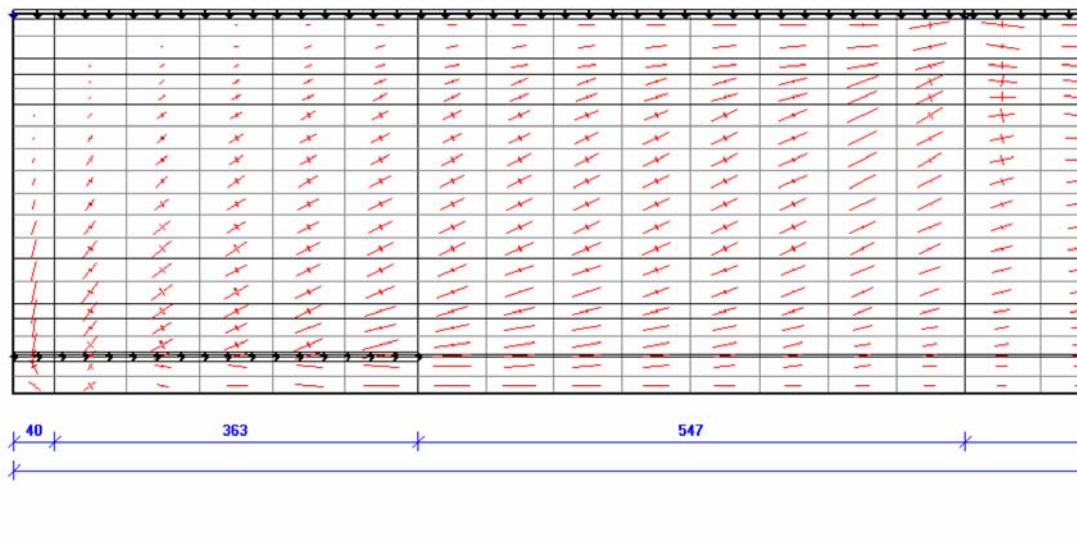
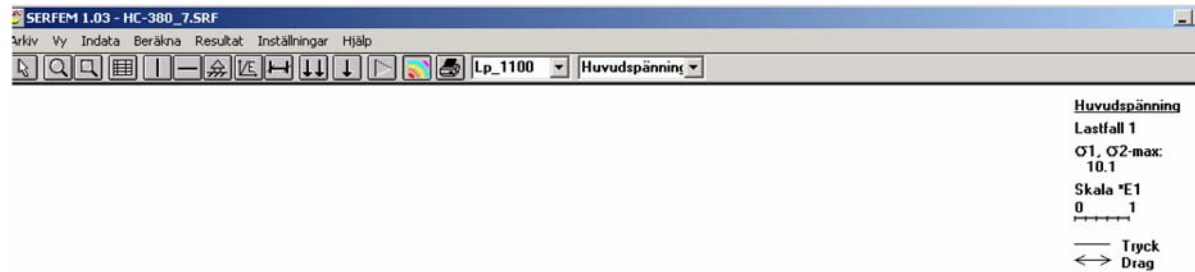


Figure 2.5:1 Principal stress in the transfer region. The slab are loaded with point load at a distance $2.5 H$ from the support. The shear load are set equal to the capacity calculated using the suggested model.

Shear torsion interaction



Analytical calculation gives very good agreement with FE-analysis

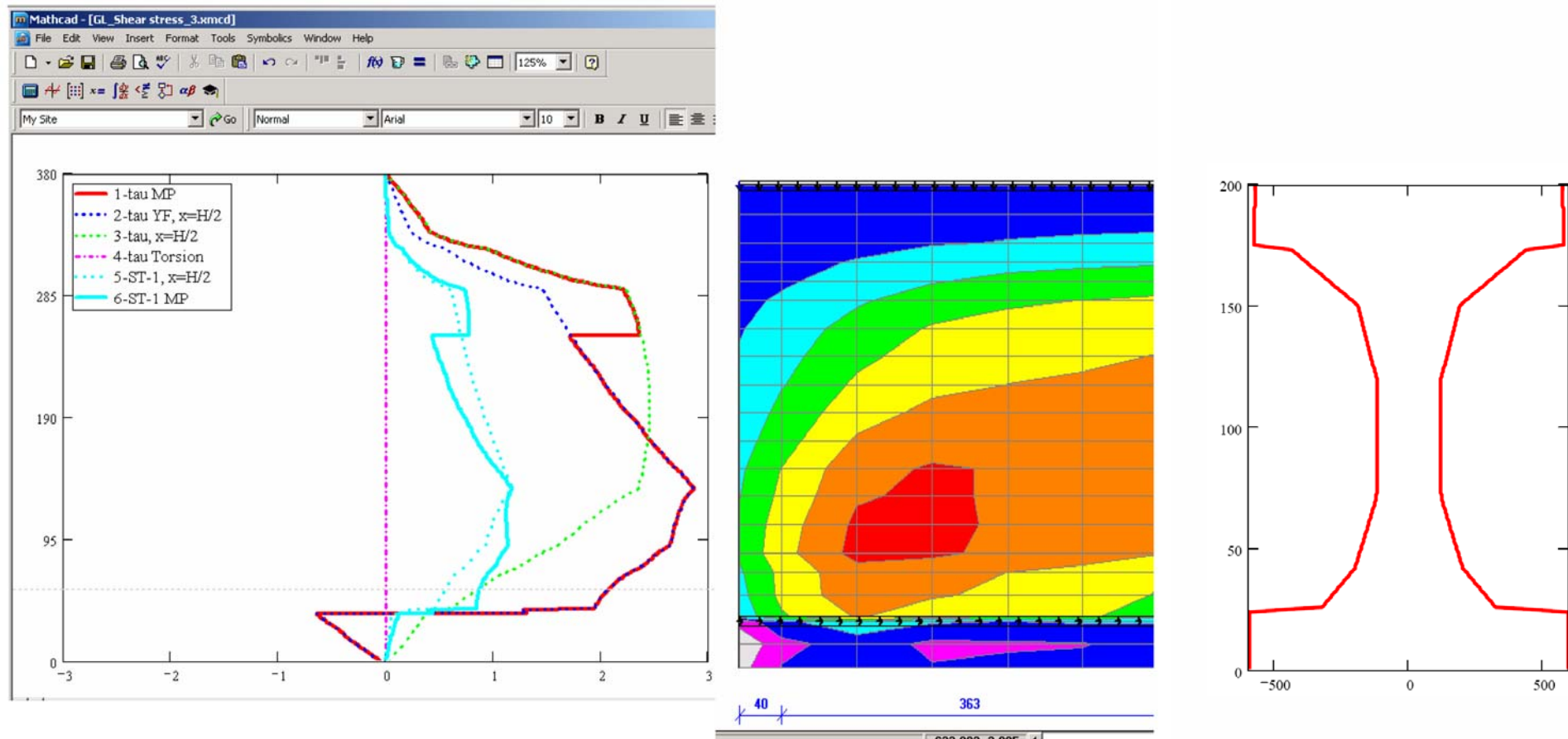


Figure 2.5:3 Comparison between the suggested model and EF-analysis, for the case HC 380 mm and 1100 mm². Initial prestress 1100 MPa and during analysis the prestress 850 MPa has been used for both cases. Max shear stress in FE-model is 2.89 MPa, which is almost identical with the suggested model.

Principal stress σ_I due to normal stress, torsion and or shear

- σ_c = Normal stress in the HC section (positive in tension)
- τ_{T_top} = Shear stress in top flange due to torsion
- τ_{T_web} = Shear stress in outer web due to torsion
- τ_{T_bot} = Shear stress in bottom flange due to torsion
- τ_{V_web} = Shear stress in outer web due to shear

Top flange of box

$$\sigma_{I_top} := \frac{\sigma_c}{2} + \sqrt{\left(\frac{\sigma_c}{2}\right)^2 + (\tau_{T_top})^2} \quad \dots(1)$$

Outer web of box

$$\sigma_{I_web} := \frac{\sigma_c}{2} + \sqrt{\left(\frac{\sigma_c}{2}\right)^2 + (\tau_{T_web} + \tau_{V_web})^2} \quad \dots(2)$$

Bottom flange of box

$$\sigma_{I_bot} := \frac{\sigma_c}{2} + \sqrt{\left(\frac{\sigma_c}{2}\right)^2 + (\tau_{T_bot})^2} \quad \dots(3)$$

For pure torsion normally the top flange is critical

with a failure condition $\sigma_{I_top} = f_{ct}$ inserted in eq 1 gives

Top flange shear stress at cracking

$$\tau_{T_top} := f_{ct} \cdot \sqrt{1 - \frac{\sigma_c}{f_{ct}}}$$

Torsion capacity at cracking on top flange

$$T := W_{top} \cdot f_{ct} \cdot \sqrt{1 - \frac{\sigma_c}{f_{ct}}}$$

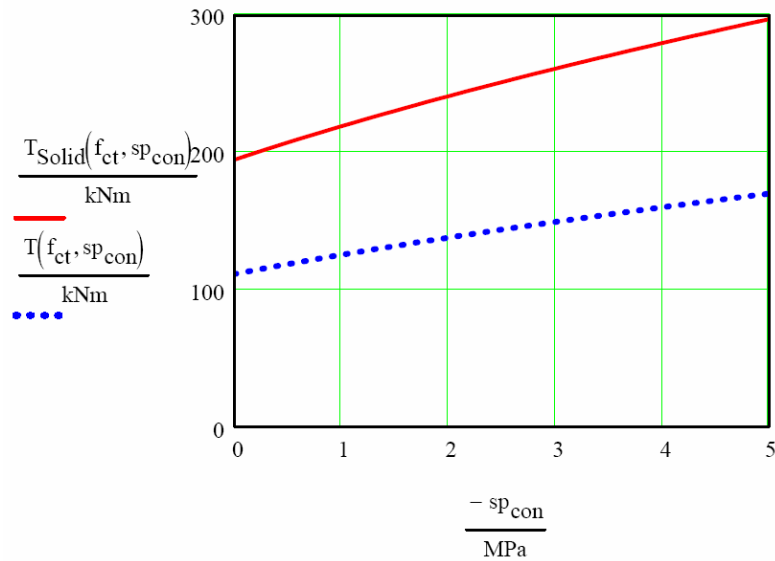
Which can be expressed

where $B_m = b - t_{web}$

$$H_m = H - (t_{fl_top} + t_{fl_bot})/2$$

$$T := 2 \cdot B_m \cdot H_m \cdot t_{fl_top} \cdot f_{ct} \cdot \sqrt{1 - \frac{\sigma_c}{f_{ct}}}$$

Shear torsion interaction

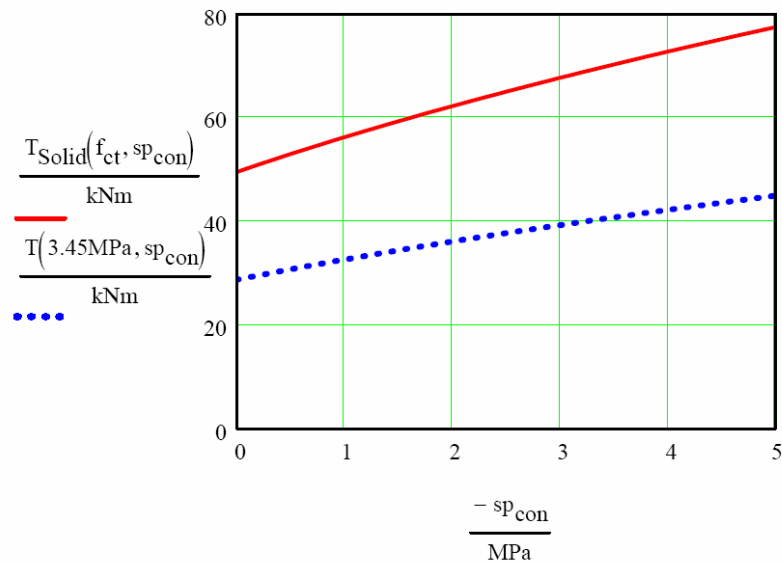


Using the above eq. for T results in
Pure torsional capacity:

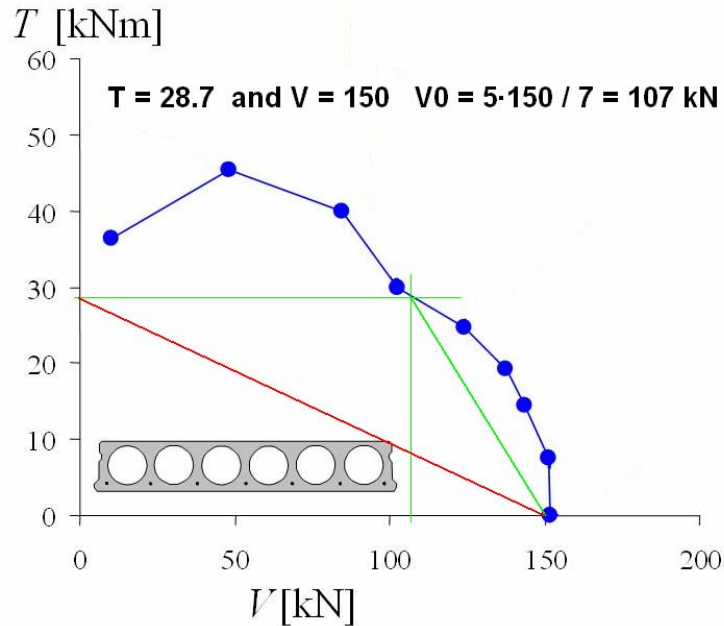
T = 111 kNm for HC 400 using $f_{ct} = 3.75 \text{ MPa}$

T = 28.7 kNm for HC 200 using $f_{ct} = 3.45 \text{ MPa}$

By including the compression stress in the top flange
the torsional capacity is increased.



Shear torsion interaction

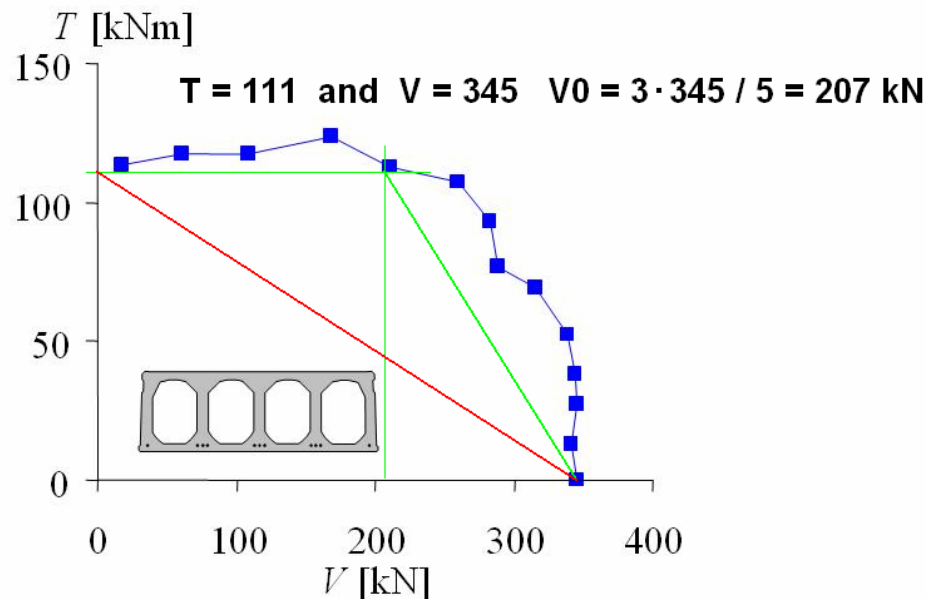


Provided pure Shear and Torsional capacity is calculated in a proper way. The interaction formula as indicated with green lines could be utilized.

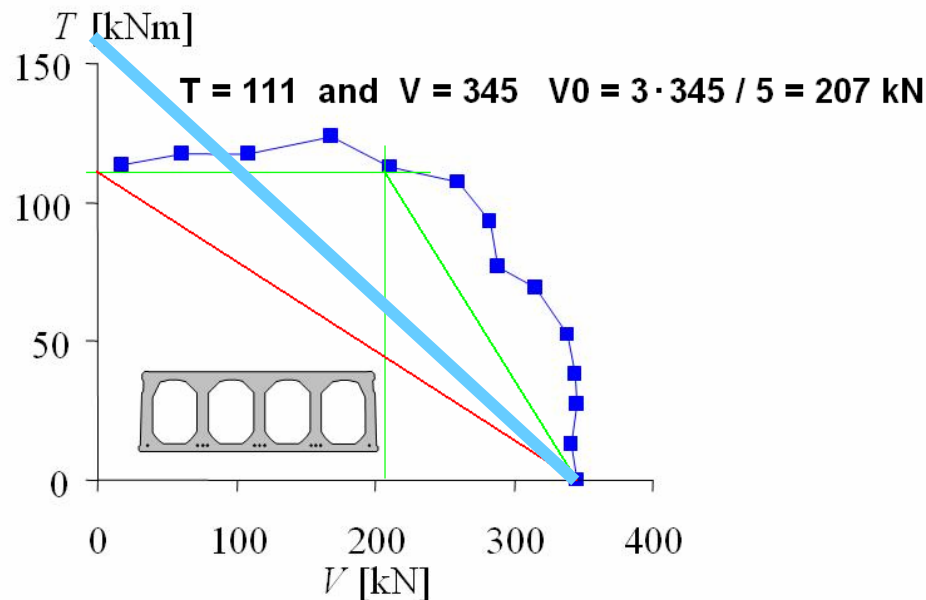
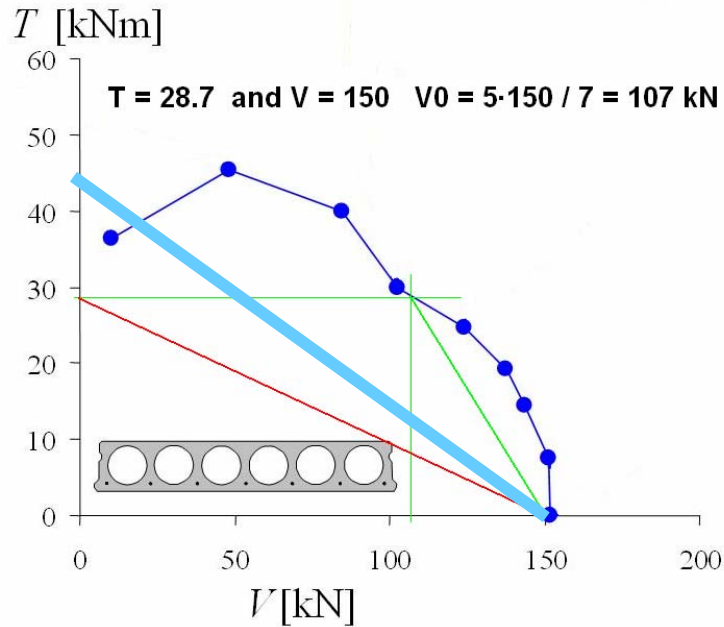
This can be used independent of Eurocode or the product standard, as it is well supported by the documented results of HOLCOTORS

Refinement can be done by:

- If top flange is critical for torsional capacity the margin not used in outer web could be included in V_0
- Calculation of V_0 should be based on the sum of web thickness in inner and outer webs
- Including compression stress in top flange which increases with increased shear (has to be used case by case for proper stress analysis and X-section change in the end due to grouting)



Shear torsion interaction



The expression in EN 1168 today

$$V_{Rdn} = V_{Rd,c} - V_{ETd}$$

$$\text{with } V_{ETd} = \frac{T_{Ed}}{2b_w} \times \frac{\Sigma b_w}{b-b_w}$$

Gives an indication on how the shearing force is reduced but it makes no check of torsional capacity. For the present sections

$$T_{\max H200} = 0.298 \times V = 44.7 \text{ kNm}$$

$$T_{\max H400} = 0.458 \times V = 158 \text{ kNm}$$

$$V \text{ (Lin Yang)} = 339.2 \text{ kN for H400 (Lp} = 0.48\text{m)}$$



Thanks for the attention

Shear torsion interaction



IPHE Seminar 2005